Author: Arthur Dryver, Ph.D.

This is a **90 minute exam, there will be no additional time**. **Open book open note, even internet, any software allowed, personal flash drive, keyboard, mouse allowed. Staff should install Excel Add-in Analysis ToolPak if students cannot.** The exam is to be taken individually, no help from others during the exam, you must select the exam files for the questions and entering answers matching the last 2 digits of your student ID. You must enter answers into the Excel Answer file in the appropriate locations. You must enter the answers in the appropriate Answer Excel File called "answer-file-...ID-XX.xlsx" in the specific cells provided and return the file at the end of the exam. **Save often to be safe. Also, graphs are required on the Excel sheets labeled graphs for the question numbers shown in the sheet names.**

Four decimal place accuracy for answers. Questions on proportions, the calculations must be done same as done for the online homework. There is no continuity correction factor this is slightly different than some software.

Questions 1-180 will be worth 4 points each. The 6 graphs you need to do will be worth 18 points each for a total of 108 points. Question 181, selecting the best test, what to do when, each part (a-f) will be worth 18 points each for a total of 108 points. A percentage is calculated using points from correct answers points divided by total points possible. Points will be deducted for not following directions. **Do your best and good luck.**

Contents

1 Calculations

1.1 Descriptive Statistics

Problem 1. Solve for the sample min, max, mean, median, variance, and standard deviation of the following stock prices of the following companies:

	y
1	15.30
2	21.40
3	18.60
4	21.80
5	15.20

Table 1: The stock prices.

Solution: The solution:

15.20
21.80
18.46
18.60
10.11
3.18

Table 2: The solution

□

Problem 2. Solve for the sample min, max, mean, median, variance, and standard deviation of the following stock prices of the following companies:

	y
1	18.30
2	20.10
3	22.60
4	16.10
5	34.90
6	13.60
7	25.20
8	18.80
9	18.80
10	22.40
11	16.70
12	10.30

Table 3: The stock prices.

Solution: The solution:

10.30
34.90
19.82
18.80
38.92
6.24

Table 4: The solution

□

Problem 3. Solve for the sample min, max, mean, median, variance, and standard deviation of the following stock prices of the following companies:

	y
1	10.30
2	17.10
3	15.90
4	10.10
5	24.40
6	16.50

Table 5: The stock prices.

Solution: The solution:

10.10
24.40
15.72
16.20
27.77
5.27

Table 6: The solution

□

Problem 4. Solve for the sample min, max, mean, median, variance, and standard deviation of the following stock prices of the following companies:

	y
1	16.20
2	21.80
3	20.00
4	19.60
5	10.20

Table 7: The stock prices.

Solution: The solution:

10.20
21.80
17.56
19.60
21.03
4.59

Table 8: The solution

$\square$

Problem 5. Solve for the sample min, max, mean, median, variance, and standard deviation of the following stock prices of the following companies:

	y
1	24.00
2	26.20
3	34.40
4	19.60
5	25.50
6	20.20
7	20.40

Table 9: The stock prices.

Solution: The solution:

19.60
34.40
24.33
24.00
26.88
5.18

Table 10: The solution

□

Problem 6. Solve for the sample min, max, mean, median, variance, and standard deviation of the following stock prices of the following companies:

	y
1	21.10
2	13.60
3	15.90
4	25.50
5	13.30
6	26.90
7	24.00
8	22.90
9	10.70

Table 11: The stock prices.

Solution: The solution:

10.70
26.90
19.32
21.10
36.06
6.01

Table 12: The solution

□

Problem 7. Solve for the sample min, max, mean, median, variance, and standard deviation of the following stock prices of the following companies:

	y
1	33.10
2	22.40
3	17.20
4	22.20
5	16.80
6	13.60
7	25.80
8	15.50

Table 13: The stock prices.

Solution: The solution:

13.60
33.10
20.82
19.70
41.39
6.43

Table 14: The solution

□

Problem 8. Solve for the sample min, max, mean, median, variance, and standard deviation of the following stock prices of the following companies:

	y
1	25.80
2	19.60
3	18.30
4	19.40
5	24.10
6	18.70
7	30.00

Table 15: The stock prices.

Solution: The solution:

18.30
30.00
22.27
19.60
19.91
4.46

Table 16: The solution

□

Problem 9. Solve for the sample min, max, mean, median, variance, and standard deviation of the following stock prices of the following companies:

	y
1	16.60
2	13.80
3	32.30
4	10.10
5	25.40
6	27.10
7	10.90
8	14.00
9	20.10
10	12.40
11	29.00

Table 17: The stock prices.

Solution: The solution:

10.10
32.30
19.25
16.60
63.02
7.94

Table 18: The solution

□

Problem 10. Solve for the sample min, max, mean, median, variance, and standard deviation of the following stock prices of the following companies:

	y
1	10.20
2	14.20
3	25.90
4	32.00
5	10.60

Table 19: The stock prices.

Solution: The solution:

10.20
32.00
18.58
14.20
96.69
9.83

Table 20: The solution

$\square$

Problem 11. Solve for the sample min, max, mean, median, variance, and standard deviation of the following stock prices of the following companies:

	y
1	24.40
2	13.20
3	17.40
4	17.00

Table 21: The stock prices.

Solution: The solution:

13.20
24.40
18.00
17.20
21.79
4.67

Table 22: The solution

□

Problem 12. Solve for the sample min, max, mean, median, variance, and standard deviation of the following stock prices of the following companies:

	y
1	16.50
2	15.00
3	27.40
4	14.30
5	19.80
6	17.90
7	17.80

Table 23: The stock prices.

Solution: The solution:

14.30
27.40
18.39
17.80
19.26
4.39

Table 24: The solution

□

Problem 13. Solve for the sample min, max, mean, median, variance, and standard deviation of the following stock prices of the following companies:

	y
1	22.60
2	19.10
3	16.00
4	20.90
5	10.30
6	28.70
7	13.00

Table 25: The stock prices.

Solution: The solution:

10.30
28.70
18.66
19.10
38.42
6.20

Table 26: The solution

□

Problem 14. Solve for the sample min, max, mean, median, variance, and standard deviation of the following stock prices of the following companies:

	y
1	19.60
2	15.20
3	17.60
4	22.70
5	15.70
6	27.70
7	30.10
8	16.20

Table 27: The stock prices.

Solution: The solution:

15.20
30.10
20.60
18.60
32.51
5.70

Table 28: The solution

□

Problem 15. Solve for the sample min, max, mean, median, variance, and standard deviation of the following stock prices of the following companies:

	y
1	18.20
2	13.90
3	19.40
4	16.00

Table 29: The stock prices.

Solution: The solution:

13.90
19.40
16.88
17.10
5.92
2.43

Table 30: The solution

□

Problem 16. Solve for the sample min, max, mean, median, variance, and standard deviation of the following stock prices of the following companies:

	y
1	10.70
2	12.30
3	27.80
4	21.10
5	23.50
6	16.00
7	26.50

Table 31: The stock prices.

Solution: The solution:

10.70
27.80
19.70
21.10
46.28
6.80

Table 32: The solution

□

Problem 17. Solve for the sample min, max, mean, median, variance, and standard deviation of the following stock prices of the following companies:

	y
1	11.90
2	12.10
3	14.90

Table 33: The stock prices.

Solution: The solution:

11.90
14.90
12.97
12.10
2.81
1.68

Table 34: The solution

□

Problem 18. Solve for the sample min, max, mean, median, variance, and standard deviation of the following stock prices of the following companies:

	y
1	10.90
2	14.30
3	14.00
4	22.90
5	10.60
6	21.20

Table 35: The stock prices.

Solution: The solution:

10.60
22.90
15.65
14.15
27.19
5.21

Table 36: The solution

□

1.2 Binomial Distribution

Problem 19. Assume the random variable(s) is from a binomial distribution with $n = 2$ and $\pi = 0.5$, and X is the number of successes. Answer the following:

(a) What is the probability $X \leq 0$:

Solution: $P(X \leq 0) = \sum_{x=0}^{x=0} \binom{2}{x} 0.5^x (1-0.5)^{2-x} = 0.25$ □

(b) What is the probability $X > 0$:

Solution: $P(X > 0) = \sum_{x=1}^{x=2} \binom{2}{x} 0.5^x (1-0.5)^{2-x} = 0.75$ □

(c) What is the probability $X \geq 0$:

Solution: $P(X \geq 0) = \sum_{x=0}^{x=2} \binom{2}{x} 0.5^x (1-0.5)^{2-x} = 1$ □

(d) What is the probability $0 < X \leq 2$:

Solution:

$$P(0 < X \leq 2) = \sum_{x=0}^{x=2} \binom{2}{x} 0.5^x (1-0.5)^{2-x} - \sum_{x=0}^{x=0} \binom{2}{x} 0.5^x (1-0.5)^{2-x} = 0.75$$

□

Problem 20. Assume the random variable(s) is from a binomial distribution with $n = 2$ and $\pi = 0.8$, and X is the number of successes. Answer the following:

(a) What is the probability $X \leq 1$:

Solution: $P(X \leq 1) = \sum_{x=0}^{x=1} \binom{2}{x} 0.8^x (1-0.8)^{2-x} = 0.36$

$\square$

(b) What is the probability $X > 1$:

Solution: $P(X > 1) = \sum_{x=2}^{x=2} \binom{2}{x} 0.8^x (1-0.8)^{2-x} = 0.64$

$\square$

(c) What is the probability $X \geq 1$:

Solution: $P(X \geq 1) = \sum_{x=1}^{x=2} \binom{2}{x} 0.8^x (1-0.8)^{2-x} = 0.96$

$\square$

(d) What is the probability $1 < X \leq 2$:

Solution:

$$P(1 < X \leq 2) = \sum_{x=0}^{x=2} \binom{2}{x} 0.8^x (1-0.8)^{2-x} - \sum_{x=0}^{x=1} \binom{2}{x} 0.8^x (1-0.8)^{2-x} = 0.64$$

$\square$

Problem 21. Assume the random variable(s) is from a binomial distribution with $n = 8$ and $\pi = 0.8$, and X is the number of successes. Answer the following:

(a) What is the probability $X \leq 5$:

Solution: $P(X \leq 5) = \sum_{x=0}^{x=5} \binom{8}{x} 0.8^x (1-0.8)^{8-x} = 0.2031$

□

(b) What is the probability $X > 5$:

Solution: $P(X > 5) = \sum_{x=6}^{x=8} \binom{8}{x} 0.8^x (1-0.8)^{8-x} = 0.7969$

□

(c) What is the probability $X \geq 5$:

Solution: $P(X \geq 5) = \sum_{x=5}^{x=8} \binom{8}{x} 0.8^x (1-0.8)^{8-x} = 0.9437$

□

(d) What is the probability $5 < X \leq 6$:

Solution:

$$P(5 < X \leq 6) = \sum_{x=0}^{x=6} \binom{8}{x} 0.8^x (1-0.8)^{8-x} - \sum_{x=0}^{x=5} \binom{8}{x} 0.8^x (1-0.8)^{8-x} = 0.2936$$

□

Problem 22. Assume the random variable(s) is from a binomial distribution with $n = 12$ and $\pi = 0.1$, and X is the number of successes. Answer the following:

(a) What is the probability $X \leq 1$:

Solution: $P(X \leq 1) = \sum_{x=0}^{x=1} \binom{12}{x} 0.1^x (1-0.1)^{12-x} = 0.659$ $\square$

(b) What is the probability $X > 1$:

Solution: $P(X > 1) = \sum_{x=2}^{x=12} \binom{12}{x} 0.1^x (1-0.1)^{12-x} = 0.341$ $\square$

(c) What is the probability $X \geq 1$:

Solution: $P(X \geq 1) = \sum_{x=1}^{x=12} \binom{12}{x} 0.1^x (1-0.1)^{12-x} = 0.7176$ $\square$

(d) What is the probability $1 < X \leq 2$:

Solution:

$$P(1 < X \leq 2) = \sum_{x=0}^{x=2} \binom{12}{x} 0.1^x (1-0.1)^{12-x} - \sum_{x=0}^{x=1} \binom{12}{x} 0.1^x (1-0.1)^{12-x} = 0.2301$$

$\square$

Problem 23. Assume the random variable(s) is from a binomial distribution with $n = 7$ and $\pi = 0.5$, and X is the number of successes. Answer the following:

(a) What is the probability $X \leq 4$:

Solution: $P(X \leq 4) = \sum_{x=0}^{x=4} \binom{7}{x} 0.5^x (1-0.5)^{7-x} = 0.7734$ □

(b) What is the probability $X > 4$:

Solution: $P(X > 4) = \sum_{x=5}^{x=7} \binom{7}{x} 0.5^x (1-0.5)^{7-x} = 0.2266$ □

(c) What is the probability $X \geq 4$:

Solution: $P(X \geq 4) = \sum_{x=4}^{x=7} \binom{7}{x} 0.5^x (1-0.5)^{7-x} = 0.5$ □

(d) What is the probability $4 < X \leq 5$:

Solution:

$$P(4 < X \leq 5) = \sum_{x=0}^{x=5} \binom{7}{x} 0.5^x (1-0.5)^{7-x} - \sum_{x=0}^{x=4} \binom{7}{x} 0.5^x (1-0.5)^{7-x} = 0.1641$$

□

Problem 24. Assume the random variable(s) is from a binomial distribution with $n = 5$ and $\pi = 0.6$, and X is the number of successes. Answer the following:

(a) What is the probability $X \leq 3$:

Solution: $P(X \leq 3) = \sum_{x=0}^{x=3} \binom{5}{x} 0.6^x (1-0.6)^{5-x} = 0.663$

□

(b) What is the probability $X > 3$:

Solution: $P(X > 3) = \sum_{x=4}^{x=5} \binom{5}{x} 0.6^x (1-0.6)^{5-x} = 0.337$

□

(c) What is the probability $X \geq 3$:

Solution: $P(X \geq 3) = \sum_{x=3}^{x=5} \binom{5}{x} 0.6^x (1-0.6)^{5-x} = 0.6826$

□

(d) What is the probability $3 < X \leq 4$:

Solution:

$$P(3 < X \leq 4) = \sum_{x=0}^{x=4} \binom{5}{x} 0.6^x (1-0.6)^{5-x} - \sum_{x=0}^{x=3} \binom{5}{x} 0.6^x (1-0.6)^{5-x} = 0.2592$$

□

Problem 25. Assume the random variable(s) is from a binomial distribution with $n = 10$ and $\pi = 0.3$, and X is the number of successes. Answer the following:

(a) What is the probability $X \leq 2$:

Solution: $P(X \leq 2) = \sum_{x=0}^{x=2} \binom{10}{x} 0.3^x (1-0.3)^{10-x} = 0.3828$

□

(b) What is the probability $X > 2$:

Solution: $P(X > 2) = \sum_{x=3}^{x=10} \binom{10}{x} 0.3^x (1-0.3)^{10-x} = 0.6172$

□

(c) What is the probability $X \geq 2$:

Solution: $P(X \geq 2) = \sum_{x=2}^{x=10} \binom{10}{x} 0.3^x (1-0.3)^{10-x} = 0.8507$

□

(d) What is the probability $2 < X \leq 3$:

Solution:

$$P(2 < X \leq 3) = \sum_{x=0}^{x=3} \binom{10}{x} 0.3^x (1-0.3)^{10-x} - \sum_{x=0}^{x=2} \binom{10}{x} 0.3^x (1-0.3)^{10-x} = 0.2668$$

□

Problem 26. Assume the random variable(s) is from a binomial distribution with $n = 5$ and $\pi = 0.8$, and X is the number of successes. Answer the following:

(a) What is the probability $X \leq 4$:

Solution: $P(X \leq 4) = \sum_{x=0}^{x=4} \binom{5}{x} 0.8^x (1-0.8)^{5-x} = 0.6723$ $\square$

(b) What is the probability $X > 4$:

Solution: $P(X > 4) = \sum_{x=5}^{x=5} \binom{5}{x} 0.8^x (1-0.8)^{5-x} = 0.3277$ $\square$

(c) What is the probability $X \geq 4$:

Solution: $P(X \geq 4) = \sum_{x=4}^{x=5} \binom{5}{x} 0.8^x (1-0.8)^{5-x} = 0.7373$ $\square$

(d) What is the probability $4 < X \leq 5$:

Solution:

$$P(4 < X \leq 5) = \sum_{x=0}^{x=5} \binom{5}{x} 0.8^x (1-0.8)^{5-x} - \sum_{x=0}^{x=4} \binom{5}{x} 0.8^x (1-0.8)^{5-x} = 0.3277$$

$\square$

Problem 27. Assume the random variable(s) is from a binomial distribution with $n = 3$ and $\pi = 0.6$, and X is the number of successes. Answer the following:

(a) What is the probability $X \leq 1$:

Solution: $P(X \leq 1) = \sum_{x=0}^{x=1} \binom{3}{x} 0.6^x (1-0.6)^{3-x} = 0.352$

□

(b) What is the probability $X > 1$:

Solution: $P(X > 1) = \sum_{x=2}^{x=3} \binom{3}{x} 0.6^x (1-0.6)^{3-x} = 0.648$

□

(c) What is the probability $X \geq 1$:

Solution: $P(X \geq 1) = \sum_{x=1}^{x=3} \binom{3}{x} 0.6^x (1-0.6)^{3-x} = 0.936$

□

(d) What is the probability $1 < X \leq 2$:

Solution:

$$P(1 < X \leq 2) = \sum_{x=0}^{x=2} \binom{3}{x} 0.6^x (1-0.6)^{3-x} - \sum_{x=0}^{x=1} \binom{3}{x} 0.6^x (1-0.6)^{3-x} = 0.432$$

□

Problem 28. Assume the random variable(s) is from a binomial distribution with $n = 11$ and $\pi = 0.5$, and X is the number of successes. Answer the following:

(a) What is the probability $X \leq 2$:

Solution: $P(X \leq 2) = \sum_{x=0}^{x=2} \binom{11}{x} 0.5^x (1-0.5)^{11-x} = 0.0327$

□

(b) What is the probability $X > 2$:

Solution: $P(X > 2) = \sum_{x=3}^{x=11} \binom{11}{x} 0.5^x (1-0.5)^{11-x} = 0.9673$

□

(c) What is the probability $X \geq 2$:

Solution: $P(X \geq 2) = \sum_{x=2}^{x=11} \binom{11}{x} 0.5^x (1-0.5)^{11-x} = 0.9941$

□

(d) What is the probability $2 < X \leq 6$:

Solution:

$$P(2 < X \leq 6) = \sum_{x=0}^{x=6} \binom{11}{x} 0.5^x (1-0.5)^{11-x} - \sum_{x=0}^{x=2} \binom{11}{x} 0.5^x (1-0.5)^{11-x} = 0.6929$$

□

Problem 29. Assume the random variable(s) is from a binomial distribution with $n = 11$ and $\pi = 0.3$, and X is the number of successes. Answer the following:

(a) What is the probability $X \leq 2$:

Solution: $P(X \leq 2) = \sum_{x=0}^{x=2} \binom{11}{x} 0.3^x (1 - 0.3)^{11-x} = 0.3127$

□

(b) What is the probability $X > 2$:

Solution: $P(X > 2) = \sum_{x=3}^{x=11} \binom{11}{x} 0.3^x (1 - 0.3)^{11-x} = 0.6873$

□

(c) What is the probability $X \geq 2$:

Solution: $P(X \geq 2) = \sum_{x=2}^{x=11} \binom{11}{x} 0.3^x (1 - 0.3)^{11-x} = 0.887$

□

(d) What is the probability $2 < X \leq 3$:

Solution:

$$P(2 < X \leq 3) = \sum_{x=0}^{x=3} \binom{11}{x} 0.3^x (1 - 0.3)^{11-x} - \sum_{x=0}^{x=2} \binom{11}{x} 0.3^x (1 - 0.3)^{11-x} = 0.2568$$

□

Problem 30. Assume the random variable(s) is from a binomial distribution with $n = 3$ and $\pi = 0.7$, and X is the number of successes. Answer the following:

(a) What is the probability $X \leq 1$:

Solution: $P(X \leq 1) = \sum_{x=0}^{x=1} \binom{3}{x} 0.7^x (1-0.7)^{3-x} = 0.216$

$\square$

(b) What is the probability $X > 1$:

Solution: $P(X > 1) = \sum_{x=2}^{x=3} \binom{3}{x} 0.7^x (1-0.7)^{3-x} = 0.784$

$\square$

(c) What is the probability $X \geq 1$:

Solution: $P(X \geq 1) = \sum_{x=1}^{x=3} \binom{3}{x} 0.7^x (1-0.7)^{3-x} = 0.973$

$\square$

(d) What is the probability $1 < X \leq 3$:

Solution:

$$P(1 < X \leq 3) = \sum_{x=0}^{x=3} \binom{3}{x} 0.7^x (1-0.7)^{3-x} - \sum_{x=0}^{x=1} \binom{3}{x} 0.7^x (1-0.7)^{3-x} = 0.784$$

$\square$

Problem 31. Assume the random variable(s) is from a binomial distribution with $n = 4$ and $\pi = 0.9$, and X is the number of successes. Answer the following:

(a) What is the probability $X \leq 3$:

Solution: $P(X \leq 3) = \sum_{x=0}^{x=3} \binom{4}{x} 0.9^x (1-0.9)^{4-x} = 0.3439$ □

(b) What is the probability $X > 3$:

Solution: $P(X > 3) = \sum_{x=4}^{x=4} \binom{4}{x} 0.9^x (1-0.9)^{4-x} = 0.6561$ □

(c) What is the probability $X \geq 3$:

Solution: $P(X \geq 3) = \sum_{x=3}^{x=4} \binom{4}{x} 0.9^x (1-0.9)^{4-x} = 0.9477$ □

(d) What is the probability $3 < X \leq 4$:

Solution:

$$P(3 < X \leq 4) = \sum_{x=0}^{x=4} \binom{4}{x} 0.9^x (1-0.9)^{4-x} - \sum_{x=0}^{x=3} \binom{4}{x} 0.9^x (1-0.9)^{4-x} = 0.6561$$

□

Problem 32. Assume the random variable(s) is from a binomial distribution with $n = 2$ and $\pi = 0.4$, and X is the number of successes. Answer the following:

(a) What is the probability $X \leq 0$:

Solution: $P(X \leq 0) = \sum_{x=0}^{x=0} \binom{2}{x} 0.4^x (1-0.4)^{2-x} = 0.36$ $\square$

(b) What is the probability $X > 0$:

Solution: $P(X > 0) = \sum_{x=1}^{x=2} \binom{2}{x} 0.4^x (1-0.4)^{2-x} = 0.64$ $\square$

(c) What is the probability $X \geq 0$:

Solution: $P(X \geq 0) = \sum_{x=0}^{x=2} \binom{2}{x} 0.4^x (1-0.4)^{2-x} = 1$ $\square$

(d) What is the probability $0 < X \leq 1$:

Solution:

$$P(0 < X \leq 1) = \sum_{x=0}^{x=1} \binom{2}{x} 0.4^x (1-0.4)^{2-x} - \sum_{x=0}^{x=0} \binom{2}{x} 0.4^x (1-0.4)^{2-x} = 0.48$$

$\square$

Problem 33. Assume the random variable(s) is from a binomial distribution with $n = 3$ and $\pi = 0.7$, and X is the number of successes. Answer the following:

(a) What is the probability $X \leq 1$:

Solution: $P(X \leq 1) = \sum_{x=0}^{x=1} \binom{3}{x} 0.7^x (1-0.7)^{3-x} = 0.216$ $\square$

(b) What is the probability $X > 1$:

Solution: $P(X > 1) = \sum_{x=2}^{x=3} \binom{3}{x} 0.7^x (1-0.7)^{3-x} = 0.784$ $\square$

(c) What is the probability $X \geq 1$:

Solution: $P(X \geq 1) = \sum_{x=1}^{x=3} \binom{3}{x} 0.7^x (1-0.7)^{3-x} = 0.973$ $\square$

(d) What is the probability $1 < X \leq 3$:

Solution:

$$P(1 < X \leq 3) = \sum_{x=0}^{x=3} \binom{3}{x} 0.7^x (1-0.7)^{3-x} - \sum_{x=0}^{x=1} \binom{3}{x} 0.7^x (1-0.7)^{3-x} = 0.784$$

$\square$

Problem 34. Assume the random variable(s) is from a binomial distribution with $n = 4$ and $\pi = 0.5$, and X is the number of successes. Answer the following:

(a) What is the probability $X \leq 2$:

Solution: $P(X \leq 2) = \sum_{x=0}^{x=2} \binom{4}{x} 0.5^x (1-0.5)^{4-x} = 0.6875$

$\square$

(b) What is the probability $X > 2$:

Solution: $P(X > 2) = \sum_{x=3}^{x=4} \binom{4}{x} 0.5^x (1-0.5)^{4-x} = 0.3125$

$\square$

(c) What is the probability $X \geq 2$:

Solution: $P(X \geq 2) = \sum_{x=2}^{x=4} \binom{4}{x} 0.5^x (1-0.5)^{4-x} = 0.6875$

$\square$

(d) What is the probability $2 < X \leq 3$:

Solution:

$$P(2 < X \leq 3) = \sum_{x=0}^{x=3} \binom{4}{x} 0.5^x (1-0.5)^{4-x} - \sum_{x=0}^{x=2} \binom{4}{x} 0.5^x (1-0.5)^{4-x} = 0.25$$

$\square$

Problem 35. Assume the random variable(s) is from a binomial distribution with $n = 9$ and $\pi = 0.6$, and X is the number of successes. Answer the following:

(a) What is the probability $X \leq 7$:

Solution: $P(X \leq 7) = \sum_{x=0}^{x=7} \binom{9}{x} 0.6^x (1 - 0.6)^{9-x} = 0.9295$

$\square$

(b) What is the probability $X > 7$:

Solution: $P(X > 7) = \sum_{x=8}^{x=9} \binom{9}{x} 0.6^x (1 - 0.6)^{9-x} = 0.0705$

$\square$

(c) What is the probability $X \geq 7$:

Solution: $P(X \geq 7) = \sum_{x=7}^{x=9} \binom{9}{x} 0.6^x (1 - 0.6)^{9-x} = 0.2318$

$\square$

(d) What is the probability $7 < X \leq 8$:

Solution:

$$P(7 < X \leq 8) = \sum_{x=0}^{x=8} \binom{9}{x} 0.6^x (1 - 0.6)^{9-x} - \sum_{x=0}^{x=7} \binom{9}{x} 0.6^x (1 - 0.6)^{9-x} = 0.0605$$

$\square$

Problem 36. Assume the random variable(s) is from a binomial distribution with $n = 7$ and $\pi = 0.7$, and X is the number of successes. Answer the following:

(a) What is the probability $X \leq 6$:

Solution: $P(X \leq 6) = \sum_{x=0}^{x=6} \binom{7}{x} 0.7^x (1-0.7)^{7-x} = 0.9176$ $\square$

(b) What is the probability $X > 6$:

Solution: $P(X > 6) = \sum_{x=7}^{x=7} \binom{7}{x} 0.7^x (1-0.7)^{7-x} = 0.0824$ $\square$

(c) What is the probability $X \geq 6$:

Solution: $P(X \geq 6) = \sum_{x=6}^{x=7} \binom{7}{x} 0.7^x (1-0.7)^{7-x} = 0.3294$ $\square$

(d) What is the probability $6 < X \leq 7$:

Solution:

$$P(6 < X \leq 7) = \sum_{x=0}^{x=7} \binom{7}{x} 0.7^x (1-0.7)^{7-x} - \sum_{x=0}^{x=6} \binom{7}{x} 0.7^x (1-0.7)^{7-x} = 0.0824$$

$\square$

1.3 Hypergeometric Distribution

Problem 37. Assume the random variable(s) is from a hypergeometric distribution with $A = 6$ number of success and population size $N = 12$, and X is the number of successes you select. You select from the population $n = 4$ items without replacement. Answer the following:

(a) What is the probability $X \leq 2$:

Solution: $P(X \leq 2) = \sum_{x=0}^{x=2} \frac{\binom{6}{x}\binom{6}{4-x}}{\binom{12}{4}} = 0.7273$ □

(b) What is the probability $X > 2$:

Solution: $P(X > 2) = \sum_{x=3}^{x=6} \frac{\binom{6}{x}\binom{6}{4-x}}{\binom{12}{4}} = 0.2727$ □

(c) What is the probability $X \geq 2$:

Solution: $P(X \geq 2) = \sum_{x=2}^{x=6} \frac{\binom{6}{x}\binom{6}{4-x}}{\binom{12}{4}} = 0.7273$ □

(d) What is the probability $2 < X \leq 3$:

Solution: $P(2 < X \leq 3) = \sum_{x=0}^{x=3} \frac{\binom{6}{x}\binom{6}{4-x}}{\binom{12}{4}} - \sum_{x=0}^{x=2} \frac{\binom{6}{x}\binom{6}{4-x}}{\binom{12}{4}} = 0.2424$ □

Problem 38. Assume the random variable(s) is from a hypergeometric distribution with $A = 6$ number of success and population size $N = 13$, and X is the number of successes you select. You select from the population $n = 13$ items without replacement. Answer the following:

(a) What is the probability $X \leq 6$:

Solution: $P(X \leq 6) = \sum_{x=0}^{x=6} \frac{\binom{6}{x}\binom{7}{13-x}}{\binom{13}{13}} = 1$ □

(b) What is the probability $X > 6$:

Solution: $P(X > 6) = \sum_{x=7}^{x=6} \frac{\binom{6}{x}\binom{7}{13-x}}{\binom{13}{13}} = 0$ □

(c) What is the probability $X \geq 6$:

Solution: $P(X \geq 6) = \sum_{x=6}^{x=6} \frac{\binom{6}{x}\binom{7}{13-x}}{\binom{13}{13}} = 1$ □

(d) What is the probability $6 < X \leq 7$:

Solution: $P(6 < X \leq 7) = \sum_{x=0}^{x=7} \frac{\binom{6}{x}\binom{7}{13-x}}{\binom{13}{13}} - \sum_{x=0}^{x=6} \frac{\binom{6}{x}\binom{7}{13-x}}{\binom{13}{13}} = 0$ □

Problem 39. Assume the random variable(s) is from a hypergeometric distribution with $A = 6$ number of success and population size $N = 16$, and X is the number of successes you select. You select from the population $n = 9$ items without replacement. Answer the following:

(a) What is the probability $X \leq 4$:

Solution: $P(X \leq 4) = \sum_{x=0}^{x=4} \frac{\binom{6}{x}\binom{10}{9-x}}{\binom{16}{9}} = 0.8794$ □

(b) What is the probability $X > 4$:

Solution: $P(X > 4) = \sum_{x=5}^{x=6} \frac{\binom{6}{x}\binom{10}{9-x}}{\binom{16}{9}} = 0.1206$ □

(c) What is the probability $X \geq 4$:

Solution: $P(X \geq 4) = \sum_{x=4}^{x=6} \frac{\binom{6}{x}\binom{10}{9-x}}{\binom{16}{9}} = 0.451$ □

(d) What is the probability $4 < X \leq 5$:

Solution: $P(4 < X \leq 5) = \sum_{x=0}^{x=5} \frac{\binom{6}{x}\binom{10}{9-x}}{\binom{16}{9}} - \sum_{x=0}^{x=4} \frac{\binom{6}{x}\binom{10}{9-x}}{\binom{16}{9}} = 0.1101$ □

Problem 40. Assume the random variable(s) is from a hypergeometric distribution with $A = 6$ number of success and population size $N = 14$, and X is the number of successes you select. You select from the population $n = 8$ items without replacement. Answer the following:

(a) What is the probability $X \leq 3$:

Solution: $P(X \leq 3) = \sum_{x=0}^{x=3} \frac{\binom{6}{x}\binom{8}{8-x}}{\binom{14}{8}} = 0.5291$ □

(b) What is the probability $X > 3$:

Solution: $P(X > 3) = \sum_{x=4}^{x=6} \frac{\binom{6}{x}\binom{8}{8-x}}{\binom{14}{8}} = 0.4709$ □

(c) What is the probability $X \geq 3$:

Solution: $P(X \geq 3) = \sum_{x=3}^{x=6} \frac{\binom{6}{x}\binom{8}{8-x}}{\binom{14}{8}} = 0.8438$ □

(d) What is the probability $3 < X \leq 4$:

Solution: $P(3 < X \leq 4) = \sum_{x=0}^{x=4} \frac{\binom{6}{x}\binom{8}{8-x}}{\binom{14}{8}} - \sum_{x=0}^{x=3} \frac{\binom{6}{x}\binom{8}{8-x}}{\binom{14}{8}} = 0.3497$ □

Problem 41. Assume the random variable(s) is from a hypergeometric distribution with $A = 9$ number of success and population size $N = 18$, and X is the number of successes you select. You select from the population $n = 4$ items without replacement. Answer the following:

(a) What is the probability $X \leq 1$:

Solution: $P(X \leq 1) = \sum_{x=0}^{x=1} \frac{\binom{9}{x}\binom{9}{4-x}}{\binom{18}{4}} = 0.2882$ $\square$

(b) What is the probability $X > 1$:

Solution: $P(X > 1) = \sum_{x=2}^{x=9} \frac{\binom{9}{x}\binom{9}{4-x}}{\binom{18}{4}} = 0.7118$ $\square$

(c) What is the probability $X \geq 1$:

Solution: $P(X \geq 1) = \sum_{x=1}^{x=9} \frac{\binom{9}{x}\binom{9}{4-x}}{\binom{18}{4}} = 0.9588$ $\square$

(d) What is the probability $1 < X \leq 2$:

Solution: $P(1 < X \leq 2) = \sum_{x=0}^{x=2} \frac{\binom{9}{x}\binom{9}{4-x}}{\binom{18}{4}} - \sum_{x=0}^{x=1} \frac{\binom{9}{x}\binom{9}{4-x}}{\binom{18}{4}} = 0.4235$ $\square$

Problem 42. Assume the random variable(s) is from a hypergeometric distribution with $A = 9$ number of success and population size $N = 15$, and X is the number of successes you select. You select from the population $n = 4$ items without replacement. Answer the following:

(a) What is the probability $X \leq 1$:

Solution: $P(X \leq 1) = \sum_{x=0}^{x=1} \frac{\binom{9}{x}\binom{6}{4-x}}{\binom{15}{4}} = 0.1429$ □

(b) What is the probability $X > 1$:

Solution: $P(X > 1) = \sum_{x=2}^{x=9} \frac{\binom{9}{x}\binom{6}{4-x}}{\binom{15}{4}} = 0.8571$ □

(c) What is the probability $X \geq 1$:

Solution: $P(X \geq 1) = \sum_{x=1}^{x=9} \frac{\binom{9}{x}\binom{6}{4-x}}{\binom{15}{4}} = 0.989$ □

(d) What is the probability $1 < X \leq 2$:

Solution: $P(1 < X \leq 2) = \sum_{x=0}^{x=2} \frac{\binom{9}{x}\binom{6}{4-x}}{\binom{15}{4}} - \sum_{x=0}^{x=1} \frac{\binom{9}{x}\binom{6}{4-x}}{\binom{15}{4}} = 0.3956$ □

Problem 43. Assume the random variable(s) is from a hypergeometric distribution with $A = 6$ number of success and population size $N = 16$, and X is the number of successes you select. You select from the population $n = 6$ items without replacement. Answer the following:

(a) What is the probability $X \leq 3$:

Solution: $P(X \leq 3) = \sum_{x=0}^{x=3} \frac{\binom{6}{x}\binom{10}{6-x}}{\binom{16}{6}} = 0.9081$ □

(b) What is the probability $X > 3$:

Solution: $P(X > 3) = \sum_{x=4}^{x=6} \frac{\binom{6}{x}\binom{10}{6-x}}{\binom{16}{6}} = 0.0919$ □

(c) What is the probability $X \geq 3$:

Solution: $P(X \geq 3) = \sum_{x=3}^{x=6} \frac{\binom{6}{x}\binom{10}{6-x}}{\binom{16}{6}} = 0.3916$ □

(d) What is the probability $3 < X \leq 5$:

Solution: $P(3 < X \leq 5) = \sum_{x=0}^{x=5} \frac{\binom{6}{x}\binom{10}{6-x}}{\binom{16}{6}} - \sum_{x=0}^{x=3} \frac{\binom{6}{x}\binom{10}{6-x}}{\binom{16}{6}} = 0.0918$ □

Problem 44. Assume the random variable(s) is from a hypergeometric distribution with $A = 6$ number of success and population size $N = 12$, and X is the number of successes you select. You select from the population $n = 9$ items without replacement. Answer the following:

(a) What is the probability $X \leq 4$:

Solution: $P(X \leq 4) = \sum_{x=0}^{x=4} \frac{\binom{6}{x}\binom{6}{9-x}}{\binom{12}{9}} = 0.5$ $\square$

(b) What is the probability $X > 4$:

Solution: $P(X > 4) = \sum_{x=5}^{x=6} \frac{\binom{6}{x}\binom{6}{9-x}}{\binom{12}{9}} = 0.5$ $\square$

(c) What is the probability $X \geq 4$:

Solution: $P(X \geq 4) = \sum_{x=4}^{x=6} \frac{\binom{6}{x}\binom{6}{9-x}}{\binom{12}{9}} = 0.9091$ $\square$

(d) What is the probability $4 < X \leq 5$:

Solution: $P(4 < X \leq 5) = \sum_{x=0}^{x=5} \frac{\binom{6}{x}\binom{6}{9-x}}{\binom{12}{9}} - \sum_{x=0}^{x=4} \frac{\binom{6}{x}\binom{6}{9-x}}{\binom{12}{9}} = 0.4091$ $\square$

Problem 45. Assume the random variable(s) is from a hypergeometric distribution with $A = 6$ number of success and population size $N = 12$, and X is the number of successes you select. You select from the population $n = 2$ items without replacement. Answer the following:

(a) What is the probability $X \leq 2$:

Solution: $P(X \leq 2) = \sum_{x=0}^{x=2} \frac{\binom{6}{x}\binom{6}{2-x}}{\binom{12}{2}} = 1$ □

(b) What is the probability $X > 2$:

Solution: $P(X > 2) = \sum_{x=3}^{x=6} \frac{\binom{6}{x}\binom{6}{2-x}}{\binom{12}{2}} = 0$ □

(c) What is the probability $X \geq 2$:

Solution: $P(X \geq 2) = \sum_{x=2}^{x=6} \frac{\binom{6}{x}\binom{6}{2-x}}{\binom{12}{2}} = 0.2273$ □

(d) What is the probability $2 < X \leq 3$:

Solution: $P(2 < X \leq 3) = \sum_{x=0}^{x=3} \frac{\binom{6}{x}\binom{6}{2-x}}{\binom{12}{2}} - \sum_{x=0}^{x=2} \frac{\binom{6}{x}\binom{6}{2-x}}{\binom{12}{2}} = 0$ □

Problem 46. Assume the random variable(s) is from a hypergeometric distribution with $A = 8$ number of success and population size $N = 16$, and X is the number of successes you select. You select from the population $n = 16$ items without replacement. Answer the following:

(a) What is the probability $X \leq 8$:

Solution: $P(X \leq 8) = \sum_{x=0}^{x=8} \frac{\binom{8}{x}\binom{8}{16-x}}{\binom{16}{16}} = 1$ □

(b) What is the probability $X > 8$:

Solution: $P(X > 8) = \sum_{x=9}^{x=8} \frac{\binom{8}{x}\binom{8}{16-x}}{\binom{16}{16}} = 0$ □

(c) What is the probability $X \geq 8$:

Solution: $P(X \geq 8) = \sum_{x=8}^{x=8} \frac{\binom{8}{x}\binom{8}{16-x}}{\binom{16}{16}} = 1$ □

(d) What is the probability $8 < X \leq 9$:

Solution: $P(8 < X \leq 9) = \sum_{x=0}^{x=9} \frac{\binom{8}{x}\binom{8}{16-x}}{\binom{16}{16}} - \sum_{x=0}^{x=8} \frac{\binom{8}{x}\binom{8}{16-x}}{\binom{16}{16}} = 0$ □

Problem 47. Assume the random variable(s) is from a hypergeometric distribution with $A = 5$ number of success and population size $N = 15$, and X is the number of successes you select. You select from the population $n = 7$ items without replacement. Answer the following:

(a) What is the probability $X \leq 2$:

Solution: $P(X \leq 2) = \sum_{x=0}^{x=2} \frac{\binom{5}{x}\binom{10}{7-x}}{\binom{15}{7}} = 0.5734$ □

(b) What is the probability $X > 2$:

Solution: $P(X > 2) = \sum_{x=3}^{x=5} \frac{\binom{5}{x}\binom{10}{7-x}}{\binom{15}{7}} = 0.4266$ □

(c) What is the probability $X \geq 2$:

Solution: $P(X \geq 2) = \sum_{x=2}^{x=5} \frac{\binom{5}{x}\binom{10}{7-x}}{\binom{15}{7}} = 0.8182$ □

(d) What is the probability $2 < X \leq 3$:

Solution: $P(2 < X \leq 3) = \sum_{x=0}^{x=3} \frac{\binom{5}{x}\binom{10}{7-x}}{\binom{15}{7}} - \sum_{x=0}^{x=2} \frac{\binom{5}{x}\binom{10}{7-x}}{\binom{15}{7}} = 0.3263$ □

Problem 48. Assume the random variable(s) is from a hypergeometric distribution with $A = 9$ number of success and population size $N = 14$, and X is the number of successes you select. You select from the population $n = 5$ items without replacement. Answer the following:

(a) What is the probability $X \leq 3$:

Solution: $P(X \leq 3) = \sum_{x=0}^{x=3} \frac{\binom{9}{x}\binom{5}{5-x}}{\binom{14}{5}} = 0.6224$ □

(b) What is the probability $X > 3$:

Solution: $P(X > 3) = \sum_{x=4}^{x=9} \frac{\binom{9}{x}\binom{5}{5-x}}{\binom{14}{5}} = 0.3776$ □

(c) What is the probability $X \geq 3$:

Solution: $P(X \geq 3) = \sum_{x=3}^{x=9} \frac{\binom{9}{x}\binom{5}{5-x}}{\binom{14}{5}} = 0.7972$ □

(d) What is the probability $3 < X \leq 5$:

Solution: $P(3 < X \leq 5) = \sum_{x=0}^{x=5} \frac{\binom{9}{x}\binom{5}{5-x}}{\binom{14}{5}} - \sum_{x=0}^{x=3} \frac{\binom{9}{x}\binom{5}{5-x}}{\binom{14}{5}} = 0.3776$ □

Problem 49. Assume the random variable(s) is from a hypergeometric distribution with $A = 5$ number of success and population size $N = 12$, and X is the number of successes you select. You select from the population $n = 12$ items without replacement. Answer the following:

(a) What is the probability $X \leq 5$:

Solution: $P(X \leq 5) = \sum_{x=0}^{x=5} \frac{\binom{5}{x}\binom{7}{12-x}}{\binom{12}{12}} = 1$ □

(b) What is the probability $X > 5$:

Solution: $P(X > 5) = \sum_{x=6}^{x=5} \frac{\binom{5}{x}\binom{7}{12-x}}{\binom{12}{12}} = 0$ □

(c) What is the probability $X \geq 5$:

Solution: $P(X \geq 5) = \sum_{x=5}^{x=5} \frac{\binom{5}{x}\binom{7}{12-x}}{\binom{12}{12}} = 1$ □

(d) What is the probability $5 < X \leq 6$:

Solution: $P(5 < X \leq 6) = \sum_{x=0}^{x=6} \frac{\binom{5}{x}\binom{7}{12-x}}{\binom{12}{12}} - \sum_{x=0}^{x=5} \frac{\binom{5}{x}\binom{7}{12-x}}{\binom{12}{12}} = 0$ □

Problem 50. Assume the random variable(s) is from a hypergeometric distribution with $A = 5$ number of success and population size $N = 13$, and X is the number of successes you select. You select from the population $n = 2$ items without replacement. Answer the following:

(a) What is the probability $X \leq 1$:

Solution: $P(X \leq 1) = \sum_{x=0}^{x=1} \frac{\binom{5}{x}\binom{8}{2-x}}{\binom{13}{2}} = 0.8718$ □

(b) What is the probability $X > 1$:

Solution: $P(X > 1) = \sum_{x=2}^{x=5} \frac{\binom{5}{x}\binom{8}{2-x}}{\binom{13}{2}} = 0.1282$ □

(c) What is the probability $X \geq 1$:

Solution: $P(X \geq 1) = \sum_{x=1}^{x=5} \frac{\binom{5}{x}\binom{8}{2-x}}{\binom{13}{2}} = 0.641$ □

(d) What is the probability $1 < X \leq 2$:

Solution: $P(1 < X \leq 2) = \sum_{x=0}^{x=2} \frac{\binom{5}{x}\binom{8}{2-x}}{\binom{13}{2}} - \sum_{x=0}^{x=1} \frac{\binom{5}{x}\binom{8}{2-x}}{\binom{13}{2}} = 0.1282$ □

Problem 51. Assume the random variable(s) is from a hypergeometric distribution with $A = 8$ number of success and population size $N = 14$, and X is the number of successes you select. You select from the population $n = 7$ items without replacement. Answer the following:

(a) What is the probability $X \leq 4$:

Solution: $P(X \leq 4) = \sum_{x=0}^{x=4} \frac{\binom{8}{x}\binom{6}{7-x}}{\binom{14}{7}} = 0.704$ □

(b) What is the probability $X > 4$:

Solution: $P(X > 4) = \sum_{x=5}^{x=8} \frac{\binom{8}{x}\binom{6}{7-x}}{\binom{14}{7}} = 0.296$ □

(c) What is the probability $X \geq 4$:

Solution: $P(X \geq 4) = \sum_{x=4}^{x=8} \frac{\binom{8}{x}\binom{6}{7-x}}{\binom{14}{7}} = 0.704$ □

(d) What is the probability $4 < X \leq 6$:

Solution: $P(4 < X \leq 6) = \sum_{x=0}^{x=6} \frac{\binom{8}{x}\binom{6}{7-x}}{\binom{14}{7}} - \sum_{x=0}^{x=4} \frac{\binom{8}{x}\binom{6}{7-x}}{\binom{14}{7}} = 0.2937$ □

Problem 52. Assume the random variable(s) is from a hypergeometric distribution with $A = 9$ number of success and population size $N = 17$, and X is the number of successes you select. You select from the population $n = 10$ items without replacement. Answer the following:

(a) What is the probability $X \leq 6$:

Solution: $P(X \leq 6) = \sum_{x=0}^{x=6} \frac{\binom{9}{x}\binom{8}{10-x}}{\binom{17}{10}} = 0.883$ □

(b) What is the probability $X > 6$:

Solution: $P(X > 6) = \sum_{x=7}^{x=9} \frac{\binom{9}{x}\binom{8}{10-x}}{\binom{17}{10}} = 0.117$ □

(c) What is the probability $X \geq 6$:

Solution: $P(X \geq 6) = \sum_{x=6}^{x=9} \frac{\binom{9}{x}\binom{8}{10-x}}{\binom{17}{10}} = 0.4194$ □

(d) What is the probability $6 < X \leq 7$:

Solution: $P(6 < X \leq 7) = \sum_{x=0}^{x=7} \frac{\binom{9}{x}\binom{8}{10-x}}{\binom{17}{10}} - \sum_{x=0}^{x=6} \frac{\binom{9}{x}\binom{8}{10-x}}{\binom{17}{10}} = 0.1037$ □

Problem 53. Assume the random variable(s) is from a hypergeometric distribution with $A = 7$ number of success and population size $N = 16$, and X is the number of successes you select. You select from the population $n = 8$ items without replacement. Answer the following:

(a) What is the probability $X \leq 2$:

Solution: $P(X \leq 2) = \sum_{x=0}^{x=2} \frac{\binom{7}{x}\binom{9}{8-x}}{\binom{16}{8}} = 0.1573$ □

(b) What is the probability $X > 2$:

Solution: $P(X > 2) = \sum_{x=3}^{x=7} \frac{\binom{7}{x}\binom{9}{8-x}}{\binom{16}{8}} = 0.8427$ □

(c) What is the probability $X \geq 2$:

Solution: $P(X \geq 2) = \sum_{x=2}^{x=7} \frac{\binom{7}{x}\binom{9}{8-x}}{\binom{16}{8}} = 0.9797$ □

(d) What is the probability $2 < X \leq 3$:

Solution: $P(2 < X \leq 3) = \sum_{x=0}^{x=3} \frac{\binom{7}{x}\binom{9}{8-x}}{\binom{16}{8}} - \sum_{x=0}^{x=2} \frac{\binom{7}{x}\binom{9}{8-x}}{\binom{16}{8}} = 0.3427$ □

Problem 54. Assume the random variable(s) is from a hypergeometric distribution with $A = 8$ number of success and population size $N = 17$, and X is the number of successes you select. You select from the population $n = 16$ items without replacement. Answer the following:

(a) What is the probability $X \leq 7$:

Solution: $P(X \leq 7) = \sum_{x=0}^{x=7} \frac{\binom{8}{x}\binom{9}{16-x}}{\binom{17}{16}} = 0.4706$ $\square$

(b) What is the probability $X > 7$:

Solution: $P(X > 7) = \sum_{x=8}^{x=8} \frac{\binom{8}{x}\binom{9}{16-x}}{\binom{17}{16}} = 0.5294$ $\square$

(c) What is the probability $X \geq 7$:

Solution: $P(X \geq 7) = \sum_{x=7}^{x=8} \frac{\binom{8}{x}\binom{9}{16-x}}{\binom{17}{16}} = 1$ $\square$

(d) What is the probability $7 < X \leq 8$:

Solution: $P(7 < X \leq 8) = \sum_{x=0}^{x=8} \frac{\binom{8}{x}\binom{9}{16-x}}{\binom{17}{16}} - \sum_{x=0}^{x=7} \frac{\binom{8}{x}\binom{9}{16-x}}{\binom{17}{16}} = 0.5294$ $\square$

1.4 Normal Distribution

Problem 55. Assume the random variable(s) is from a normal distribution with $\mu = 7$ and $\sigma = 2.7$. Answer the following:

(a) What is the probability $X \leq 4.89$:

Solution: $P(X \leq 4.89) = P(Z \leq \frac{4.89-7}{2.7}) = 0.2173$ $\square$

(b) What is the probability $X \geq 4.89$:

Solution: $P(X \geq 4.89) = P(Z \geq \frac{4.89-7}{2.7}) = 0.7827$ $\square$

(c) What is the probability $4.89 \leq X \leq 6.71$:

Solution: $P(4.89 \leq X \leq 6.71) = P(\frac{4.89-7}{2.7} \leq Z \leq \frac{6.71-7}{2.7}) = 0.24$ $\square$

Problem 56. Assume the random variable(s) is from a normal distribution with $\mu = 8.1$ and $\sigma = 2.6$. Answer the following:

(a) What is the probability $X \leq 10.81$:

Solution: $P(X \leq 10.81) = P(Z \leq \frac{10.81-8.1}{2.6}) = 0.8514$ $\square$

(b) What is the probability $X \geq 10.81$:

Solution: $P(X \geq 10.81) = P(Z \geq \frac{10.81-8.1}{2.6}) = 0.1486$ $\square$

(c) What is the probability $10.81 \leq X \leq 11.5$:

Solution: $P(10.81 \leq X \leq 11.5) = P(\frac{10.81-8.1}{2.6} \leq Z \leq \frac{11.5-8.1}{2.6}) = 0.0531$ $\square$

Problem 57. Assume the random variable(s) is from a normal distribution with $\mu = 4$ and $\sigma = 1.2$. Answer the following:

(a) What is the probability $X \leq 2.7$:

Solution: $P(X \leq 2.7) = P(Z \leq \frac{2.7-4}{1.2}) = 0.1393$ □

(b) What is the probability $X \geq 2.7$:

Solution: $P(X \geq 2.7) = P(Z \geq \frac{2.7-4}{1.2}) = 0.8607$ □

(c) What is the probability $2.7 \leq X \leq 4.77$:

Solution: $P(2.7 \leq X \leq 4.77) = P(\frac{2.7-4}{1.2} \leq Z \leq \frac{4.77-4}{1.2}) = 0.6001$ □

Problem 58. Assume the random variable(s) is from a normal distribution with $\mu = 2.6$ and $\sigma = 1.8$. Answer the following:

(a) What is the probability $X \leq 3.36$:

Solution: $P(X \leq 3.36) = P(Z \leq \frac{3.36-2.6}{1.8}) = 0.6636$ $\square$

(b) What is the probability $X \geq 3.36$:

Solution: $P(X \geq 3.36) = P(Z \geq \frac{3.36-2.6}{1.8}) = 0.3364$ $\square$

(c) What is the probability $3.36 \leq X \leq 6.18$:

Solution: $P(3.36 \leq X \leq 6.18) = P(\frac{3.36-2.6}{1.8} \leq Z \leq \frac{6.18-2.6}{1.8}) = 0.3131$ $\square$

Problem 59. Assume the random variable(s) is from a normal distribution with $\mu = 0.8$ and $\sigma = 2.2$. Answer the following:

(a) What is the probability $X \leq -0.68$:

Solution: $P(X \leq -0.68) = P(Z \leq \frac{-0.68-0.8}{2.2}) = 0.2506$ $\square$

(b) What is the probability $X \geq -0.68$:

Solution: $P(X \geq -0.68) = P(Z \geq \frac{-0.68-0.8}{2.2}) = 0.7494$ $\square$

(c) What is the probability $-0.68 \leq X \leq -0.18$:

Solution: $P(-0.68 \leq X \leq -0.18) = P(\frac{-0.68-0.8}{2.2} \leq Z \leq \frac{-0.18-0.8}{2.2}) = 0.0774$ $\square$

Problem 60. Assume the random variable(s) is from a normal distribution with $\mu = 2.7$ and $\sigma = 2.9$. Answer the following:

(a) What is the probability $X \leq 1.95$:

Solution: $P(X \leq 1.95) = P(Z \leq \frac{1.95-2.7}{2.9}) = 0.398$

$\square$

(b) What is the probability $X \geq 1.95$:

Solution: $P(X \geq 1.95) = P(Z \geq \frac{1.95-2.7}{2.9}) = 0.602$

$\square$

(c) What is the probability $1.95 \leq X \leq 5.33$:

Solution: $P(1.95 \leq X \leq 5.33) = P(\frac{1.95-2.7}{2.9} \leq Z \leq \frac{5.33-2.7}{2.9}) = 0.4198$

$\square$

Problem 61. Assume the random variable(s) is from a normal distribution with $\mu = 5.5$ and $\sigma = 2.1$. Answer the following:

(a) What is the probability $X \leq 5.88$:

Solution: $P(X \leq 5.88) = P(Z \leq \frac{5.88-5.5}{2.1}) = 0.5718$ □

(b) What is the probability $X \geq 5.88$:

Solution: $P(X \geq 5.88) = P(Z \geq \frac{5.88-5.5}{2.1}) = 0.4282$ □

(c) What is the probability $5.88 \leq X \leq 7.33$:

Solution: $P(5.88 \leq X \leq 7.33) = P(\frac{5.88-5.5}{2.1} \leq Z \leq \frac{7.33-5.5}{2.1}) = 0.2364$ □

Problem 62. Assume the random variable(s) is from a normal distribution with $\mu = 1.7$ and $\sigma = 1.8$. Answer the following:

(a) What is the probability $X \leq -2.22$:

Solution: $P(X \leq -2.22) = P(Z \leq \frac{-2.22-1.7}{1.8}) = 0.0147$ □

(b) What is the probability $X \geq -2.22$:

Solution: $P(X \geq -2.22) = P(Z \geq \frac{-2.22-1.7}{1.8}) = 0.9853$ □

(c) What is the probability $-2.22 \leq X \leq 1.93$:

Solution: $P(-2.22 \leq X \leq 1.93) = P(\frac{-2.22-1.7}{1.8} \leq Z \leq \frac{1.93-1.7}{1.8}) = 0.5361$ □

Problem 63. Assume the random variable(s) is from a normal distribution with $\mu = 8.4$ and $\sigma = 0.5$. Answer the following:

(a) What is the probability $X \leq 8.79$:

Solution: $P(X \leq 8.79) = P(Z \leq \frac{8.79-8.4}{0.5}) = 0.7823$ $\square$

(b) What is the probability $X \geq 8.79$:

Solution: $P(X \geq 8.79) = P(Z \geq \frac{8.79-8.4}{0.5}) = 0.2177$ $\square$

(c) What is the probability $8.79 \leq X \leq 9.07$:

Solution: $P(8.79 \leq X \leq 9.07) = P(\frac{8.79-8.4}{0.5} \leq Z \leq \frac{9.07-8.4}{0.5}) = 0.1276$ $\square$

Problem 64. Assume the random variable(s) is from a normal distribution with $\mu = 5.5$ and $\sigma = 1$. Answer the following:

(a) What is the probability $X \leq 4.84$:

Solution: $P(X \leq 4.84) = P(Z \leq \frac{4.84-5.5}{1}) = 0.2546$ $\square$

(b) What is the probability $X \geq 4.84$:

Solution: $P(X \geq 4.84) = P(Z \geq \frac{4.84-5.5}{1}) = 0.7454$ $\square$

(c) What is the probability $4.84 \leq X \leq 6.47$:

Solution: $P(4.84 \leq X \leq 6.47) = P(\frac{4.84-5.5}{1} \leq Z \leq \frac{6.47-5.5}{1}) = 0.5793$ $\square$

Problem 65. Assume the random variable(s) is from a normal distribution with $\mu = 5$ and $\sigma = 2.5$. Answer the following:

(a) What is the probability $X \leq 0.85$:

Solution: $P(X \leq 0.85) = P(Z \leq \frac{0.85-5}{2.5}) = 0.0485$ $\square$

(b) What is the probability $X \geq 0.85$:

Solution: $P(X \geq 0.85) = P(Z \geq \frac{0.85-5}{2.5}) = 0.9515$ $\square$

(c) What is the probability $0.85 \leq X \leq 4.49$:

Solution: $P(0.85 \leq X \leq 4.49) = P(\frac{0.85-5}{2.5} \leq Z \leq \frac{4.49-5}{2.5}) = 0.3707$ $\square$

Problem 66. Assume the random variable(s) is from a normal distribution with $\mu = 4.2$ and $\sigma = 1.5$. Answer the following:

(a) What is the probability $X \leq 2.54$:

Solution: $P(X \leq 2.54) = P(Z \leq \frac{2.54-4.2}{1.5}) = 0.1342$ $\square$

(b) What is the probability $X \geq 2.54$:

Solution: $P(X \geq 2.54) = P(Z \geq \frac{2.54-4.2}{1.5}) = 0.8658$ $\square$

(c) What is the probability $2.54 \leq X \leq 6.24$:

Solution: $P(2.54 \leq X \leq 6.24) = P(\frac{2.54-4.2}{1.5} \leq Z \leq \frac{6.24-4.2}{1.5}) = 0.7789$ $\square$

Problem 67. Assume the random variable(s) is from a normal distribution with $\mu = 0.2$ and $\sigma = 2.8$. Answer the following:

(a) What is the probability $X \leq 0.98$:

Solution: $P(X \leq 0.98) = P(Z \leq \frac{0.98-0.2}{2.8}) = 0.6097$ □

(b) What is the probability $X \geq 0.98$:

Solution: $P(X \geq 0.98) = P(Z \geq \frac{0.98-0.2}{2.8}) = 0.3903$ □

(c) What is the probability $0.98 \leq X \leq 3.59$:

Solution: $P(0.98 \leq X \leq 3.59) = P(\frac{0.98-0.2}{2.8} \leq Z \leq \frac{3.59-0.2}{2.8}) = 0.2773$ □

Problem 68. Assume the random variable(s) is from a normal distribution with $\mu = 1.3$ and $\sigma = 2.1$. Answer the following:

(a) What is the probability $X \leq 2.29$:

Solution: $P(X \leq 2.29) = P(Z \leq \frac{2.29-1.3}{2.1}) = 0.6813$ $\square$

(b) What is the probability $X \geq 2.29$:

Solution: $P(X \geq 2.29) = P(Z \geq \frac{2.29-1.3}{2.1}) = 0.3187$ $\square$

(c) What is the probability $2.29 \leq X \leq 4.65$:

Solution: $P(2.29 \leq X \leq 4.65) = P(\frac{2.29-1.3}{2.1} \leq Z \leq \frac{4.65-1.3}{2.1}) = 0.2633$ $\square$

Problem 69. Assume the random variable(s) is from a normal distribution with $\mu = 9.4$ and $\sigma = 2.8$. Answer the following:

(a) What is the probability $X \leq 9.37$:

Solution: $P(X \leq 9.37) = P(Z \leq \frac{9.37-9.4}{2.8}) = 0.4957$ □

(b) What is the probability $X \geq 9.37$:

Solution: $P(X \geq 9.37) = P(Z \geq \frac{9.37-9.4}{2.8}) = 0.5043$ □

(c) What is the probability $9.37 \leq X \leq 11.69$:

Solution: $P(9.37 \leq X \leq 11.69) = P(\frac{9.37-9.4}{2.8} \leq Z \leq \frac{11.69-9.4}{2.8}) = 0.2976$ □

Problem 70. Assume the random variable(s) is from a normal distribution with $\mu = 8$ and $\sigma = 2.8$. Answer the following:

(a) What is the probability $X \leq 8.38$:

Solution: $P(X \leq 8.38) = P(Z \leq \frac{8.38-8}{2.8}) = 0.554$ $\square$

(b) What is the probability $X \geq 8.38$:

Solution: $P(X \geq 8.38) = P(Z \geq \frac{8.38-8}{2.8}) = 0.446$ $\square$

(c) What is the probability $8.38 \leq X \leq 10.3$:

Solution: $P(8.38 \leq X \leq 10.3) = P(\frac{8.38-8}{2.8} \leq Z \leq \frac{10.3-8}{2.8}) = 0.2403$ $\square$

Problem 71. Assume the random variable(s) is from a normal distribution with $\mu = 2.2$ and $\sigma = 0.4$. Answer the following:

(a) What is the probability $X \leq 1.89$:

Solution: $P(X \leq 1.89) = P(Z \leq \frac{1.89-2.2}{0.4}) = 0.2192$

$\square$

(b) What is the probability $X \geq 1.89$:

Solution: $P(X \geq 1.89) = P(Z \geq \frac{1.89-2.2}{0.4}) = 0.7808$

$\square$

(c) What is the probability $1.89 \leq X \leq 1.97$:

Solution: $P(1.89 \leq X \leq 1.97) = P(\frac{1.89-2.2}{0.4} \leq Z \leq \frac{1.97-2.2}{0.4}) = 0.0635$

$\square$

Problem 72. Assume the random variable(s) is from a normal distribution with $\mu = 2.7$ and $\sigma = 2.8$. Answer the following:

(a) What is the probability $X \leq 2.47$:

Solution: $P(X \leq 2.47) = P(Z \leq \frac{2.47-2.7}{2.8}) = 0.4673$ $\square$

(b) What is the probability $X \geq 2.47$:

Solution: $P(X \geq 2.47) = P(Z \geq \frac{2.47-2.7}{2.8}) = 0.5327$ $\square$

(c) What is the probability $2.47 \leq X \leq 3.4$:

Solution: $P(2.47 \leq X \leq 3.4) = P(\frac{2.47-2.7}{2.8} \leq Z \leq \frac{3.4-2.7}{2.8}) = 0.1314$ $\square$

1.5 One-Sample Test of Proportions Examples

Problem 73. You are investigating the probability of the stock market having a positive day. A positive day is defined as the SET (Stock Exchange of Thailand) rises. You randomly select 114 days from the past 20 years and find 61 positive days. You assume each day is independent of one another and that the true unknown probably that the SET rises is constant although unknown for the past 20 years.

Test if the true probability the SET rises π is *less than* 0.58. Use $\alpha = 0.05$. Do not use a continuity correction factor.

Solution:

$$z - statistic = \frac{\hat{p} - \pi}{\sqrt{\pi(1-\pi)/n}} = \frac{0.5351 - 0.58}{\sqrt{0.58 * (1 - 0.58)/114}} = -0.9716$$

The p-value = 0.1656 and reject if p-value < 0.05

$\square$

Problem 74. You are investigating the probability of the stock market having a positive day. A positive day is defined as the SET (Stock Exchange of Thailand) rises. You randomly select 104 days from the past 20 years and find 43 positive days. You assume each day is independent of one another and that the true unknown probably that the SET rises is constant although unknown for the past 20 years.

Test if the true probability the SET rises π is *less than* 0.46. Use $\alpha = 0.05$. Do not use a continuity correction factor.

Solution:

$$z - statistic = \frac{\hat{p} - \pi}{\sqrt{\pi(1-\pi)/n}} = \frac{0.4135 - 0.46}{\sqrt{0.46 * (1 - 0.46)/104}} = -0.9523$$

The p-value = 0.1705 and reject if p-value < 0.05

□

Problem 75. You are investigating the probability of the stock market having a positive day. A positive day is defined as the SET (Stock Exchange of Thailand) rises. You randomly select 98 days from the past 20 years and find 43 positive days. You assume each day is independent of one another and that the true unknown probably that the SET rises is constant although unknown for the past 20 years.

Test if the true probability the SET rises π is *not equal to* 0.6. Use $\alpha = 0.05$. Do not use a continuity correction factor.

Solution:

$$z - statistic = \frac{\hat{p} - \pi}{\sqrt{\pi(1-\pi)/n}} = \frac{0.4388 - 0.6}{\sqrt{0.6 * (1 - 0.6)/98}} = -3.2579$$

The p-value = 0.0011 and reject if p-value < 0.05

□

Problem 76. You are investigating the probability of the stock market having a positive day. A positive day is defined as the SET (Stock Exchange of Thailand) rises. You randomly select 111 days from the past 20 years and find 39 positive days. You assume each day is independent of one another and that the true unknown probably that the SET rises is constant although unknown for the past 20 years.

Test if the true probability the SET rises π is *not equal to* 0.57. Use $\alpha = 0.05$. Do not use a continuity correction factor.

Solution:

$$z - statistic = \frac{\hat{p} - \pi}{\sqrt{\pi(1-\pi)/n}} = \frac{0.3514 - 0.57}{\sqrt{0.57 * (1 - 0.57)/111}} = -4.653$$

The p-value = 0 and reject if p-value < 0.05

□

Problem 77. You are investigating the probability of the stock market having a positive day. A positive day is defined as the SET (Stock Exchange of Thailand) rises. You randomly select 89 days from the past 20 years and find 38 positive days. You assume each day is independent of one another and that the true unknown probably that the SET rises is constant although unknown for the past 20 years.

Test if the true probability the SET rises π is *not equal to* 0.47. Use $\alpha = 0.05$. Do not use a continuity correction factor.

Solution:

$$z - statistic = \frac{\hat{p} - \pi}{\sqrt{\pi(1-\pi)/n}} = \frac{0.427 - 0.47}{\sqrt{0.47 * (1 - 0.47)/89}} = -0.8134$$

The p-value = 0.416 and reject if p-value < 0.05

□

Problem 78. You are investigating the probability of the stock market having a positive day. A positive day is defined as the SET (Stock Exchange of Thailand) rises. You randomly select 102 days from the past 20 years and find 49 positive days. You assume each day is independent of one another and that the true unknown probably that the SET rises is constant although unknown for the past 20 years.

Test if the true probability the SET rises π is *not equal to* 0.42. Use $\alpha = 0.05$. Do not use a continuity correction factor.

Solution:

$$z - statistic = \frac{\hat{p} - \pi}{\sqrt{\pi(1-\pi)/n}} = \frac{0.4804 - 0.42}{\sqrt{0.42 * (1 - 0.42)/102}} = 1.2358$$

The p-value = 0.2165 and reject if p-value < 0.05

$\square$

Problem 79. You are investigating the probability of the stock market having a positive day. A positive day is defined as the SET (Stock Exchange of Thailand) rises. You randomly select 99 days from the past 20 years and find 34 positive days. You assume each day is independent of one another and that the true unknown probably that the SET rises is constant although unknown for the past 20 years.

Test if the true probability the SET rises π is *less than* 0.48. Use $\alpha = 0.05$. Do not use a continuity correction factor.

Solution:

$$z - statistic = \frac{\hat{p} - \pi}{\sqrt{\pi(1-\pi)/n}} = \frac{0.3434 - 0.48}{\sqrt{0.48 * (1 - 0.48)/99}} = -2.7198$$

The p-value = 0.0033 and reject if p-value < 0.05

□

Problem 80. You are investigating the probability of the stock market having a positive day. A positive day is defined as the SET (Stock Exchange of Thailand) rises. You randomly select 97 days from the past 20 years and find 48 positive days. You assume each day is independent of one another and that the true unknown probably that the SET rises is constant although unknown for the past 20 years.

Test if the true probability the SET rises π is *not equal to* 0.5. Use $\alpha = 0.05$. Do not use a continuity correction factor.

Solution:

$$z - statistic = \frac{\hat{p} - \pi}{\sqrt{\pi(1-\pi)/n}} = \frac{0.4948 - 0.5}{\sqrt{0.5 * (1-0.5)/97}} = -0.1015$$

The p-value $= 0.9191$ and reject if p-value < 0.05

□

Problem 81. You are investigating the probability of the stock market having a positive day. A positive day is defined as the SET (Stock Exchange of Thailand) rises. You randomly select 104 days from the past 20 years and find 51 positive days. You assume each day is independent of one another and that the true unknown probably that the SET rises is constant although unknown for the past 20 years.

Test if the true probability the SET rises π is *greater than* 0.46. Use $\alpha = 0.05$. Do not use a continuity correction factor.

Solution:

$$z - statistic = \frac{\hat{p} - \pi}{\sqrt{\pi(1-\pi)/n}} = \frac{0.4904 - 0.46}{\sqrt{0.46 * (1 - 0.46)/104}} = 0.6217$$

The p-value = 0.2671 and reject if p-value < 0.05

□

Problem 82. You are investigating the probability of the stock market having a positive day. A positive day is defined as the SET (Stock Exchange of Thailand) rises. You randomly select 93 days from the past 20 years and find 43 positive days. You assume each day is independent of one another and that the true unknown probably that the SET rises is constant although unknown for the past 20 years.

Test if the true probability the SET rises π is *not equal to* 0.46. Use $\alpha = 0.05$. Do not use a continuity correction factor.

Solution:

$$z - statistic = \frac{\hat{p} - \pi}{\sqrt{\pi(1-\pi)/n}} = \frac{0.4624 - 0.46}{\sqrt{0.46 * (1 - 0.46)/93}} = 0.0458$$

The p-value = 0.9635 and reject if p-value < 0.05

□

Problem 83. You are investigating the probability of the stock market having a positive day. A positive day is defined as the SET (Stock Exchange of Thailand) rises. You randomly select 104 days from the past 20 years and find 54 positive days. You assume each day is independent of one another and that the true unknown probably that the SET rises is constant although unknown for the past 20 years.

Test if the true probability the SET rises π is *greater than* 0.46. Use $\alpha = 0.05$. Do not use a continuity correction factor.

Solution:

$$z - statistic = \frac{\hat{p} - \pi}{\sqrt{\pi(1-\pi)/n}} = \frac{0.5192 - 0.46}{\sqrt{0.46 * (1 - 0.46)/104}} = 1.212$$

The p-value = 0.1128 and reject if p-value < 0.05

$\square$

Problem 84. You are investigating the probability of the stock market having a positive day. A positive day is defined as the SET (Stock Exchange of Thailand) rises. You randomly select 105 days from the past 20 years and find 48 positive days. You assume each day is independent of one another and that the true unknown probably that the SET rises is constant although unknown for the past 20 years.

Test if the true probability the SET rises π is *greater than* 0.41. Use $\alpha = 0.05$. Do not use a continuity correction factor.

Solution:

$$z - statistic = \frac{\hat{p} - \pi}{\sqrt{\pi(1-\pi)/n}} = \frac{0.4571 - 0.41}{\sqrt{0.41 * (1 - 0.41)/105}} = 0.9822$$

The p-value = 0.163 and reject if p-value < 0.05

$\square$

Problem 85. You are investigating the probability of the stock market having a positive day. A positive day is defined as the SET (Stock Exchange of Thailand) rises. You randomly select 104 days from the past 20 years and find 59 positive days. You assume each day is independent of one another and that the true unknown probably that the SET rises is constant although unknown for the past 20 years.

Test if the true probability the SET rises π is *not equal to* 0.46. Use $\alpha = 0.05$. Do not use a continuity correction factor.

Solution:

$$z - statistic = \frac{\hat{p} - \pi}{\sqrt{\pi(1-\pi)/n}} = \frac{0.5673 - 0.46}{\sqrt{0.46 * (1 - 0.46)/104}} = 2.1957$$

The p-value = 0.0281 and reject if p-value < 0.05

□

Problem 86. You are investigating the probability of the stock market having a positive day. A positive day is defined as the SET (Stock Exchange of Thailand) rises. You randomly select 102 days from the past 20 years and find 49 positive days. You assume each day is independent of one another and that the true unknown probably that the SET rises is constant although unknown for the past 20 years.

Test if the true probability the SET rises π is *not equal to* 0.48. Use $\alpha = 0.05$. Do not use a continuity correction factor.

Solution:

$$z - statistic = \frac{\hat{p} - \pi}{\sqrt{\pi(1-\pi)/n}} = \frac{0.4804 - 0.48}{\sqrt{0.48 * (1 - 0.48)/102}} = 0.0079$$

The p-value = 0.9937 and reject if p-value < 0.05

□

Problem 87. You are investigating the probability of the stock market having a positive day. A positive day is defined as the SET (Stock Exchange of Thailand) rises. You randomly select 95 days from the past 20 years and find 35 positive days. You assume each day is independent of one another and that the true unknown probably that the SET rises is constant although unknown for the past 20 years.

Test if the true probability the SET rises π is *less than* 0.42. Use $\alpha = 0.05$. Do not use a continuity correction factor.

Solution:

$$z - statistic = \frac{\hat{p} - \pi}{\sqrt{\pi(1-\pi)/n}} = \frac{0.3684 - 0.42}{\sqrt{0.42 * (1-0.42)/95}} = -1.0186$$

The p-value = 0.1542 and reject if p-value < 0.05

□

Problem 88. You are investigating the probability of the stock market having a positive day. A positive day is defined as the SET (Stock Exchange of Thailand) rises. You randomly select 98 days from the past 20 years and find 59 positive days. You assume each day is independent of one another and that the true unknown probably that the SET rises is constant although unknown for the past 20 years.

Test if the true probability the SET rises π is *greater than* 0.5. Use $\alpha = 0.05$. Do not use a continuity correction factor.

Solution:

$$z - statistic = \frac{\hat{p} - \pi}{\sqrt{\pi(1-\pi)/n}} = \frac{0.602 - 0.5}{\sqrt{0.5 * (1-0.5)/98}} = 2.0203$$

The p-value = 0.0217 and reject if p-value < 0.05

□

Problem 89. You are investigating the probability of the stock market having a positive day. A positive day is defined as the SET (Stock Exchange of Thailand) rises. You randomly select 101 days from the past 20 years and find 56 positive days. You assume each day is independent of one another and that the true unknown probably that the SET rises is constant although unknown for the past 20 years.

Test if the true probability the SET rises π is *greater than* 0.49. Use $\alpha = 0.05$. Do not use a continuity correction factor.

Solution:

$$z - statistic = \frac{\hat{p} - \pi}{\sqrt{\pi(1-\pi)/n}} = \frac{0.5545 - 0.49}{\sqrt{0.49 * (1 - 0.49)/101}} = 1.2958$$

The p-value = 0.0975 and reject if p-value < 0.05

$\square$

Problem 90. You are investigating the probability of the stock market having a positive day. A positive day is defined as the SET (Stock Exchange of Thailand) rises. You randomly select 98 days from the past 20 years and find 41 positive days. You assume each day is independent of one another and that the true unknown probably that the SET rises is constant although unknown for the past 20 years.

Test if the true probability the SET rises π is *less than* 0.47. Use $\alpha = 0.05$. Do not use a continuity correction factor.

Solution:

$$z - statistic = \frac{\hat{p} - \pi}{\sqrt{\pi(1-\pi)/n}} = \frac{0.4184 - 0.47}{\sqrt{0.47 * (1 - 0.47)/98}} = -1.0241$$

The p-value = 0.1529 and reject if p-value < 0.05

□

1.6 Two-Sample Test of Proportions Examples

Problem 91. You are investigating the probability of the stock market having a positive day. A positive day is defined as the SET (Stock Exchange of Thailand) for $\hat{p}_1$ and for $\hat{p}_2$ the TWSE (Taiwan Stock Exchange) rises. You randomly select 105 days from the past 20 years for the SET and find 50 positive days. You also sample 106 days from the past 20 years for the TWSE and find 49 positive days. You assume each day is independent of one another, the stock exchanges movements are independent and that the true unknown probably that the SET rises and the TWSE rises is constant although unknown for the past 20 years.

Test if the true probability the SET rises is *greater than* the TWSE. Use $\alpha = 0.05$. Do not use a continuity correction factor.

Solution:

$$\begin{aligned}
\hat{p}_1 &= \tfrac{50}{105} = & 0.4762 \\
\hat{p}_2 &= \tfrac{49}{106} = & 0.4623 \\
\hat{p} &= \tfrac{50+49}{105+106} = & 0.4692
\end{aligned}$$

$$\begin{aligned}
z - statistic &= \tfrac{\hat{p}_1 - \hat{p}_2}{\sqrt{\hat{p}(1-\hat{p})*(1/n_1+1/n_2)}} \\
&= \tfrac{0.4762-0.4623}{\sqrt{0.4692*(1-0.4692)*0.018958}} \\
&= 0.2027
\end{aligned}$$

The p-value = 0.4197 and reject if p-value < 0.05

□

Problem 92. You are investigating the probability of the stock market having a positive day. A positive day is defined as the SET (Stock Exchange of Thailand) for $\hat{p}_1$ and for $\hat{p}_2$ the TWSE (Taiwan Stock Exchange) rises. You randomly select 85 days from the past 20 years for the SET and find 46 positive days. You also sample 99 days from the past 20 years for the TWSE and find 58 positive days. You assume each day is independent of one another, the stock exchanges movements are independent and that the true unknown probably that the SET rises and the TWSE rises is constant although unknown for the past 20 years.

Test if the true probability the SET rises is *not equal to* the TWSE. Use $\alpha = 0.05$. Do not use a continuity correction factor.

Solution:

$$\begin{aligned} \hat{p}_1 &= \tfrac{46}{85} = & 0.5412 \\ \hat{p}_2 &= \tfrac{58}{99} = & 0.5859 \\ \hat{p} &= \tfrac{46+58}{85+99} = & 0.5652 \end{aligned}$$

$$\begin{aligned} z - statistic &= \tfrac{\hat{p}_1 - \hat{p}_2}{\sqrt{\hat{p}(1-\hat{p})*(1/n_1+1/n_2)}} \\ &= \tfrac{0.5412-0.5859}{\sqrt{0.5652*(1-0.5652)*0.021866}} \\ &= -0.6095 \end{aligned}$$

The p-value = 0.5422 and reject if p-value < 0.05

$\square$

Problem 93. You are investigating the probability of the stock market having a positive day. A positive day is defined as the SET (Stock Exchange of Thailand) for $\hat{p}_1$ and for $\hat{p}_2$ the TWSE (Taiwan Stock Exchange) rises. You randomly select 101 days from the past 20 years for the SET and find 46 positive days. You also sample 86 days from the past 20 years for the TWSE and find 39 positive days. You assume each day is independent of one another, the stock exchanges movements are independent and that the true unknown probably that the SET rises and the TWSE rises is constant although unknown for the past 20 years.

Test if the true probability the SET rises is *greater than* the TWSE. Use $\alpha = 0.05$. Do not use a continuity correction factor.

Solution:

$$
\begin{aligned}
\hat{p}_1 &= \tfrac{46}{101} = & 0.4554 \\
\hat{p}_2 &= \tfrac{39}{86} = & 0.4535 \\
\hat{p} &= \tfrac{46+39}{101+86} = & 0.4545
\end{aligned}
$$

$$
\begin{aligned}
z - statistic &= \frac{\hat{p}_1 - \hat{p}_2}{\sqrt{\hat{p}(1-\hat{p}) * (1/n_1 + 1/n_2)}} \\
&= \frac{0.4554 - 0.4535}{\sqrt{0.4545 * (1 - 0.4545) * 0.021529}} \\
&= 0.0268
\end{aligned}
$$

The p-value = 0.4893 and reject if p-value < 0.05

□

Problem 94. You are investigating the probability of the stock market having a positive day. A positive day is defined as the SET (Stock Exchange of Thailand) for $\hat{p}_1$ and for $\hat{p}_2$ the TWSE (Taiwan Stock Exchange) rises. You randomly select 95 days from the past 20 years for the SET and find 46 positive days. You also sample 91 days from the past 20 years for the TWSE and find 42 positive days. You assume each day is independent of one another, the stock exchanges movements are independent and that the true unknown probably that the SET rises and the TWSE rises is constant although unknown for the past 20 years.

Test if the true probability the SET rises is *greater than* the TWSE. Use $\alpha = 0.05$. Do not use a continuity correction factor.

Solution:

$$\begin{aligned}
\hat{p}_1 &= \tfrac{46}{95} = & 0.4842 \\
\hat{p}_2 &= \tfrac{42}{91} = & 0.4615 \\
\hat{p} &= \tfrac{46+42}{95+91} = & 0.4731
\end{aligned}$$

$$\begin{aligned}
z - statistic &= \tfrac{\hat{p}_1 - \hat{p}_2}{\sqrt{\hat{p}(1-\hat{p}) * (1/n_1 + 1/n_2)}} \\
&= \tfrac{0.4842 - 0.4615}{\sqrt{0.4731 * (1-0.4731) * 0.021515}} \\
&= 0.3096
\end{aligned}$$

The p-value $= 0.3784$ and reject if p-value < 0.05

□

Problem 95. You are investigating the probability of the stock market having a positive day. A positive day is defined as the SET (Stock Exchange of Thailand) for $\hat{p}_1$ and for $\hat{p}_2$ the TWSE (Taiwan Stock Exchange) rises. You randomly select 102 days from the past 20 years for the SET and find 57 positive days. You also sample 92 days from the past 20 years for the TWSE and find 41 positive days. You assume each day is independent of one another, the stock exchanges movements are independent and that the true unknown probably that the SET rises and the TWSE rises is constant although unknown for the past 20 years.

Test if the true probability the SET rises is *greater than* the TWSE. Use $\alpha = 0.05$. Do not use a continuity correction factor.

Solution:

$$\begin{aligned}
\hat{p}_1 &= \tfrac{57}{102} = & 0.5588 \\
\hat{p}_2 &= \tfrac{41}{92} = & 0.4457 \\
\hat{p} &= \tfrac{57+41}{102+92} = & 0.5052
\end{aligned}$$

$$\begin{aligned}
z - statistic &= \tfrac{\hat{p}_1 - \hat{p}_2}{\sqrt{\hat{p}(1-\hat{p})*(1/n_1+1/n_2)}} \\
&= \tfrac{0.5588-0.4457}{\sqrt{0.5052*(1-0.5052)*0.020673}} \\
&= 1.5743
\end{aligned}$$

The p-value = 0.0577 and reject if p-value < 0.05

□

Problem 96. You are investigating the probability of the stock market having a positive day. A positive day is defined as the SET (Stock Exchange of Thailand) for $\hat{p}_1$ and for $\hat{p}_2$ the TWSE (Taiwan Stock Exchange) rises. You randomly select 100 days from the past 20 years for the SET and find 54 positive days. You also sample 92 days from the past 20 years for the TWSE and find 46 positive days. You assume each day is independent of one another, the stock exchanges movements are independent and that the true unknown probably that the SET rises and the TWSE rises is constant although unknown for the past 20 years.

Test if the true probability the SET rises is *greater than* the TWSE. Use $\alpha = 0.05$. Do not use a continuity correction factor.

Solution:

$$\begin{aligned}
\hat{p}_1 &= \tfrac{54}{100} = & 0.54 \\
\hat{p}_2 &= \tfrac{46}{92} = & 0.5 \\
\hat{p} &= \tfrac{54+46}{100+92} = & 0.5208
\end{aligned}$$

$$\begin{aligned}
z - statistic &= \tfrac{\hat{p}_1 - \hat{p}_2}{\sqrt{\hat{p}(1-\hat{p})*(1/n_1+1/n_2)}} \\
&= \tfrac{0.54-0.5}{\sqrt{0.5208*(1-0.5208)*0.02087}} \\
&= 0.5543
\end{aligned}$$

The p-value $= 0.2897$ and reject if p-value < 0.05

□

Problem 97. You are investigating the probability of the stock market having a positive day. A positive day is defined as the SET (Stock Exchange of Thailand) for $\hat{p}_1$ and for $\hat{p}_2$ the TWSE (Taiwan Stock Exchange) rises. You randomly select 91 days from the past 20 years for the SET and find 49 positive days. You also sample 97 days from the past 20 years for the TWSE and find 47 positive days. You assume each day is independent of one another, the stock exchanges movements are independent and that the true unknown probably that the SET rises and the TWSE rises is constant although unknown for the past 20 years.

Test if the true probability the SET rises is *not equal to* the TWSE. Use $\alpha = 0.05$. Do not use a continuity correction factor.

Solution:

$$\begin{aligned}
\hat{p}_1 &= \tfrac{49}{91} = & 0.5385 \\
\hat{p}_2 &= \tfrac{47}{97} = & 0.4845 \\
\hat{p} &= \tfrac{49+47}{91+97} = & 0.5106
\end{aligned}$$

$$\begin{aligned}
z - statistic &= \tfrac{\hat{p}_1 - \hat{p}_2}{\sqrt{\hat{p}(1-\hat{p})*(1/n_1+1/n_2)}} \\
&= \tfrac{0.5385-0.4845}{\sqrt{0.5106*(1-0.5106)*0.021298}} \\
&= 0.7392
\end{aligned}$$

The p-value $= 0.4598$ and reject if p-value < 0.05

□

Problem 98. You are investigating the probability of the stock market having a positive day. A positive day is defined as the SET (Stock Exchange of Thailand) for $\hat{p}_1$ and for $\hat{p}_2$ the TWSE (Taiwan Stock Exchange) rises. You randomly select 107 days from the past 20 years for the SET and find 51 positive days. You also sample 112 days from the past 20 years for the TWSE and find 60 positive days. You assume each day is independent of one another, the stock exchanges movements are independent and that the true unknown probably that the SET rises and the TWSE rises is constant although unknown for the past 20 years.

Test if the true probability the SET rises is *not equal to* the TWSE. Use $\alpha = 0.05$. Do not use a continuity correction factor.

Solution:

$$\begin{aligned} \hat{p}_1 &= \tfrac{51}{107} = & 0.4766 \\ \hat{p}_2 &= \tfrac{60}{112} = & 0.5357 \\ \hat{p} &= \tfrac{51+60}{107+112} = & 0.5068 \end{aligned}$$

$$\begin{aligned} z - statistic &= \tfrac{\hat{p}_1 - \hat{p}_2}{\sqrt{\hat{p}(1-\hat{p})*(1/n_1+1/n_2)}} \\ &= \tfrac{0.4766-0.5357}{\sqrt{0.5068*(1-0.5068)*0.018274}} \\ &= -0.8741 \end{aligned}$$

The p-value = 0.382 and reject if p-value < 0.05

□

Problem 99. You are investigating the probability of the stock market having a positive day. A positive day is defined as the SET (Stock Exchange of Thailand) for $\hat{p}_1$ and for $\hat{p}_2$ the TWSE (Taiwan Stock Exchange) rises. You randomly select 97 days from the past 20 years for the SET and find 50 positive days. You also sample 94 days from the past 20 years for the TWSE and find 47 positive days. You assume each day is independent of one another, the stock exchanges movements are independent and that the true unknown probably that the SET rises and the TWSE rises is constant although unknown for the past 20 years.

Test if the true probability the SET rises is *greater than* the TWSE. Use $\alpha = 0.05$. Do not use a continuity correction factor.

Solution:

$$\begin{aligned} \hat{p}_1 &= \tfrac{50}{97} = & 0.5155 \\ \hat{p}_2 &= \tfrac{47}{94} = & 0.5 \\ \hat{p} &= \tfrac{50+47}{97+94} = & 0.5079 \end{aligned}$$

$$\begin{aligned} z - statistic &= \tfrac{\hat{p}_1 - \hat{p}_2}{\sqrt{\hat{p}(1-\hat{p}) * (1/n_1 + 1/n_2)}} \\ &= \tfrac{0.5155 - 0.5}{\sqrt{0.5079 * (1 - 0.5079) * 0.020948}} \\ &= 0.2137 \end{aligned}$$

The p-value = 0.4154 and reject if p-value < 0.05

□

Problem 100. You are investigating the probability of the stock market having a positive day. A positive day is defined as the SET (Stock Exchange of Thailand) for $\hat{p}_1$ and for $\hat{p}_2$ the TWSE (Taiwan Stock Exchange) rises. You randomly select 109 days from the past 20 years for the SET and find 47 positive days. You also sample 91 days from the past 20 years for the TWSE and find 43 positive days. You assume each day is independent of one another, the stock exchanges movements are independent and that the true unknown probably that the SET rises and the TWSE rises is constant although unknown for the past 20 years.

Test if the true probability the SET rises is *not equal to* the TWSE. Use $\alpha = 0.05$. Do not use a continuity correction factor.

Solution:

$$\begin{aligned}
\hat{p}_1 &= \tfrac{47}{109} = & 0.4312 \\
\hat{p}_2 &= \tfrac{43}{91} = & 0.4725 \\
\hat{p} &= \tfrac{47+43}{109+91} = & 0.45
\end{aligned}$$

$$\begin{aligned}
z - statistic &= \tfrac{\hat{p}_1 - \hat{p}_2}{\sqrt{\hat{p}(1-\hat{p})*(1/n_1+1/n_2)}} \\
&= \tfrac{0.4312-0.4725}{\sqrt{0.45*(1-0.45)*0.020163}} \\
&= -0.5851
\end{aligned}$$

The p-value = 0.5585 and reject if p-value < 0.05

□

Problem 101. You are investigating the probability of the stock market having a positive day. A positive day is defined as the SET (Stock Exchange of Thailand) for $\hat{p}_1$ and for $\hat{p}_2$ the TWSE (Taiwan Stock Exchange) rises. You randomly select 109 days from the past 20 years for the SET and find 54 positive days. You also sample 94 days from the past 20 years for the TWSE and find 44 positive days. You assume each day is independent of one another, the stock exchanges movements are independent and that the true unknown probably that the SET rises and the TWSE rises is constant although unknown for the past 20 years.

Test if the true probability the SET rises is *greater than* the TWSE. Use $\alpha = 0.05$. Do not use a continuity correction factor.

Solution:

$$\begin{aligned}
\hat{p}_1 &= \tfrac{54}{109} = & 0.4954 \\
\hat{p}_2 &= \tfrac{44}{94} = & 0.4681 \\
\hat{p} &= \tfrac{54+44}{109+94} = & 0.4828
\end{aligned}$$

$$\begin{aligned}
z - statistic &= \tfrac{\hat{p}_1 - \hat{p}_2}{\sqrt{\hat{p}(1-\hat{p}) * (1/n_1 + 1/n_2)}} \\
&= \tfrac{0.4954 - 0.4681}{\sqrt{0.4828 * (1 - 0.4828) * 0.019813}} \\
&= 0.3885
\end{aligned}$$

The p-value = 0.3488 and reject if p-value < 0.05

$\square$

Problem 102. You are investigating the probability of the stock market having a positive day. A positive day is defined as the SET (Stock Exchange of Thailand) for $\hat{p}_1$ and for $\hat{p}_2$ the TWSE (Taiwan Stock Exchange) rises. You randomly select 96 days from the past 20 years for the SET and find 43 positive days. You also sample 97 days from the past 20 years for the TWSE and find 55 positive days. You assume each day is independent of one another, the stock exchanges movements are independent and that the true unknown probably that the SET rises and the TWSE rises is constant although unknown for the past 20 years.

Test if the true probability the SET rises is *not equal to* the TWSE. Use $\alpha = 0.05$. Do not use a continuity correction factor.

Solution:

$$\begin{aligned} \hat{p}_1 &= \tfrac{43}{96} = & 0.4479 \\ \hat{p}_2 &= \tfrac{55}{97} = & 0.567 \\ \hat{p} &= \tfrac{43+55}{96+97} = & 0.5078 \end{aligned}$$

$$\begin{aligned} z - statistic &= \tfrac{\hat{p}_1 - \hat{p}_2}{\sqrt{\hat{p}(1-\hat{p})*(1/n_1+1/n_2)}} \\ &= \tfrac{0.4479-0.567}{\sqrt{0.5078*(1-0.5078)*0.020726}} \\ &= -1.6547 \end{aligned}$$

The p-value = 0.098 and reject if p-value < 0.05

□

Problem 103. You are investigating the probability of the stock market having a positive day. A positive day is defined as the SET (Stock Exchange of Thailand) for $\hat{p}_1$ and for $\hat{p}_2$ the TWSE (Taiwan Stock Exchange) rises. You randomly select 97 days from the past 20 years for the SET and find 51 positive days. You also sample 107 days from the past 20 years for the TWSE and find 50 positive days. You assume each day is independent of one another, the stock exchanges movements are independent and that the true unknown probably that the SET rises and the TWSE rises is constant although unknown for the past 20 years.

Test if the true probability the SET rises is *greater than* the TWSE. Use $\alpha = 0.05$. Do not use a continuity correction factor.

Solution:

$$\begin{aligned} \hat{p}_1 &= \tfrac{51}{97} = & 0.5258 \\ \hat{p}_2 &= \tfrac{50}{107} = & 0.4673 \\ \hat{p} &= \tfrac{51+50}{97+107} = & 0.4951 \end{aligned}$$

$$\begin{aligned} z - statistic &= \tfrac{\hat{p}_1 - \hat{p}_2}{\sqrt{\hat{p}(1-\hat{p})*(1/n_1+1/n_2)}} \\ &= \tfrac{0.5258-0.4673}{\sqrt{0.4951*(1-0.4951)*0.019655}} \\ &= 0.8343 \end{aligned}$$

The p-value = 0.202 and reject if p-value < 0.05

$\square$

Problem 104. You are investigating the probability of the stock market having a positive day. A positive day is defined as the SET (Stock Exchange of Thailand) for $\hat{p}_1$ and for $\hat{p}_2$ the TWSE (Taiwan Stock Exchange) rises. You randomly select 104 days from the past 20 years for the SET and find 53 positive days. You also sample 98 days from the past 20 years for the TWSE and find 49 positive days. You assume each day is independent of one another, the stock exchanges movements are independent and that the true unknown probably that the SET rises and the TWSE rises is constant although unknown for the past 20 years.

Test if the true probability the SET rises is *greater than* the TWSE. Use $\alpha = 0.05$. Do not use a continuity correction factor.

Solution:

$$\begin{aligned}
\hat{p}_1 &= \tfrac{53}{104} = & 0.5096 \\
\hat{p}_2 &= \tfrac{49}{98} = & 0.5 \\
\hat{p} &= \tfrac{53+49}{104+98} = & 0.505
\end{aligned}$$

$$\begin{aligned}
z - statistic &= \tfrac{\hat{p}_1 - \hat{p}_2}{\sqrt{\hat{p}(1-\hat{p})*(1/n_1+1/n_2)}} \\
&= \tfrac{0.5096-0.5}{\sqrt{0.505*(1-0.505)*0.019819}} \\
&= 0.1366
\end{aligned}$$

The p-value $= 0.4457$ and reject if p-value < 0.05

□

Problem 105. You are investigating the probability of the stock market having a positive day. A positive day is defined as the SET (Stock Exchange of Thailand) for $\hat{p}_1$ and for $\hat{p}_2$ the TWSE (Taiwan Stock Exchange) rises. You randomly select 93 days from the past 20 years for the SET and find 41 positive days. You also sample 109 days from the past 20 years for the TWSE and find 53 positive days. You assume each day is independent of one another, the stock exchanges movements are independent and that the true unknown probably that the SET rises and the TWSE rises is constant although unknown for the past 20 years.

Test if the true probability the SET rises is *less than* the TWSE. Use $\alpha = 0.05$. Do not use a continuity correction factor.

Solution:

$$\begin{aligned} \hat{p}_1 &= \tfrac{41}{93} = & 0.4409 \\ \hat{p}_2 &= \tfrac{53}{109} = & 0.4862 \\ \hat{p} &= \tfrac{41+53}{93+109} = & 0.4653 \end{aligned}$$

$$\begin{aligned} z - statistic &= \tfrac{\hat{p}_1 - \hat{p}_2}{\sqrt{\hat{p}(1-\hat{p})*(1/n_1+1/n_2)}} \\ &= \tfrac{0.4409-0.4862}{\sqrt{0.4653*(1-0.4653)*0.019927}} \\ &= -0.6445 \end{aligned}$$

The p-value = 0.2596 and reject if p-value $<$ 0.05

□

Problem 106. You are investigating the probability of the stock market having a positive day. A positive day is defined as the SET (Stock Exchange of Thailand) for $\hat{p}_1$ and for $\hat{p}_2$ the TWSE (Taiwan Stock Exchange) rises. You randomly select 101 days from the past 20 years for the SET and find 51 positive days. You also sample 109 days from the past 20 years for the TWSE and find 49 positive days. You assume each day is independent of one another, the stock exchanges movements are independent and that the true unknown probably that the SET rises and the TWSE rises is constant although unknown for the past 20 years.

Test if the true probability the SET rises is *greater than* the TWSE. Use $\alpha = 0.05$. Do not use a continuity correction factor.

Solution:

$$\begin{aligned} \hat{p}_1 &= \tfrac{51}{101} = & 0.505 \\ \hat{p}_2 &= \tfrac{49}{109} = & 0.4495 \\ \hat{p} &= \tfrac{51+49}{101+109} = & 0.4762 \end{aligned}$$

$$\begin{aligned} z - statistic &= \tfrac{\hat{p}_1 - \hat{p}_2}{\sqrt{\hat{p}(1-\hat{p})*(1/n_1+1/n_2)}} \\ &= \tfrac{0.505-0.4495}{\sqrt{0.4762*(1-0.4762)*0.019075}} \\ &= 0.8033 \end{aligned}$$

The p-value $= 0.2109$ and reject if p-value < 0.05

□

Problem 107. You are investigating the probability of the stock market having a positive day. A positive day is defined as the SET (Stock Exchange of Thailand) for $\hat{p}_1$ and for $\hat{p}_2$ the TWSE (Taiwan Stock Exchange) rises. You randomly select 110 days from the past 20 years for the SET and find 51 positive days. You also sample 101 days from the past 20 years for the TWSE and find 45 positive days. You assume each day is independent of one another, the stock exchanges movements are independent and that the true unknown probably that the SET rises and the TWSE rises is constant although unknown for the past 20 years.

Test if the true probability the SET rises is *not equal to* the TWSE. Use $\alpha = 0.05$. Do not use a continuity correction factor.

Solution:

$$
\begin{aligned}
\hat{p}_1 &= \tfrac{51}{110} = & 0.4636 \\
\hat{p}_2 &= \tfrac{45}{101} = & 0.4455 \\
\hat{p} &= \tfrac{51+45}{110+101} = & 0.455
\end{aligned}
$$

$$
\begin{aligned}
z - statistic &= \tfrac{\hat{p}_1 - \hat{p}_2}{\sqrt{\hat{p}(1-\hat{p})*(1/n_1+1/n_2)}} \\
&= \tfrac{0.4636-0.4455}{\sqrt{0.455*(1-0.455)*0.018992}} \\
&= 0.2636
\end{aligned}
$$

The p-value $= 0.7921$ and reject if p-value < 0.05

$\square$

Problem 108. You are investigating the probability of the stock market having a positive day. A positive day is defined as the SET (Stock Exchange of Thailand) for $\hat{p}_1$ and for $\hat{p}_2$ the TWSE (Taiwan Stock Exchange) rises. You randomly select 96 days from the past 20 years for the SET and find 42 positive days. You also sample 100 days from the past 20 years for the TWSE and find 54 positive days. You assume each day is independent of one another, the stock exchanges movements are independent and that the true unknown probably that the SET rises and the TWSE rises is constant although unknown for the past 20 years.

Test if the true probability the SET rises is *less than* the TWSE. Use $\alpha = 0.05$. Do not use a continuity correction factor.

Solution:

$$\begin{aligned} \hat{p}_1 &= \tfrac{42}{96} = & 0.4375 \\ \hat{p}_2 &= \tfrac{54}{100} = & 0.54 \\ \hat{p} &= \tfrac{42+54}{96+100} = & 0.4898 \end{aligned}$$

$$\begin{aligned} z - statistic &= \tfrac{\hat{p}_1 - \hat{p}_2}{\sqrt{\hat{p}(1-\hat{p})*(1/n_1+1/n_2)}} \\ &= \tfrac{0.4375-0.54}{\sqrt{0.4898*(1-0.4898)*0.020417}} \\ &= -1.435 \end{aligned}$$

The p-value = 0.0756 and reject if p-value < 0.05

□

1.7 One-Sample T-test Examples

Problem 109. Use the data below to answer the following questions.

	y
1	503.70
2	506.12
3	509.83
4	506.00

Table 37: Randomly selected bank savings account data in U.S. dollars

Test if the population mean is *not equal to* $\mu = 507.73$. Use $\alpha = 0.05$.

Solution:

$$t - statistic = \frac{\bar{x} - \mu}{s/\sqrt{n}} = \frac{506.4125 - 507.73}{2.5359/\sqrt{4}} = -1.0391$$

The p-value $= 0.3752$ and reject if p-value < 0.05 $\square$

Problem 110. Use the data below to answer the following questions.

	y
1	495.93
2	491.79
3	500.85
4	496.28
5	470.76

Table 38: Randomly selected bank savings account data in U.S. dollars

Test if the population mean is *greater than* $\mu = 495.38$. Use $\alpha = 0.05$.

Solution:

$$t - statistic = \frac{\bar{x} - \mu}{s/\sqrt{n}} = \frac{491.122 - 495.38}{11.826/\sqrt{5}} = -0.8051$$

The p-value $= 0.7671$ and reject if p-value < 0.05 $\square$

Problem 111. Use the data below to answer the following questions.

	y
1	521.01
2	497.16
3	482.61
4	490.04
5	497.28
6	503.88

Table 39: Randomly selected bank savings account data in U.S. dollars

Test if the population mean is *not equal to* $\mu = 488.62$. Use $\alpha = 0.05$.

Solution:

$$t - statistic = \frac{\bar{x} - \mu}{s/\sqrt{n}} = \frac{498.6633 - 488.62}{13.1365/\sqrt{6}} = 1.8727$$

The p-value $= 0.12$ and reject if p-value < 0.05 $\square$

Problem 112. Use the data below to answer the following questions.

	y
1	488.97
2	512.83
3	486.40
4	488.01
5	493.17
6	484.79
7	517.16
8	503.48
9	515.18

Table 40: Randomly selected bank savings account data in U.S. dollars

Test if the population mean is *not equal to* $\mu = 476.27$. Use $\alpha = 0.05$.

Solution:

$$t - statistic = \frac{\bar{x} - \mu}{s/\sqrt{n}} = \frac{498.8878 - 476.27}{13.3176/\sqrt{9}} = 5.095$$

The p-value $= 9e - 04$ and reject if p-value < 0.05 $\square$

Problem 113. Use the data below to answer the following questions.

	y
1	491.90
2	502.38
3	500.61
4	487.64
5	499.77
6	496.66
7	508.78

Table 41: Randomly selected bank savings account data in U.S. dollars

Test if the population mean is *greater than* $\mu = 486.86$. Use $\alpha = 0.05$.

Solution:

$$t - statistic = \frac{\bar{x} - \mu}{s/\sqrt{n}} = \frac{498.2486 - 486.86}{6.9671/\sqrt{7}} = 4.3248$$

The p-value = 0.0025 and reject if p-value < 0.05 □

Problem 114. Use the data below to answer the following questions.

	y
1	502.88
2	504.55
3	496.29
4	496.48
5	504.66

Table 42: Randomly selected bank savings account data in U.S. dollars

Test if the population mean is *less than* $\mu = 493.54$. Use $\alpha = 0.05$.

Solution:

$$t - statistic = \frac{\bar{x} - \mu}{s/\sqrt{n}} = \frac{500.972 - 493.54}{4.2469/\sqrt{5}} = 3.9131$$

The p-value = 0.9913 and reject if p-value < 0.05 □

Problem 115. Use the data below to answer the following questions.

	y
1	502.78
2	522.77
3	494.47
4	488.39
5	491.74
6	503.71

Table 43: Randomly selected bank savings account data in U.S. dollars

Test if the population mean is *not equal to* $\mu = 514.62$. Use $\alpha = 0.05$.

Solution:

$$t - statistic = \frac{\bar{x} - \mu}{s/\sqrt{n}} = \frac{500.6433 - 514.62}{12.4184/\sqrt{6}} = -2.7569$$

The p-value = 0.04 and reject if p-value < 0.05 $\square$

Problem 116. Use the data below to answer the following questions.

	y
1	491.18
2	491.56
3	506.02
4	495.00
5	505.77
6	506.40
7	482.66

Table 44: Randomly selected bank savings account data in U.S. dollars

Test if the population mean is *not equal to* $\mu = 514.42$. Use $\alpha = 0.05$.

Solution:

$$t - statistic = \frac{\bar{x} - \mu}{s/\sqrt{n}} = \frac{496.9414 - 514.42}{9.3069/\sqrt{7}} = -4.9688$$

The p-value = 0.0025 and reject if p-value < 0.05 $\square$

Problem 117. Use the data below to answer the following questions.

	y
1	510.91
2	500.78
3	482.34

Table 45: Randomly selected bank savings account data in U.S. dollars

Test if the population mean is *not equal to* $\mu = 457.63$. Use $\alpha = 0.05$.

Solution:

$$t - statistic = \frac{\bar{x} - \mu}{s/\sqrt{n}} = \frac{498.01 - 457.63}{14.485/\sqrt{3}} = 4.8285$$

The p-value = 0.0403 and reject if p-value < 0.05 $\square$

Problem 118. Use the data below to answer the following questions.

	y
1	500.38
2	488.29
3	510.16
4	504.22
5	498.45
6	501.35

Table 46: Randomly selected bank savings account data in U.S. dollars

Test if the population mean is *greater than* $\mu = 509.98$. Use $\alpha = 0.05$.

Solution:

$$t - statistic = \frac{\bar{x} - \mu}{s/\sqrt{n}} = \frac{500.475 - 509.98}{7.2274/\sqrt{6}} = -3.2214$$

The p-value = 0.9883 and reject if p-value < 0.05 $\square$

Problem 119. Use the data below to answer the following questions.

	y
1	497.32
2	501.21
3	502.19
4	487.42
5	512.20
6	482.14
7	503.43
8	492.31
9	502.91
10	501.72
11	495.04
12	503.42
13	510.71
14	484.57

Table 47: Randomly selected bank savings account data in U.S. dollars

Test if the population mean is *less than* $\mu = 506.42$. Use $\alpha = 0.05$.

Solution:

$$t - statistic = \frac{\bar{x} - \mu}{s/\sqrt{n}} = \frac{498.3279 - 506.42}{9.0799/\sqrt{14}} = -3.3346$$

The p-value = 0.0027 and reject if p-value < 0.05 $\square$

Problem 120. Use the data below to answer the following questions.

	y
1	493.67
2	499.32
3	498.37
4	491.68
5	499.57

Table 48: Randomly selected bank savings account data in U.S. dollars

Test if the population mean is *greater than* $\mu = 486.32$. Use $\alpha = 0.05$.

Solution:

$$t - statistic = \frac{\bar{x} - \mu}{s/\sqrt{n}} = \frac{496.522 - 486.32}{3.6095/\sqrt{5}} = 6.3201$$

The p-value $= 0.0016$ and reject if p-value < 0.05 $\square$

Problem 121. Use the data below to answer the following questions.

	y
1	506.01
2	511.21
3	498.97
4	492.84
5	511.84
6	510.72

Table 49: Randomly selected bank savings account data in U.S. dollars

Test if the population mean is *not equal to* $\mu = 510.58$. Use $\alpha = 0.05$.

Solution:

$$t - statistic = \frac{\bar{x} - \mu}{s/\sqrt{n}} = \frac{505.265 - 510.58}{7.7832/\sqrt{6}} = -1.6727$$

The p-value $= 0.1552$ and reject if p-value < 0.05 $\square$

Problem 122. Use the data below to answer the following questions.

	y
1	499.56
2	484.87
3	504.77
4	511.55
5	509.35

Table 50: Randomly selected bank savings account data in U.S. dollars

Test if the population mean is *not equal to* $\mu = 491.69$. Use $\alpha = 0.05$.

Solution:

$$t - statistic = \frac{\bar{x} - \mu}{s/\sqrt{n}} = \frac{502.02 - 491.69}{10.6335/\sqrt{5}} = 2.1722$$

The p-value = 0.0956 and reject if p-value < 0.05 $\square$

Problem 123. Use the data below to answer the following questions.

	y
1	502.22
2	496.09
3	506.88
4	518.43

Table 51: Randomly selected bank savings account data in U.S. dollars

Test if the population mean is *not equal to* $\mu = 490.03$. Use $\alpha = 0.05$.

Solution:

$$t - statistic = \frac{\bar{x} - \mu}{s/\sqrt{n}} = \frac{505.905 - 490.03}{9.447/\sqrt{4}} = 3.3608$$

The p-value = 0.0437 and reject if p-value < 0.05 $\square$

Problem 124. Use the data below to answer the following questions.

	y
1	498.13
2	497.80
3	482.43
4	522.53
5	487.48

Table 52: Randomly selected bank savings account data in U.S. dollars

Test if the population mean is *not equal to* $\mu = 499.55$. Use $\alpha = 0.05$.

Solution:

$$t - statistic = \frac{\bar{x} - \mu}{s/\sqrt{n}} = \frac{497.674 - 499.55}{15.4462/\sqrt{5}} = -0.2716$$

The p-value $= 0.7994$ and reject if p-value < 0.05 □

Problem 125. Use the data below to answer the following questions.

	y
1	491.50
2	487.01
3	517.27

Table 53: Randomly selected bank savings account data in U.S. dollars

Test if the population mean is *not equal to* $\mu = 483.67$. Use $\alpha = 0.05$.

Solution:

$$t - statistic = \frac{\bar{x} - \mu}{s/\sqrt{n}} = \frac{498.5933 - 483.67}{16.3295/\sqrt{3}} = 1.5829$$

The p-value = 0.2543 and reject if p-value < 0.05 □

Problem 126. Use the data below to answer the following questions.

	y
1	489.29
2	507.99
3	511.04
4	515.16

Table 54: Randomly selected bank savings account data in U.S. dollars

Test if the population mean is *not equal to* $\mu = 494.75$. Use $\alpha = 0.05$.

Solution:

$$t - statistic = \frac{\bar{x} - \mu}{s/\sqrt{n}} = \frac{505.87 - 494.75}{11.4371/\sqrt{4}} = 1.9445$$

The p-value = 0.1471 and reject if p-value < 0.05 $\square$

1.8 Two Sample T-test - Equal variances assumed Examples

Problem 127. Use the data below to answer the following questions. Assume the data comes from a normal distribution. Also assume the variances are equal.

	y
1	7334.47
2	7535.39
3	7860.15
4	7624.32
5	7586.52
6	7581.16
7	7144.51

Table 55: Randomly selected bank savings account data in U.S. dollars of women.

	x
1	7648.25
2	8429.93
3	7955.88
4	7610.89
5	7597.50
6	7470.67
7	7767.20
8	7565.33
9	8029.09

Table 56: Randomly selected bank savings account data in U.S. dollars of men.

Test if the population mean savings of women is *not equal to* men. Use $\alpha = 0.05$.

Solution:

Variable	**Levels**	**n**	average	s^2	Sum of Squares (SS)
thedat	Female	7	7523.7886	51716.7735	310300.6411
	Male	9	7786.0822	92031.2836	736250.2690
	all	16	7671.3287	87829.5250	1317442.8746

Table 57: Savings of men and women with descriptive statistics.

$$s_p = \sqrt{\frac{(n_1-1)s_1^2 + (n_2-1)s_2^2}{n_1+n_2-2}} = \sqrt{\frac{(7-1)*51716.7735 + (9-1)*92031.2836}{7+9-2}} = 273.4111$$

$$t = \frac{\bar{x}_1 - \bar{x}_2}{s_p\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}} = \frac{7523.7886 - 7786.0822}{273.4111\sqrt{\frac{1}{7}+\frac{1}{9}}} = -1.9036$$

The degrees of freedom $= n_1 + n_2 - 2 = 14$. The p-value $= 0.0777$ and reject if p-value < 0.05 □

Problem 128. Use the data below to answer the following questions. Assume the data comes from a normal distribution. Also assume the variances are equal.

	y
1	7291.63
2	7628.50
3	7377.57
4	7507.96
5	7437.07
6	7745.26
7	7544.37
8	7551.28
9	7323.37

Table 58: Randomly selected bank savings account data in U.S. dollars of women.

	x
1	7258.15
2	7778.20
3	7588.29
4	7435.82
5	7602.18

Table 59: Randomly selected bank savings account data in U.S. dollars of men.

Test if the population mean savings of women is *less than* men. Use $\alpha = 0.05$.

Solution:

Variable	**Levels**	**n**	average	s^2	Sum of Squares (SS)
thedat	Female	9	7489.6678	21741.3906	173931.1248
	Male	5	7532.5280	38237.8144	152951.2575
	all	14	7504.9750	25599.0016	332787.0208

Table 60: Savings of men and women with descriptive statistics.

$$s_p = \sqrt{\frac{(n_1-1)s_1^2 + (n_2-1)s_2^2}{n_1+n_2-2}} = \sqrt{\frac{(9-1)*21741.3906 + (5-1)*38237.8144}{9+5-2}} = 165.046$$

$$t = \frac{\bar{x}_1 - \bar{x}_2}{s_p\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}} = \frac{7489.6678 - 7532.528}{165.046\sqrt{\frac{1}{9}+\frac{1}{5}}} = -0.4656$$

The degrees of freedom $= n_1 + n_2 - 2 = 12$. The p-value $= 0.3249$ and reject if p-value < 0.05 $\square$

Problem 129. Use the data below to answer the following questions. Assume the data comes from a normal distribution. Also assume the variances are equal.

	y
1	7454.15
2	7934.17
3	7560.18

Table 61: Randomly selected bank savings account data in U.S. dollars of women.

	x
1	7629.47
2	7289.62
3	7330.98
4	7692.76

Table 62: Randomly selected bank savings account data in U.S. dollars of men.

Test if the population mean savings of women is *greater than* men. Use $\alpha = 0.05$.
Solution:

Variable	**Levels**	**n**	average	s^2	Sum of Squares (SS)
thedat	Female	3	7649.5000	63588.3469	127176.6938
	Male	4	7485.7075	41976.4337	125929.3011
	all	7	7555.9043	49849.4705	299096.8230

Table 63: Savings of men and women with descriptive statistics.

$$s_p = \sqrt{\frac{(n_1-1)s_1^2 + (n_2-1)s_2^2}{n_1+n_2-2}} = \sqrt{\frac{(3-1)*63588.3469 + (4-1)*41976.4337}{3+4-2}} = 224.9916$$

$$t = \frac{\bar{x}_1 - \bar{x}_2}{s_p\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}} = \frac{7649.5 - 7485.7075}{224.9916\sqrt{\frac{1}{3}+\frac{1}{4}}} = 0.9532$$

The degrees of freedom $= n_1 + n_2 - 2 = 5$. The p-value $= 0.1921$ and reject if p-value < 0.05 $\square$

Problem 130. Use the data below to answer the following questions. Assume the data comes from a normal distribution. Also assume the variances are equal.

	y
1	7580.18
2	7504.75
3	7397.05
4	7571.72
5	7305.87
6	7151.03

Table 64: Randomly selected bank savings account data in U.S. dollars of women.

	x
1	7389.41
2	7967.91
3	7577.15
4	7532.09

Table 65: Randomly selected bank savings account data in U.S. dollars of men.

Test if the population mean savings of women is *less than* men. Use $\alpha = 0.05$.
Solution:

Variable	**Levels**	**n**	average	s^2	Sum of Squares (SS)
thedat	Female	6	7418.4333	28348.3294	141741.6469
	Male	4	7616.6400	61244.0828	183732.2484
	all	10	7497.7160	46640.0015	419760.0138

Table 66: Savings of men and women with descriptive statistics.

$$s_p = \sqrt{\frac{(n_1-1)s_1^2 + (n_2-1)s_2^2}{n_1+n_2-2}} = \sqrt{\frac{(6-1)*28348.3294 + (4-1)*61244.0828}{6+4-2}} = 201.7033$$

$$t = \frac{\bar{x}_1 - \bar{x}_2}{s_p\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}} = \frac{7418.4333 - 7616.64}{201.7033\sqrt{\frac{1}{6}+\frac{1}{4}}} = -1.5223$$

The degrees of freedom $= n_1 + n_2 - 2 = 8$. The p-value $= 0.0832$ and reject if p-value < 0.05 $\square$

Problem 131. Use the data below to answer the following questions. Assume the data comes from a normal distribution. Also assume the variances are equal.

	y
1	7494.23
2	7680.58
3	7713.14
4	7287.35
5	8088.99

Table 67: Randomly selected bank savings account data in U.S. dollars of women.

	x
1	8069.19
2	7443.07
3	7860.28
4	7872.33
5	8069.96
6	7523.39
7	7895.39
8	7517.39

Table 68: Randomly selected bank savings account data in U.S. dollars of men.

Test if the population mean savings of women is *greater than* men. Use $\alpha = 0.05$.
Solution:

Variable	**Levels**	**n**	average	s^2	Sum of Squares (SS)
thedat	Female	5	7652.8580	88343.1227	353372.4907
	Male	8	7781.3750	63473.0887	444311.6212
	all	13	7731.9454	70708.7066	848504.4789

Table 69: Savings of men and women with descriptive statistics.

$$s_p = \sqrt{\frac{(n_1-1)s_1^2 + (n_2-1)s_2^2}{n_1+n_2-2}} = \sqrt{\frac{(5-1)*88343.1227 + (8-1)*63473.0887}{5+8-2}} = 269.2893$$

$$t = \frac{\bar{x}_1 - \bar{x}_2}{s_p\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}} = \frac{7652.858 - 7781.375}{269.2893\sqrt{\frac{1}{5}+\frac{1}{8}}} = -0.8371$$

The degrees of freedom $= n_1 + n_2 - 2 = 11$. The p-value $= 0.7898$ and reject if p-value < 0.05 $\square$

Problem 132. Use the data below to answer the following questions. Assume the data comes from a normal distribution. Also assume the variances are equal.

	y
1	7361.83
2	7195.10
3	7707.37
4	7394.57
5	7513.74
6	7544.16

Table 70: Randomly selected bank savings account data in U.S. dollars of women.

	x
1	7934.76
2	7423.54
3	7439.84
4	7464.08
5	7761.71
6	7585.64

Table 71: Randomly selected bank savings account data in U.S. dollars of men.

Test if the population mean savings of women is *less than* men. Use $\alpha = 0.05$.
Solution:

Variable	**Levels**	**n**	average	s^2	Sum of Squares (SS)
thedat	Female	6	7452.7950	30988.3563	154941.7817
	Male	6	7601.5950	42733.7862	213668.9308
	all	12	7527.1950	39548.6393	435035.0325

Table 72: Savings of men and women with descriptive statistics.

$$s_p = \sqrt{\frac{(n_1-1)s_1^2 + (n_2-1)s_2^2}{n_1+n_2-2}} = \sqrt{\frac{(6-1)*30988.3563 + (6-1)*42733.7862}{6+6-2}} = 191.9924$$

$$t = \frac{\bar{x}_1 - \bar{x}_2}{s_p\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}} = \frac{7452.795 - 7601.595}{191.9924\sqrt{\frac{1}{6}+\frac{1}{6}}} = -1.3424$$

The degrees of freedom $= n_1 + n_2 - 2 = 10$. The p-value $= 0.1046$ and reject if p-value < 0.05 $\square$

Problem 133. Use the data below to answer the following questions. Assume the data comes from a normal distribution. Also assume the variances are equal.

	y
1	7600.19
2	7646.54
3	7612.56
4	7201.34

Table 73: Randomly selected bank savings account data in U.S. dollars of women.

	x
1	7611.22
2	7442.58
3	7992.76
4	7609.64
5	7405.74
6	7554.90
7	7410.78

Table 74: Randomly selected bank savings account data in U.S. dollars of men.

Test if the population mean savings of women is *less than* men. Use $\alpha = 0.05$.

Solution:

Variable	**Levels**	**n**	average	s^2	Sum of Squares (SS)
thedat	Female	4	7515.1575	44153.5192	132460.5577
	Male	7	7575.3743	41765.0796	250590.4774
	all	11	7553.4773	39228.1009	392281.0092

Table 75: Savings of men and women with descriptive statistics.

$$s_p = \sqrt{\frac{(n_1-1)s_1^2 + (n_2-1)s_2^2}{n_1+n_2-2}} = \sqrt{\frac{(4-1)*44153.5192 + (7-1)*41765.0796}{4+7-2}} = 206.3037$$

$$t = \frac{\bar{x}_1 - \bar{x}_2}{s_p\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}} = \frac{7515.1575 - 7575.3743}{206.3037\sqrt{\frac{1}{4}+\frac{1}{7}}} = -0.4657$$

The degrees of freedom $= n_1 + n_2 - 2 = 9$. The p-value $= 0.3263$ and reject if p-value < 0.05 $\square$

Problem 134. Use the data below to answer the following questions. Assume the data comes from a normal distribution. Also assume the variances are equal.

	y
1	7337.53
2	7212.56
3	7267.81
4	7586.44
5	7668.40
6	7305.24

Table 76: Randomly selected bank savings account data in U.S. dollars of women.

	x
1	7557.62
2	7483.56
3	7805.23
4	7227.38
5	7524.98

Table 77: Randomly selected bank savings account data in U.S. dollars of men.

Test if the population mean savings of women is *less than* men. Use $\alpha = 0.05$.

Solution:

Variable	**Levels**	**n**	average	s^2	Sum of Squares (SS)
thedat	Female	6	7396.3300	34441.5057	172207.5284
	Male	5	7519.7540	42437.5633	169750.2531
	all	11	7452.4318	38350.3646	383503.6464

Table 78: Savings of men and women with descriptive statistics.

$$s_p = \sqrt{\frac{(n_1-1)s_1^2 + (n_2-1)s_2^2}{n_1+n_2-2}} = \sqrt{\frac{(6-1)*34441.5057 + (5-1)*42437.5633}{6+5-2}} = 194.9239$$

$$t = \frac{\bar{x}_1 - \bar{x}_2}{s_p\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}} = \frac{7396.33 - 7519.754}{194.9239\sqrt{\frac{1}{6}+\frac{1}{5}}} = -1.0457$$

The degrees of freedom $= n_1 + n_2 - 2 = 9$. The p-value $= 0.1615$ and reject if p-value < 0.05 $\square$

Problem 135. Use the data below to answer the following questions. Assume the data comes from a normal distribution. Also assume the variances are equal.

	y
1	7289.21
2	7377.72

Table 79: Randomly selected bank savings account data in U.S. dollars of women.

	x
1	7411.94
2	7728.33
3	7587.07
4	7903.65
5	7315.69
6	7636.12
7	7588.46

Table 80: Randomly selected bank savings account data in U.S. dollars of men.

Test if the population mean savings of women is *less than* men. Use $\alpha = 0.05$.

Solution:

Variable	**Levels**	**n**	average	s^2	Sum of Squares (SS)
thedat	Female	2	7333.4650	3917.0101	3917.0101
	Male	7	7595.8943	37726.2773	226357.6638
	all	9	7537.5767	42175.5539	337404.4316

Table 81: Savings of men and women with descriptive statistics.

$$s_p = \sqrt{\frac{(n_1-1)s_1^2 + (n_2-1)s_2^2}{n_1+n_2-2}} = \sqrt{\frac{(2-1)*3917.0101 + (7-1)*37726.2773}{2+7-2}} = 181.3736$$

$$t = \frac{\bar{x}_1 - \bar{x}_2}{s_p\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}} = \frac{7333.465 - 7595.8943}{181.3736\sqrt{\frac{1}{2}+\frac{1}{7}}} = -1.8046$$

The degrees of freedom $= n_1 + n_2 - 2 = 7$. The p-value $= 0.0571$ and reject if p-value < 0.05 $\square$

Problem 136. Use the data below to answer the following questions. Assume the data comes from a normal distribution. Also assume the variances are equal.

	y
1	7193.22
2	7396.11
3	7195.90
4	7542.17
5	7296.16
6	7527.38
7	7185.09
8	7605.11
9	7571.50

Table 82: Randomly selected bank savings account data in U.S. dollars of women.

	x
1	7708.10
2	7508.19
3	8131.92
4	7875.43
5	7644.54
6	7542.00
7	7680.40

Table 83: Randomly selected bank savings account data in U.S. dollars of men.

Test if the population mean savings of women is *less than* men. Use $\alpha = 0.05$.

Solution:

Variable	**Levels**	**n**	average	s^2	Sum of Squares (SS)
thedat	Female	9	7390.2933	31058.9156	248471.3252
	Male	7	7727.2257	46237.0921	277422.5524
	all	16	7537.7013	64859.4920	972892.3806

Table 84: Savings of men and women with descriptive statistics.

$$s_p = \sqrt{\frac{(n_1-1)s_1^2 + (n_2-1)s_2^2}{n_1+n_2-2}} = \sqrt{\frac{(9-1)*31058.9156 + (7-1)*46237.0921}{9+7-2}} = 193.814$$

$$t = \frac{\bar{x}_1 - \bar{x}_2}{s_p\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}} = \frac{7390.2933 - 7727.2257}{193.814\sqrt{\frac{1}{9}+\frac{1}{7}}} = -3.4496$$

The degrees of freedom $= n_1 + n_2 - 2 = 14$. The p-value $= 0.002$ and reject if p-value < 0.05 $\square$

Problem 137. Use the data below to answer the following questions. Assume the data comes from a normal distribution. Also assume the variances are equal.

	y
1	7397.17
2	7361.14
3	7516.81
4	7532.94
5	7297.18
6	7712.34
7	7516.89
8	7303.97

Table 85: Randomly selected bank savings account data in U.S. dollars of women.

	x
1	7622.11
2	7534.71
3	7537.75
4	7528.89
5	7658.11
6	7722.40
7	7794.02

Table 86: Randomly selected bank savings account data in U.S. dollars of men.

Test if the population mean savings of women is *less than* men. Use $\alpha = 0.05$.

Solution:

Variable	**Levels**	**n**	average	s^2	Sum of Squares (SS)
thedat	Female	8	7454.8050	19974.3264	139820.2850
	Male	7	7628.2843	10680.9363	64085.6176
	all	15	7535.7620	22590.0573	316260.8028

Table 87: Savings of men and women with descriptive statistics.

$$s_p = \sqrt{\frac{(n_1-1)s_1^2 + (n_2-1)s_2^2}{n_1+n_2-2}} = \sqrt{\frac{(8-1)*19974.3264 + (7-1)*10680.9363}{8+7-2}} = 125.24$$

$$t = \frac{\bar{x}_1 - \bar{x}_2}{s_p\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}} = \frac{7454.805 - 7628.2843}{125.24\sqrt{\frac{1}{8}+\frac{1}{7}}} = -2.6764$$

The degrees of freedom $= n_1 + n_2 - 2 = 13$. The p-value $= 0.0095$ and reject if p-value < 0.05 $\square$

Problem 138. Use the data below to answer the following questions. Assume the data comes from a normal distribution. Also assume the variances are equal.

	y
1	7577.02
2	7562.21
3	7486.49
4	7373.27
5	7518.79
6	7307.79

Table 88: Randomly selected bank savings account data in U.S. dollars of women.

	x
1	7234.70
2	7322.23
3	7497.31
4	7636.43
5	7531.03

Table 89: Randomly selected bank savings account data in U.S. dollars of men.

Test if the population mean savings of women is *greater than* men. Use $\alpha = 0.05$.
Solution:

Variable	**Levels**	**n**	average	s^2	Sum of Squares (SS)
thedat	Female	6	7470.9283	11654.3910	58271.9549
	Male	5	7444.3400	26519.8317	106079.3268
	all	11	7458.8427	16627.9298	166279.2984

Table 90: Savings of men and women with descriptive statistics.

$$s_p = \sqrt{\frac{(n_1-1)s_1^2 + (n_2-1)s_2^2}{n_1+n_2-2}} = \sqrt{\frac{(6-1)*11654.391 + (5-1)*26519.8317}{6+5-2}} = 135.1342$$

$$t = \frac{\bar{x}_1 - \bar{x}_2}{s_p\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}} = \frac{7470.9283 - 7444.34}{135.1342\sqrt{\frac{1}{6} + \frac{1}{5}}} = 0.3249$$

The degrees of freedom $= n_1 + n_2 - 2 = 9$. The p-value $= 0.3763$ and reject if p-value < 0.05 $\square$

Problem 139. Use the data below to answer the following questions. Assume the data comes from a normal distribution. Also assume the variances are equal.

	y
1	7418.83
2	7537.74
3	7692.50
4	7442.59
5	7542.10
6	7399.96
7	7576.73

Table 91: Randomly selected bank savings account data in U.S. dollars of women.

	x
1	7339.19
2	7571.66
3	7880.87
4	7345.59

Table 92: Randomly selected bank savings account data in U.S. dollars of men.

Test if the population mean savings of women is *greater than* men. Use $\alpha = 0.05$.

Solution:

Variable	**Levels**	**n**	average	s^2	Sum of Squares (SS)
thedat	Female	7	7515.7786	10715.0326	64290.1959
	Male	4	7534.3275	65061.9692	195185.9077
	all	11	7522.5236	26035.1900	260351.8997

Table 93: Savings of men and women with descriptive statistics.

$$s_p = \sqrt{\frac{(n_1-1)s_1^2 + (n_2-1)s_2^2}{n_1+n_2-2}} = \sqrt{\frac{(7-1)*10715.0326 + (4-1)*65061.9692}{7+4-2}} = 169.796$$

$$t = \frac{\bar{x}_1 - \bar{x}_2}{s_p\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}} = \frac{7515.7786 - 7534.3275}{169.796\sqrt{\frac{1}{7}+\frac{1}{4}}} = -0.1743$$

The degrees of freedom $= n_1 + n_2 - 2 = 9$. The p-value $= 0.5673$ and reject if p-value < 0.05 □

Problem 140. Use the data below to answer the following questions. Assume the data comes from a normal distribution. Also assume the variances are equal.

	y
1	7378.57
2	7699.59

Table 94: Randomly selected bank savings account data in U.S. dollars of women.

	x
1	7562.63
2	7463.68
3	7321.19
4	7299.27
5	7563.61
6	7129.61

Table 95: Randomly selected bank savings account data in U.S. dollars of men.

Test if the population mean savings of women is *not equal to* men. Use $\alpha = 0.05$.
Solution:

Variable	**Levels**	**n**	average	s^2	Sum of Squares (SS)
thedat	Female	2	7539.0800	51526.9202	51526.9202
	Male	6	7389.9983	29227.9985	146139.9925
	all	8	7427.2688	33000.7040	231004.9277

Table 96: Savings of men and women with descriptive statistics.

$$s_p = \sqrt{\frac{(n_1-1)s_1^2 + (n_2-1)s_2^2}{n_1+n_2-2}} = \sqrt{\frac{(2-1)*51526.9202 + (6-1)*29227.9985}{2+6-2}} = 181.5062$$

$$t = \frac{\bar{x}_1 - \bar{x}_2}{s_p\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}} = \frac{7539.08 - 7389.9983}{181.5062\sqrt{\frac{1}{2}+\frac{1}{6}}} = 1.006$$

The degrees of freedom $= n_1 + n_2 - 2 = 6$. The p-value $= 0.3533$ and reject if p-value < 0.05 □

Problem 141. Use the data below to answer the following questions. Assume the data comes from a normal distribution. Also assume the variances are equal.

	y
1	7790.45
2	7743.19
3	7809.53
4	7289.82

Table 97: Randomly selected bank savings account data in U.S. dollars of women.

	x
1	7654.56
2	7760.05
3	7634.11
4	7804.58
5	7428.61

Table 98: Randomly selected bank savings account data in U.S. dollars of men.

Test if the population mean savings of women is *greater than* men. Use $\alpha = 0.05$.
Solution:

Variable	**Levels**	**n**	average	s^2	Sum of Squares (SS)
thedat	Female	4	7658.2475	61105.9823	183317.9469
	Male	5	7656.3820	21272.2868	85089.1471
	all	9	7657.2111	33551.8534	268414.8275

Table 99: Savings of men and women with descriptive statistics.

$$s_p = \sqrt{\frac{(n_1-1)s_1^2 + (n_2-1)s_2^2}{n_1+n_2-2}} = \sqrt{\frac{(4-1)*61105.9823 + (5-1)*21272.2868}{4+5-2}} = 195.8159$$

$$t = \frac{\bar{x}_1 - \bar{x}_2}{s_p\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}} = \frac{7658.2475 - 7656.382}{195.8159\sqrt{\frac{1}{4}+\frac{1}{5}}} = 0.0142$$

The degrees of freedom $= n_1 + n_2 - 2 = 7$. The p-value $= 0.4945$ and reject if p-value < 0.05 $\square$

Problem 142. Use the data below to answer the following questions. Assume the data comes from a normal distribution. Also assume the variances are equal.

	y
1	7194.71
2	7470.77
3	7708.78
4	7274.66
5	7336.19
6	7514.94

Table 100: Randomly selected bank savings account data in U.S. dollars of women.

	x
1	7822.85
2	7239.17
3	7422.10
4	7236.87

Table 101: Randomly selected bank savings account data in U.S. dollars of men.

Test if the population mean savings of women is *less than* men. Use $\alpha = 0.05$.

Solution:

Variable	**Levels**	**n**	average	s^2	Sum of Squares (SS)
thedat	Female	6	7416.6750	34764.4334	173822.1669
	Male	4	7430.2475	76036.1911	228108.5733
	all	10	7422.1040	44708.0945	402372.8508

Table 102: Savings of men and women with descriptive statistics.

$$s_p = \sqrt{\frac{(n_1-1)s_1^2 + (n_2-1)s_2^2}{n_1+n_2-2}} = \sqrt{\frac{(6-1)*34764.4334 + (4-1)*76036.1911}{6+4-2}} = 224.1458$$

$$t = \frac{\bar{x}_1 - \bar{x}_2}{s_p\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}} = \frac{7416.675 - 7430.2475}{224.1458\sqrt{\frac{1}{6}+\frac{1}{4}}} = -0.0938$$

The degrees of freedom $= n_1 + n_2 - 2 = 8$. The p-value $= 0.4638$ and reject if p-value < 0.05 $\square$

Problem 143. Use the data below to answer the following questions. Assume the data comes from a normal distribution. Also assume the variances are equal.

	y
1	7748.15
2	7544.96
3	7760.37

Table 103: Randomly selected bank savings account data in U.S. dollars of women.

	x
1	6837.75
2	7081.33
3	7040.63
4	7397.79
5	6900.19

Table 104: Randomly selected bank savings account data in U.S. dollars of men.

Test if the population mean savings of women is *less than* men. Use $\alpha = 0.05$.
Solution:

Variable	**Levels**	**n**	average	s^2	Sum of Squares (SS)
thedat	Female	3	7684.4933	14639.4954	29278.9909
	Male	5	7051.5380	47377.1303	189508.5213
	all	8	7288.8962	138567.6233	969973.3634

Table 105: Savings of men and women with descriptive statistics.

$$s_p = \sqrt{\frac{(n_1-1)s_1^2 + (n_2-1)s_2^2}{n_1+n_2-2}} = \sqrt{\frac{(3-1)*14639.4954 + (5-1)*47377.1303}{3+5-2}} = 190.957$$

$$t = \frac{\bar{x}_1 - \bar{x}_2}{s_p\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}} = \frac{7684.4933 - 7051.538}{190.957\sqrt{\frac{1}{3}+\frac{1}{5}}} = 4.5388$$

The degrees of freedom $= n_1 + n_2 - 2 = 6$. The p-value $= 0.998$ and reject if p-value < 0.05 □

Problem 144. Use the data below to answer the following questions. Assume the data comes from a normal distribution. Also assume the variances are equal.

	y
1	7262.23
2	7419.17
3	7204.97
4	7658.37
5	7701.28
6	7771.73

Table 106: Randomly selected bank savings account data in U.S. dollars of women.

	x
1	7385.27
2	7577.68
3	7347.39
4	7735.31
5	7609.05
6	7584.18
7	7296.33
8	7452.62
9	7644.56

Table 107: Randomly selected bank savings account data in U.S. dollars of men.

Test if the population mean savings of women is *less than* men. Use $\alpha = 0.05$.
Solution:

Variable	**Levels**	**n**	average	s^2	Sum of Squares (SS)
thedat	Female	6	7502.9583	57898.0281	289490.1405
	Male	9	7514.7100	22439.0511	179512.4084
	all	15	7510.0093	33535.6939	469499.7149

Table 108: Savings of men and women with descriptive statistics.

$$s_p = \sqrt{\frac{(n_1-1)s_1^2 + (n_2-1)s_2^2}{n_1+n_2-2}} = \sqrt{\frac{(6-1)*57898.0281 + (9-1)*22439.0511}{6+9-2}} = 189.9398$$

$$t = \frac{\bar{x}_1 - \bar{x}_2}{s_p\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}} = \frac{7502.9583 - 7514.71}{189.9398\sqrt{\frac{1}{6}+\frac{1}{9}}} = -0.1174$$

The degrees of freedom $= n_1 + n_2 - 2 = 13$. The p-value $= 0.4542$ and reject if p-value < 0.05 □

1.9 Paired T-test Examples

Problem 145. Use the data below to answer the following questions.

	Before	After
1	81.38	82.38
2	76.57	77.78
3	80.59	85.43

Table 109: Before and after training exam scores.

Test if the population mean exam score after is *greater than* before training. Use $\alpha = 0.05$.
Solution:

Before	After	d_i
81.38	82.38	1.00
76.57	77.78	1.21
80.59	85.43	4.84

Table 110: Before and after training exam scores with differences.

$$t - statistic = \frac{\bar{d} - 0}{s_d/\sqrt{n}} = \frac{2.35 - 0}{2.159/\sqrt{3}} = 1.8853$$

The p-value $= 0.1$ and reject if p-value < 0.05 □

Problem 146. Use the data below to answer the following questions.

	Before	After
1	75.21	75.92
2	75.69	75.84
3	78.70	81.07
4	74.92	78.77
5	75.75	75.83
6	76.67	78.26

Table 111: Before and after training exam scores.

Test if the population mean exam score after is *greater than* before training. Use $\alpha = 0.05$.
Solution:

Before	After	d_i
75.21	75.92	0.71
75.69	75.84	0.15
78.70	81.07	2.37
74.92	78.77	3.85
75.75	75.83	0.08
76.67	78.26	1.59

Table 112: Before and after training exam scores with differences.

$$t - statistic = \frac{\bar{d} - 0}{s_d/\sqrt{n}} = \frac{1.4583 - 0}{1.4656/\sqrt{6}} = 2.4373$$

The p-value = 0.0294 and reject if p-value < 0.05 $\square$

Problem 147. Use the data below to answer the following questions.

	Before	After
1	78.24	88.37
2	73.61	76.56
3	77.86	79.36
4	80.35	80.94
5	72.91	75.61
6	81.64	82.12
7	80.10	87.82
8	72.91	78.99

Table 113: Before and after training exam scores.

Test if the population mean exam score after is *not equal to* before training. Use $\alpha = 0.05$.
Solution:

Before	After	d_i
78.24	88.37	10.13
73.61	76.56	2.95
77.86	79.36	1.50
80.35	80.94	0.59
72.91	75.61	2.70
81.64	82.12	0.48
80.10	87.82	7.72
72.91	78.99	6.08

Table 114: Before and after training exam scores with differences.

$$t - statistic = \frac{\bar{d} - 0}{s_d/\sqrt{n}} = \frac{4.0188 - 0}{3.5617/\sqrt{8}} = 3.1914$$

The p-value = 0.0152 and reject if p-value < 0.05 $\square$

Problem 148. Use the data below to answer the following questions.

	Before	After
1	73.18	78.93
2	77.14	79.59
3	71.44	76.96
4	78.13	81.21

Table 115: Before and after training exam scores.

Test if the population mean exam score after is *not equal to* before training. Use $\alpha = 0.05$.
Solution:

Before	After	d_i
73.18	78.93	5.75
77.14	79.59	2.45
71.44	76.96	5.52
78.13	81.21	3.08

Table 116: Before and after training exam scores with differences.

$$t - statistic = \frac{\bar{d} - 0}{s_d/\sqrt{n}} = \frac{4.2 - 0}{1.6795/\sqrt{4}} = 5.0016$$

The p-value $= 0.0154$ and reject if p-value < 0.05 $\square$

Problem 149. Use the data below to answer the following questions.

	Before	After
1	77.30	80.89
2	71.13	75.41
3	75.53	76.94
4	76.56	77.94
5	75.54	76.18
6	80.90	82.29

Table 117: Before and after training exam scores.

Test if the population mean exam score after is *greater than* before training. Use $\alpha = 0.05$.

Solution:

Before	After	d_i
77.30	80.89	3.59
71.13	75.41	4.28
75.53	76.94	1.41
76.56	77.94	1.38
75.54	76.18	0.64
80.90	82.29	1.39

Table 118: Before and after training exam scores with differences.

$$t - statistic = \frac{\bar{d} - 0}{s_d/\sqrt{n}} = \frac{2.115 - 0}{1.4561/\sqrt{6}} = 3.5579$$

The p-value = 0.0081 and reject if p-value < 0.05 □

Problem 150. Use the data below to answer the following questions.

	Before	After
1	68.13	72.22
2	78.39	84.45
3	75.95	77.28
4	70.00	74.43
5	80.52	82.55
6	78.57	89.41
7	74.00	83.93

Table 119: Before and after training exam scores.

Test if the population mean exam score after is *not equal to* before training. Use $\alpha = 0.05$.
Solution:

Before	After	d_i
68.13	72.22	4.09
78.39	84.45	6.06
75.95	77.28	1.33
70.00	74.43	4.43
80.52	82.55	2.03
78.57	89.41	10.84
74.00	83.93	9.93

Table 120: Before and after training exam scores with differences.

$$t - statistic = \frac{\bar{d} - 0}{s_d/\sqrt{n}} = \frac{5.53 - 0}{3.6745/\sqrt{7}} = 3.9818$$

The p-value = 0.0073 and reject if p-value < 0.05 $\square$

Problem 151. Use the data below to answer the following questions.

	Before	After
1	72.93	77.26
2	74.80	78.57
3	72.46	76.49
4	70.55	72.73
5	71.05	71.17
6	71.91	72.59
7	73.55	74.64

Table 121: Before and after training exam scores.

Test if the population mean exam score after is *not equal to* before training. Use $\alpha = 0.05$.
Solution:

Before	After	d_i
72.93	77.26	4.33
74.80	78.57	3.77
72.46	76.49	4.03
70.55	72.73	2.18
71.05	71.17	0.12
71.91	72.59	0.68
73.55	74.64	1.09

Table 122: Before and after training exam scores with differences.

$$t - statistic = \frac{\bar{d} - 0}{s_d/\sqrt{n}} = \frac{2.3143 - 0}{1.7382/\sqrt{7}} = 3.5226$$

The p-value = 0.0125 and reject if p-value < 0.05 $\square$

Problem 152. Use the data below to answer the following questions.

	Before	After
1	74.24	85.09
2	82.34	83.13
3	77.53	87.17
4	73.64	76.11
5	78.44	80.14

Table 123: Before and after training exam scores.

Test if the population mean exam score after is *not equal to* before training. Use $\alpha = 0.05$.
Solution:

Before	After	d_i
74.24	85.09	10.85
82.34	83.13	0.79
77.53	87.17	9.64
73.64	76.11	2.47
78.44	80.14	1.70

Table 124: Before and after training exam scores with differences.

$$t - statistic = \frac{\bar{d} - 0}{s_d/\sqrt{n}} = \frac{5.09 - 0}{4.7625/\sqrt{5}} = 2.3898$$

The p-value = 0.0752 and reject if p-value < 0.05 $\square$

Problem 153. Use the data below to answer the following questions.

	Before	After
1	73.08	79.16
2	73.42	78.12
3	75.26	77.65
4	76.63	77.05
5	75.79	79.94
6	80.10	82.29
7	76.60	80.29
8	80.60	80.94

Table 125: Before and after training exam scores.

Test if the population mean exam score after is *greater than* before training. Use $\alpha = 0.05$.
Solution:

Before	After	d_i
73.08	79.16	6.08
73.42	78.12	4.70
75.26	77.65	2.39
76.63	77.05	0.42
75.79	79.94	4.15
80.10	82.29	2.19
76.60	80.29	3.69
80.60	80.94	0.34

Table 126: Before and after training exam scores with differences.

$$t - statistic = \frac{\bar{d} - 0}{s_d/\sqrt{n}} = \frac{2.995 - 0}{2.0331/\sqrt{8}} = 4.1666$$

The p-value = 0.0021 and reject if p-value < 0.05 $\square$

Problem 154. Use the data below to answer the following questions.

	Before	After
1	69.63	72.75
2	84.83	97.08
3	70.13	70.83
4	67.51	72.52
5	78.21	81.66
6	71.76	78.66
7	69.34	77.83
8	73.46	84.80
9	77.25	89.93

Table 127: Before and after training exam scores.

Test if the population mean exam score after is *greater than* before training. Use $\alpha = 0.05$.
Solution:

Before	After	d_i
69.63	72.75	3.12
84.83	97.08	12.25
70.13	70.83	0.70
67.51	72.52	5.01
78.21	81.66	3.45
71.76	78.66	6.90
69.34	77.83	8.49
73.46	84.80	11.34
77.25	89.93	12.68

Table 128: Before and after training exam scores with differences.

$$t - statistic = \frac{\bar{d} - 0}{s_d/\sqrt{n}} = \frac{7.1044 - 0}{4.3603/\sqrt{9}} = 4.888$$

The p-value $= 6e - 04$ and reject if p-value < 0.05 $\square$

Problem 155. Use the data below to answer the following questions.

	Before	After
1	69.64	83.31
2	75.48	78.61
3	80.62	90.10
4	76.48	89.73
5	80.76	88.18

Table 129: Before and after training exam scores.

Test if the population mean exam score after is *greater than* before training. Use $\alpha = 0.05$.
Solution:

Before	After	d_i
69.64	83.31	13.67
75.48	78.61	3.13
80.62	90.10	9.48
76.48	89.73	13.25
80.76	88.18	7.42

Table 130: Before and after training exam scores with differences.

$$t - statistic = \frac{\bar{d} - 0}{s_d/\sqrt{n}} = \frac{9.39 - 0}{4.3673/\sqrt{5}} = 4.8077$$

The p-value = 0.0043 and reject if p-value < 0.05 □

Problem 156. Use the data below to answer the following questions.

	Before	After
1	65.87	75.95
2	79.75	91.97
3	72.02	74.64

Table 131: Before and after training exam scores.

Test if the population mean exam score after is *greater than* before training. Use $\alpha = 0.05$.
Solution:

Before	After	d_i
65.87	75.95	10.08
79.75	91.97	12.22
72.02	74.64	2.62

Table 132: Before and after training exam scores with differences.

$$t - statistic = \frac{\bar{d} - 0}{s_d/\sqrt{n}} = \frac{8.3067 - 0}{5.0397/\sqrt{3}} = 2.8548$$

The p-value = 0.052 and reject if p-value < 0.05 $\square$

Problem 157. Use the data below to answer the following questions.

	Before	After
1	79.09	80.92
2	77.96	82.76
3	74.28	78.15
4	76.16	77.21
5	74.59	79.84

Table 133: Before and after training exam scores.

Test if the population mean exam score after is *less than* before training. Use $\alpha = 0.05$.
Solution:

Before	After	d_i
79.09	80.92	1.83
77.96	82.76	4.80
74.28	78.15	3.87
76.16	77.21	1.05
74.59	79.84	5.25

Table 134: Before and after training exam scores with differences.

$$t - statistic = \frac{\bar{d} - 0}{s_d/\sqrt{n}} = \frac{3.36 - 0}{1.8427/\sqrt{5}} = 4.0772$$

The p-value $= 0.9924$ and reject if p-value < 0.05 $\square$

Problem 158. Use the data below to answer the following questions.

	Before	After
1	79.49	85.00
2	83.61	83.78
3	73.75	78.54
4	77.59	84.11
5	74.76	77.92
6	73.89	75.47
7	80.07	82.67

Table 135: Before and after training exam scores.

Test if the population mean exam score after is *greater than* before training. Use $\alpha = 0.05$.
Solution:

Before	After	d_i
79.49	85.00	5.51
83.61	83.78	0.17
73.75	78.54	4.79
77.59	84.11	6.52
74.76	77.92	3.16
73.89	75.47	1.58
80.07	82.67	2.60

Table 136: Before and after training exam scores with differences.

$$t - statistic = \frac{\bar{d} - 0}{s_d/\sqrt{n}} = \frac{3.4757 - 0}{2.2554/\sqrt{7}} = 4.0772$$

The p-value = 0.0033 and reject if p-value < 0.05 $\square$

Problem 159. Use the data below to answer the following questions.

	Before	After
1	74.52	76.70
2	78.37	83.82
3	77.41	79.20
4	74.89	81.53
5	70.03	74.48
6	76.09	78.72
7	77.47	80.13
8	74.48	84.30

Table 137: Before and after training exam scores.

Test if the population mean exam score after is *greater than* before training. Use $\alpha = 0.05$.
Solution:

Before	After	d_i
74.52	76.70	2.18
78.37	83.82	5.45
77.41	79.20	1.79
74.89	81.53	6.64
70.03	74.48	4.45
76.09	78.72	2.63
77.47	80.13	2.66
74.48	84.30	9.82

Table 138: Before and after training exam scores with differences.

$$t - statistic = \frac{\bar{d} - 0}{s_d/\sqrt{n}} = \frac{4.4525 - 0}{2.7614/\sqrt{8}} = 4.5605$$

The p-value = 0.0013 and reject if p-value < 0.05 $\square$

Problem 160. Use the data below to answer the following questions.

	Before	After
1	79.25	80.74
2	69.52	76.17
3	70.62	76.85
4	71.33	80.97
5	73.55	79.02

Table 139: Before and after training exam scores.

Test if the population mean exam score after is *less than* before training. Use $\alpha = 0.05$.
Solution:

Before	After	d_i
79.25	80.74	1.49
69.52	76.17	6.65
70.62	76.85	6.23
71.33	80.97	9.64
73.55	79.02	5.47

Table 140: Before and after training exam scores with differences.

$$t - statistic = \frac{\bar{d} - 0}{s_d/\sqrt{n}} = \frac{5.896 - 0}{2.928/\sqrt{5}} = 4.5027$$

The p-value = 0.9946 and reject if p-value < 0.05 □

Problem 161. Use the data below to answer the following questions.

	Before	After
1	73.11	80.87
2	73.92	76.03
3	69.72	73.09
4	77.63	79.81

Table 141: Before and after training exam scores.

Test if the population mean exam score after is *not equal to* before training. Use $\alpha = 0.05$.

Solution:

Before	After	d_i
73.11	80.87	7.76
73.92	76.03	2.11
69.72	73.09	3.37
77.63	79.81	2.18

Table 142: Before and after training exam scores with differences.

$$t - statistic = \frac{\bar{d} - 0}{s_d/\sqrt{n}} = \frac{3.855 - 0}{2.6668/\sqrt{4}} = 2.8911$$

The p-value = 0.063 and reject if p-value < 0.05 $\square$

Problem 162. Use the data below to answer the following questions.

	Before	After
1	83.11	96.10
2	81.44	100.00
3	67.28	76.32
4	64.65	84.29

Table 143: Before and after training exam scores.

Test if the population mean exam score after is *greater than* before training. Use $\alpha = 0.05$.
Solution:

Before	After	d_i
83.11	96.10	12.99
81.44	100.00	18.56
67.28	76.32	9.04
64.65	84.29	19.64

Table 144: Before and after training exam scores with differences.

$$t - statistic = \frac{\bar{d} - 0}{s_d/\sqrt{n}} = \frac{15.0575 - 0}{4.9582/\sqrt{4}} = 6.0738$$

The p-value = 0.0045 and reject if p-value < 0.05 $\square$

1.10 Simple Linear Regression - Test $\beta_1 = 0$

Problem 163. Test if the slope equals zero or not for the simple linear regression model.

	x	y
1	6.00	-25.70
2	3.00	-0.70
3	7.00	4.00
4	3.00	1.10
5	4.00	14.00

Table 145: The independent and response variable.

Solution: The regression table and other pertinent information

	Estimate	Std. Error	t value	Pr(>|t|)
(Intercept)	10.8833	21.4911	0.51	0.6474
x	-2.6833	4.4053	-0.61	0.5855

Table 146: Linear regression model output.

	x	y	yhat	resid
1	6.0000	-25.7000	-5.2167	-20.4833
2	3.0000	-0.7000	2.8333	-3.5333
3	7.0000	4.0000	-7.9000	11.9000
4	3.0000	1.1000	2.8333	-1.7333
5	4.0000	14.0000	0.1500	13.8500

$$\begin{aligned}
MSE &= 256.16278 \\
\sum_{i=1}^{n}(x_i - \bar{x})^2 &= 13.2 \\
s(\hat{\beta}_1) &= \sqrt{MSE \div \sum_{i=1}^{n}(x_i - \bar{x})^2} = 4.40525
\end{aligned}$$

The t-value equals: $t^* = \frac{\hat{\beta}_1 - 0}{s(\hat{\beta}_1)} = -0.60912$

The t-values - lower and upper for .95; n-2: -3.1824 and 3.1824

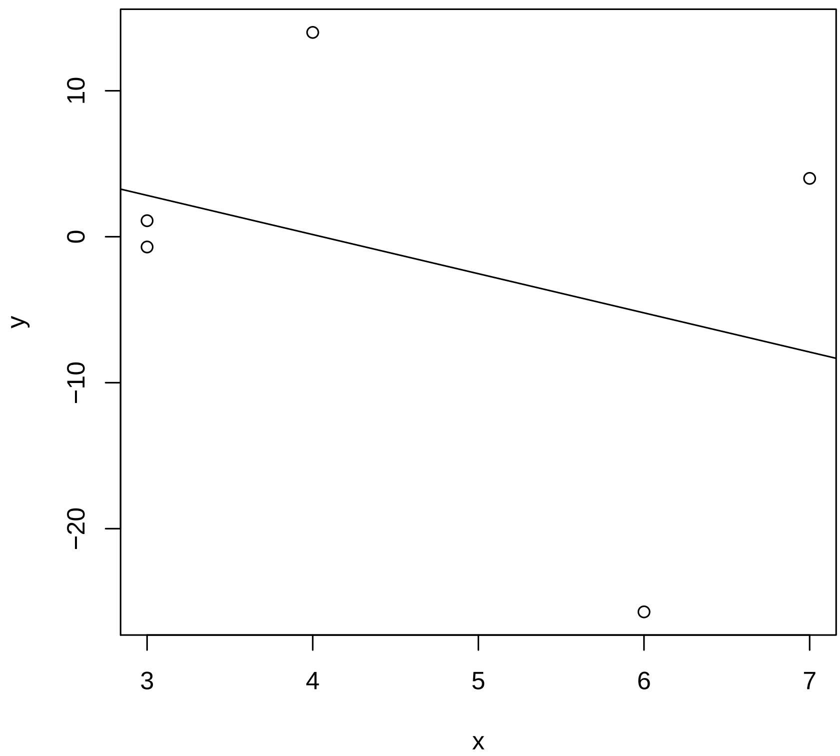

Figure 1: Scatter Plot with Regression Line

□

Problem 164. Test if the slope equals zero or not for the simple linear regression model.

	x	y
1	5.00	-3.00
2	3.00	2.80
3	3.00	-7.50
4	6.00	3.70
5	1.00	16.50
6	8.00	-33.80

Table 147: The independent and response variable.

Solution: The regression table and other pertinent information

	Estimate	Std. Error	t value	Pr(>|t|)
(Intercept)	19.7532	9.9760	1.98	0.1188
x	-5.3777	2.0363	-2.64	0.0575

Table 148: Linear regression model output.

	x	y	yhat	resid
1	5.0000	-3.0000	-7.1351	4.1351
2	3.0000	2.8000	3.6202	-0.8202
3	3.0000	-7.5000	3.6202	-11.1202
4	6.0000	3.7000	-12.5128	16.2128
5	1.0000	16.5000	14.3755	2.1245
6	8.0000	-33.8000	-23.2681	-10.5319

$$\begin{aligned}
MSE &= 129.92984 \\
\sum_{i=1}^{n}(x_i - \bar{x})^2 &= 31.3333333333333 \\
s(\hat{\beta}_1) &= \sqrt{MSE \div \sum_{i=1}^{n}(x_i - \bar{x})^2} = 2.03634
\end{aligned}$$

The t-value equals: $t^* = \frac{\hat{\beta}_1 - 0}{s(\hat{\beta}_1)} = -2.64084$

The t-values - lower and upper for .95; n-2: -2.7764 and 2.7764

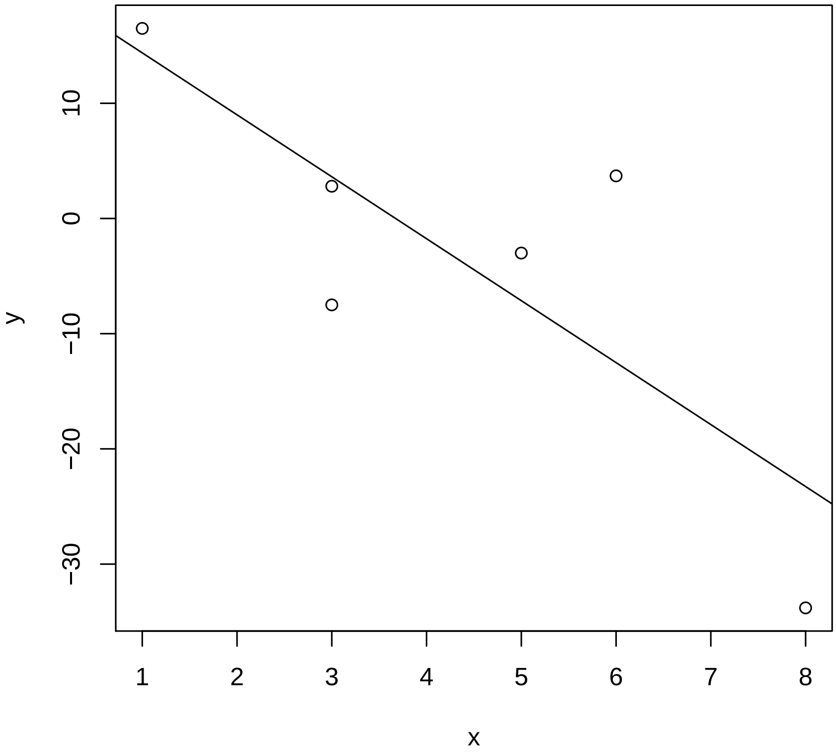

Figure 2: Scatter Plot with Regression Line

□

Problem 165. Test if the slope equals zero or not for the simple linear regression model.

	x	y
1	8.00	9.20
2	9.00	-18.80
3	5.00	2.50
4	3.00	-30.00
5	7.00	10.90
6	6.00	-36.10

Table 149: The independent and response variable.

Solution: The regression table and other pertinent information

	Estimate	Std. Error	t value	Pr(>|t|)
(Intercept)	-32.4143	29.4291	-1.10	0.3325
x	3.4786	4.4366	0.78	0.4768

Table 150: Linear regression model output.

	x	y	yhat	resid
1	8.0000	9.2000	-4.5857	13.7857
2	9.0000	-18.8000	-1.1071	-17.6929
3	5.0000	2.5000	-15.0214	17.5214
4	3.0000	-30.0000	-21.9786	-8.0214
5	7.0000	10.9000	-8.0643	18.9643
6	6.0000	-36.1000	-11.5429	-24.5571

$$
\begin{aligned}
MSE &= 459.28107 \\
\sum_{i=1}^{n}(x_i - \bar{x})^2 &= 23.3333333333333 \\
s(\hat{\beta}_1) &= \sqrt{MSE \div \sum_{i=1}^{n}(x_i - \bar{x})^2} = 4.43661
\end{aligned}
$$

The t-value equals: $t^* = \frac{\hat{\beta}_1 - 0}{s(\hat{\beta}_1)} = 0.78406$

The t-values - lower and upper for .95; n-2: -2.7764 and 2.7764

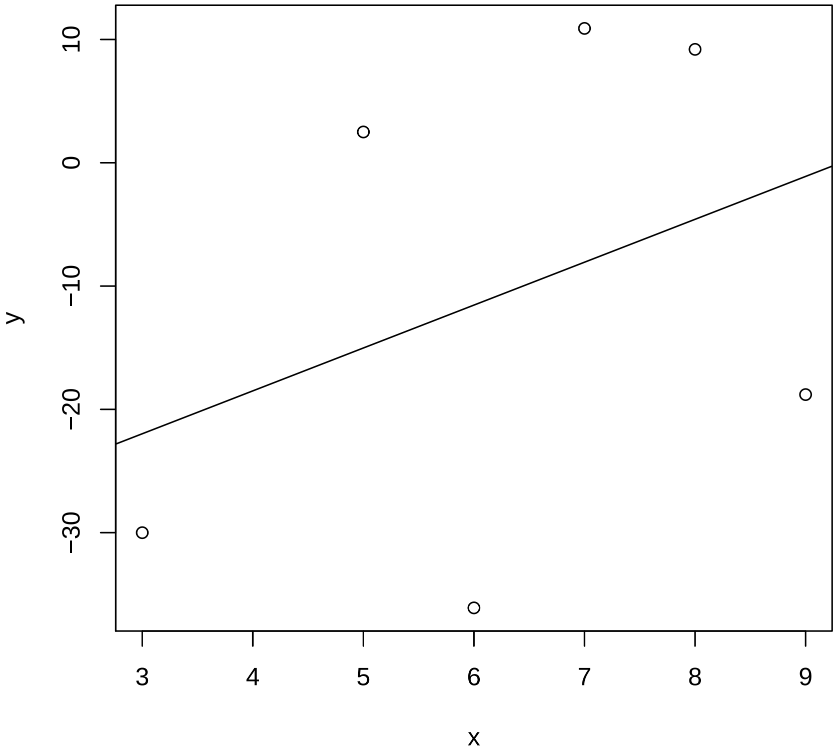

Figure 3: Scatter Plot with Regression Line

□

Problem 166. Test if the slope equals zero or not for the simple linear regression model.

	x	y
1	3.00	-16.80
2	8.00	21.70
3	8.00	-0.20
4	1.00	-21.40
5	9.00	15.10
6	4.00	14.40

Table 151: The independent and response variable.

Solution: The regression table and other pertinent information

	Estimate	Std. Error	t value	Pr(>|t|)
(Intercept)	-20.8227	11.2146	-1.86	0.1369
x	4.1738	1.7920	2.33	0.0803

Table 152: Linear regression model output.

	x	y	yhat	resid
1	3.0000	-16.8000	-8.3012	-8.4988
2	8.0000	21.7000	12.5679	9.1321
3	8.0000	-0.2000	12.5679	-12.7679
4	1.0000	-21.4000	-16.6489	-4.7511
5	9.0000	15.1000	16.7417	-1.6417
6	4.0000	14.4000	-4.1274	18.5274

$$\begin{aligned}
MSE &= 171.79417 \\
\sum_{i=1}^{n}(x_i - \bar{x})^2 &= 53.5 \\
s(\hat{\beta}_1) &= \sqrt{MSE \div \sum_{i=1}^{n}(x_i - \bar{x})^2} = 1.79196
\end{aligned}$$

The t-value equals: $t^* = \frac{\hat{\beta}_1 - 0}{s(\hat{\beta}_1)} = 2.3292$

The t-values - lower and upper for .95; n-2: -2.7764 and 2.7764

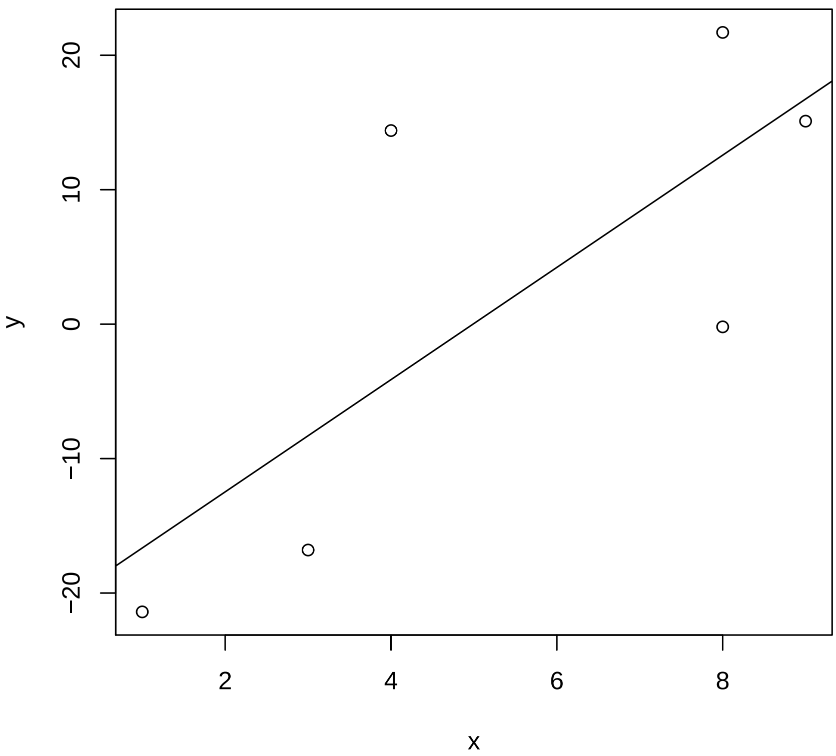

Figure 4: Scatter Plot with Regression Line

□

Problem 167. Test if the slope equals zero or not for the simple linear regression model.

	x	y
1	4.00	-20.10
2	9.00	-31.70
3	8.00	-15.30
4	1.00	16.10
5	5.00	23.20

Table 153: The independent and response variable.

Solution: The regression table and other pertinent information

	Estimate	Std. Error	t value	Pr(>|t|)
(Intercept)	21.4243	19.5032	1.10	0.3522
x	-4.9971	3.1891	-1.57	0.2151

Table 154: Linear regression model output.

	x	y	yhat	resid
1	4.0000	-20.1000	1.4359	-21.5359
2	9.0000	-31.7000	-23.5495	-8.1505
3	8.0000	-15.3000	-18.5524	3.2524
4	1.0000	16.1000	16.4272	-0.3272
5	5.0000	23.2000	-3.5612	26.7612

$$
\begin{aligned}
MSE &= 419.02388 \\
\sum_{i=1}^{n}(x_i - \bar{x})^2 &= 41.2 \\
s(\hat{\beta}_1) &= \sqrt{MSE \div \sum_{i=1}^{n}(x_i - \bar{x})^2} = 3.18912
\end{aligned}
$$

The t-value equals: $t^* = \frac{\hat{\beta}_1 - 0}{s(\hat{\beta}_1)} = -1.56692$

The t-values - lower and upper for .95; n-2: -3.1824 and 3.1824

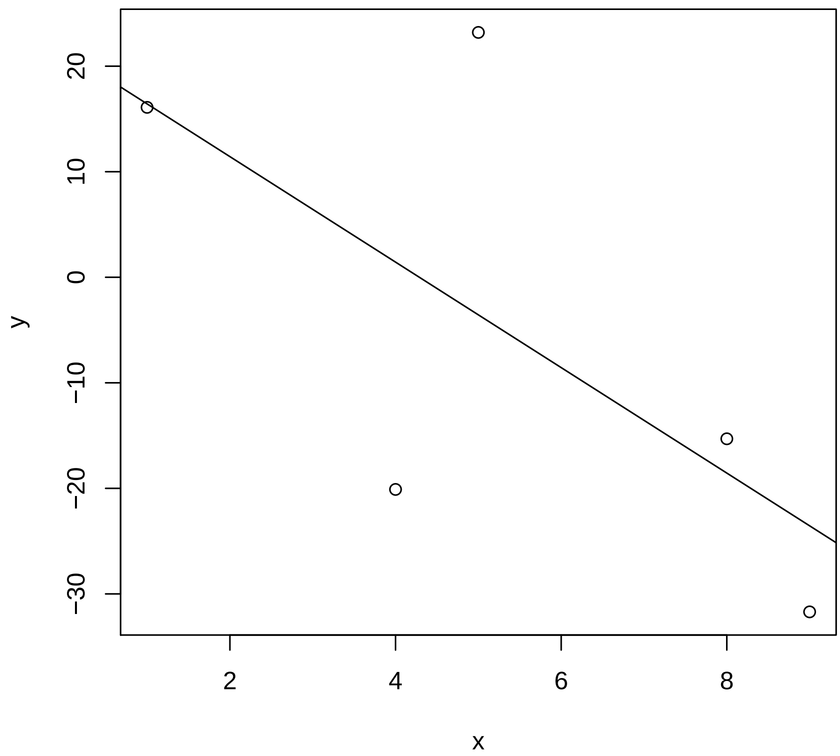

Figure 5: Scatter Plot with Regression Line

□

Problem 168. Test if the slope equals zero or not for the simple linear regression model.

	x	y
1	1.00	-2.30
2	4.00	19.40
3	2.00	1.80
4	2.00	17.00
5	6.00	-13.50
6	8.00	34.90
7	8.00	-3.60

Table 155: The independent and response variable.

Solution: The regression table and other pertinent information

	Estimate	Std. Error	t value	Pr(>|t|)
(Intercept)	3.8619	13.0738	0.30	0.7796
x	0.8602	2.5160	0.34	0.7463

Table 156: Linear regression model output.

	x	y	yhat	resid
1	1.0000	-2.3000	4.7221	-7.0221
2	4.0000	19.4000	7.3028	12.0972
3	2.0000	1.8000	5.5823	-3.7823
4	2.0000	17.0000	5.5823	11.4177
5	6.0000	-13.5000	9.0232	-22.5232
6	8.0000	34.9000	10.7436	24.1564
7	8.0000	-3.6000	10.7436	-14.3436

$$
\begin{aligned}
MSE &= 327.37735 \\
\sum_{i=1}^{n}(x_i - \bar{x})^2 &= 51.7142857142857 \\
s(\hat{\beta}_1) &= \sqrt{MSE \div \sum_{i=1}^{n}(x_i - \bar{x})^2} = 2.51605
\end{aligned}
$$

The t-value equals: $t^* = \frac{\hat{\beta}_1 - 0}{s(\hat{\beta}_1)} = 0.34189$

The t-values - lower and upper for .95; n-2: -2.5706 and 2.5706

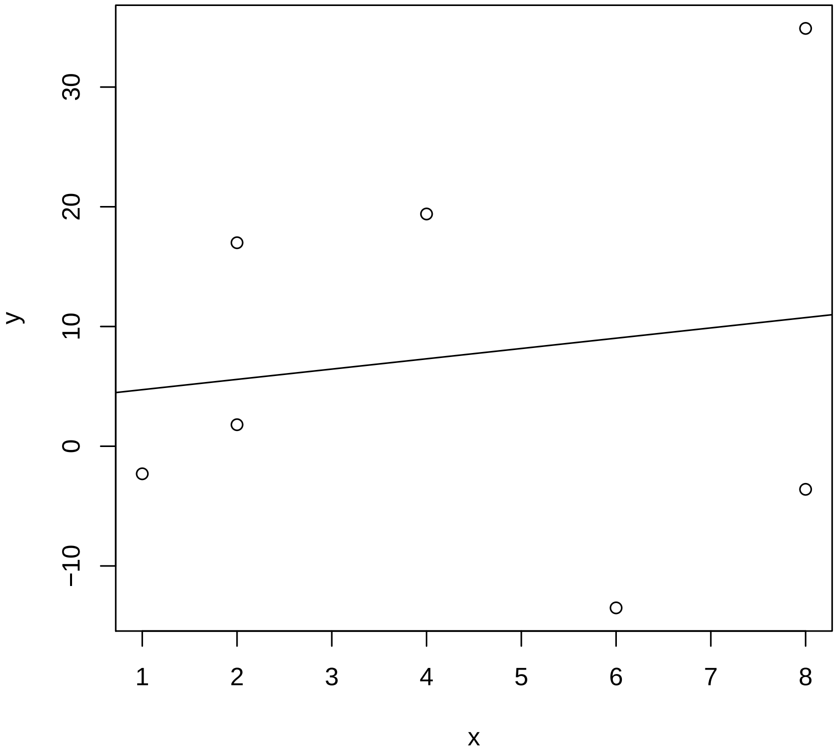

Figure 6: Scatter Plot with Regression Line

□

Problem 169. Test if the slope equals zero or not for the simple linear regression model.

	x	y
1	7.00	32.50
2	0.00	0.70
3	1.00	-10.10
4	5.00	8.20
5	0.00	-8.90
6	7.00	-28.70
7	4.00	-25.40

Table 157: The independent and response variable.

Solution: The regression table and other pertinent information

	Estimate	Std. Error	t value	Pr(>|t|)
(Intercept)	-8.3653	13.3155	-0.63	0.5574
x	1.1191	2.9774	0.38	0.7224

Table 158: Linear regression model output.

	x	y	yhat	resid
1	7.0000	32.5000	-0.5319	33.0319
2	0.0000	0.7000	-8.3653	9.0653
3	1.0000	-10.1000	-7.2463	-2.8537
4	5.0000	8.2000	-2.7700	10.9700
5	0.0000	-8.9000	-8.3653	-0.5347
6	7.0000	-28.7000	-0.5319	-28.1681
7	4.0000	-25.4000	-3.8891	-21.5109

$$
\begin{aligned}
MSE &= 511.64381 \\
\sum_{i=1}^{n}(x_i - \bar{x})^2 &= 57.7142857142857 \\
s(\hat{\beta}_1) &= \sqrt{MSE \div \sum_{i=1}^{n}(x_i - \bar{x})^2} = 2.97743
\end{aligned}
$$

The t-value equals: $t^* = \frac{\hat{\beta}_1 - 0}{s(\hat{\beta}_1)} = 0.37585$

The t-values - lower and upper for .95; n-2: -2.5706 and 2.5706

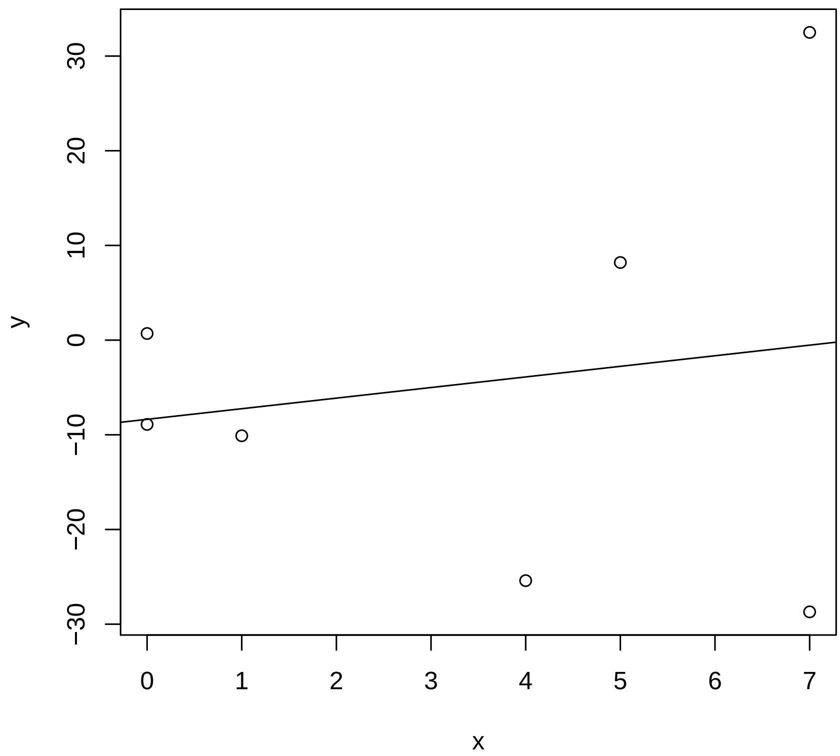

Figure 7: Scatter Plot with Regression Line

□

Problem 170. Test if the slope equals zero or not for the simple linear regression model.

	x	y
1	5.00	2.20
2	7.00	-43.70
3	3.00	-10.50
4	3.00	-3.60
5	9.00	-7.80
6	9.00	-47.90
7	2.00	16.90

Table 159: The independent and response variable.

Solution: The regression table and other pertinent information

	Estimate	Std. Error	t value	Pr(>|t|)
(Intercept)	17.1917	15.8122	1.09	0.3265
x	-5.6511	2.6045	-2.17	0.0822

Table 160: Linear regression model output.

	x	y	yhat	resid
1	5.0000	2.2000	-11.0638	13.2638
2	7.0000	-43.7000	-22.3660	-21.3340
3	3.0000	-10.5000	0.2384	-10.7384
4	3.0000	-3.6000	0.2384	-3.8384
5	9.0000	-7.8000	-33.6682	25.8682
6	9.0000	-47.9000	-33.6682	-14.2318
7	2.0000	16.9000	5.8895	11.0105

$$
\begin{aligned}
MSE &= 350.8107 \\
\sum_{i=1}^{n}(x_i - \bar{x})^2 &= 51.7142857142857 \\
s(\hat{\beta}_1) &= \sqrt{MSE \div \sum_{i=1}^{n}(x_i - \bar{x})^2} = 2.60454
\end{aligned}
$$

The t-value equals: $t^* = \frac{\hat{\beta}_1 - 0}{s(\hat{\beta}_1)} = -2.16971$

The t-values - lower and upper for .95; n-2: -2.5706 and 2.5706

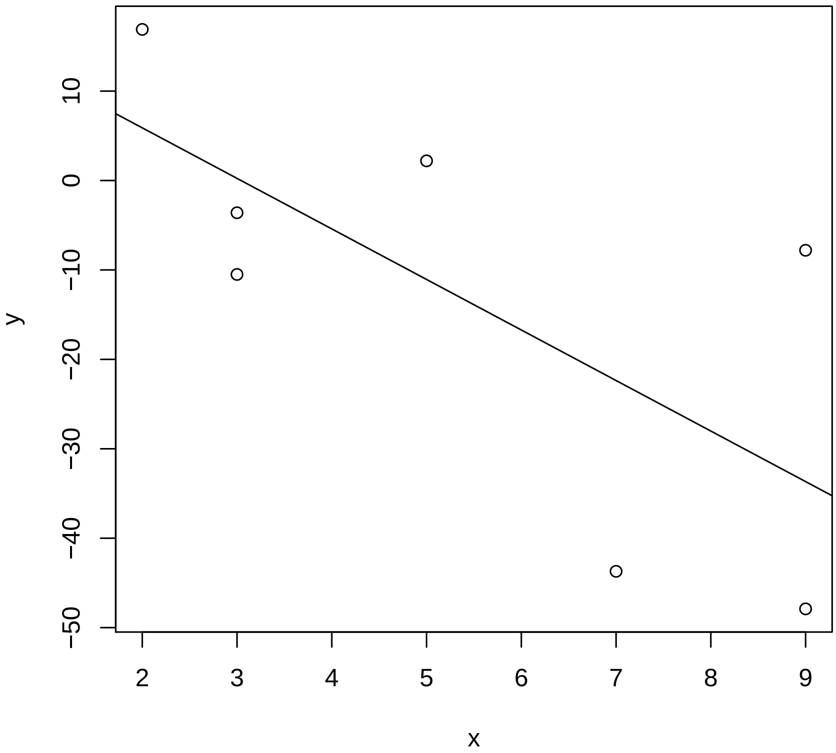

Figure 8: Scatter Plot with Regression Line

□

Problem 171. Test if the slope equals zero or not for the simple linear regression model.

	x	y
1	4.00	-6.30
2	0.00	-2.60
3	8.00	-18.50
4	3.00	21.30
5	4.00	-12.00

Table 161: The independent and response variable.

Solution: The regression table and other pertinent information

	Estimate	Std. Error	t value	Pr(>|t|)
(Intercept)	6.6354	12.0641	0.55	0.6206
x	-2.6988	2.6326	-1.03	0.3807

Table 162: Linear regression model output.

	x	y	yhat	resid
1	4.0000	-6.3000	-4.1598	-2.1402
2	0.0000	-2.6000	6.6354	-9.2354
3	8.0000	-18.5000	-14.9549	-3.5451
4	3.0000	21.3000	-1.4610	22.7610
5	4.0000	-12.0000	-4.1598	-7.8402

$$\begin{aligned} MSE &= 227.32398 \\ \sum_{i=1}^{n}(x_i - \bar{x})^2 &= 32.8 \\ s(\hat{\beta}_1) &= \sqrt{MSE \div \sum_{i=1}^{n}(x_i - \bar{x})^2} = 2.63261 \end{aligned}$$

The t-value equals: $t^* = \frac{\hat{\beta}_1 - 0}{s(\hat{\beta}_1)} = -1.02514$

The t-values - lower and upper for .95; n-2: -3.1824 and 3.1824

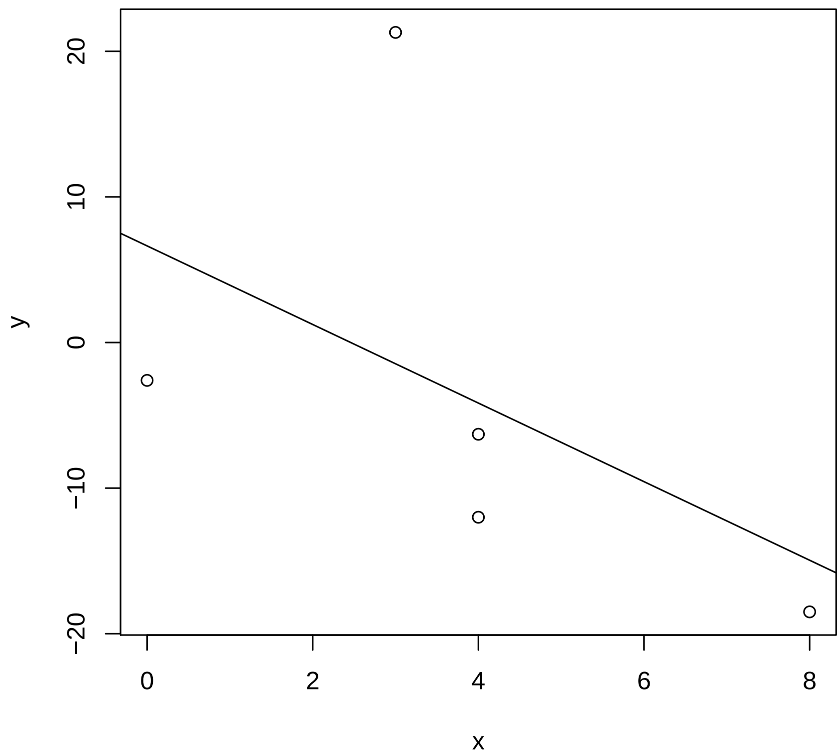

Figure 9: Scatter Plot with Regression Line

□

Problem 172. Test if the slope equals zero or not for the simple linear regression model.

	x	y
1	9.00	-19.90
2	2.00	22.50
3	6.00	0.70
4	0.00	-2.80
5	5.00	-40.80

Table 163: The independent and response variable.

Solution: The regression table and other pertinent information

	Estimate	Std. Error	t value	Pr(>|t|)
(Intercept)	5.9431	18.6177	0.32	0.7705
x	-3.1825	3.4454	-0.92	0.4238

Table 164: Linear regression model output.

	x	y	yhat	resid
1	9.0000	-19.9000	-22.6996	2.7996
2	2.0000	22.5000	-0.4220	22.9220
3	6.0000	0.7000	-13.1520	13.8520
4	0.0000	-2.8000	5.9431	-8.7431
5	5.0000	-40.8000	-9.9695	-30.8305

$$\begin{aligned} MSE &= 584.03099 \\ \sum_{i=1}^{n}(x_i - \bar{x})^2 &= 49.2 \\ s(\hat{\beta}_1) &= \sqrt{MSE \div \sum_{i=1}^{n}(x_i - \bar{x})^2} = 3.44537 \end{aligned}$$

The t-value equals: $t^* = \frac{\hat{\beta}_1 - 0}{s(\hat{\beta}_1)} = -0.92371$

The t-values - lower and upper for .95; n-2: -3.1824 and 3.1824

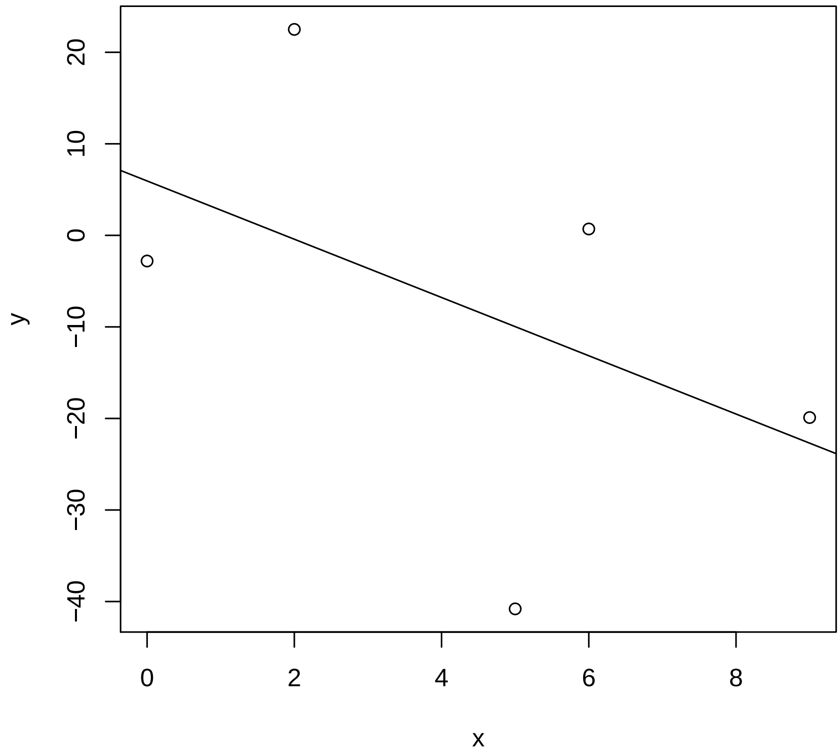

Figure 10: Scatter Plot with Regression Line

□

Problem 173. Test if the slope equals zero or not for the simple linear regression model.

	x	y
1	2.00	-13.50
2	9.00	-42.20
3	9.00	-57.20
4	1.00	-6.10
5	3.00	7.50
6	1.00	-12.70

Table 165: The independent and response variable.

Solution: The regression table and other pertinent information

	Estimate	Std. Error	t value	Pr(>|t|)
(Intercept)	2.2062	8.5240	0.26	0.8085
x	-5.4975	1.5694	-3.50	0.0248

Table 166: Linear regression model output.

	x	y	yhat	resid
1	2.0000	-13.5000	-8.7888	-4.7112
2	9.0000	-42.2000	-47.2712	5.0712
3	9.0000	-57.2000	-47.2712	-9.9288
4	1.0000	-6.1000	-3.2913	-2.8087
5	3.0000	7.5000	-14.2863	21.7863
6	1.0000	-12.7000	-3.2913	-9.4087

$$\begin{aligned}
MSE &= 179.38697 \\
\sum_{i=1}^{n}(x_i - \bar{x})^2 &= 72.8333333333333 \\
s(\hat{\beta}_1) &= \sqrt{MSE \div \sum_{i=1}^{n}(x_i - \bar{x})^2} = 1.56939
\end{aligned}$$

The t-value equals: $t^* = \frac{\hat{\beta}_1 - 0}{s(\hat{\beta}_1)} = -3.50295$

The t-values - lower and upper for .95; n-2: -2.7764 and 2.7764

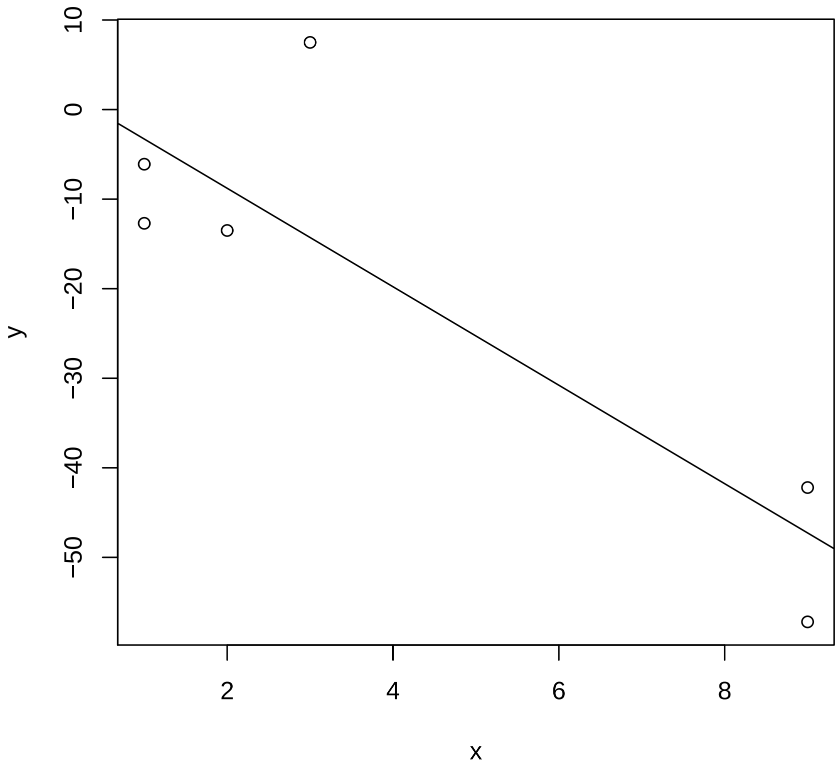

Figure 11: Scatter Plot with Regression Line

□

Problem 174. Test if the slope equals zero or not for the simple linear regression model.

	x	y
1	6.00	5.00
2	1.00	9.50
3	5.00	-4.00
4	2.00	-6.50
5	6.00	12.80

Table 167: The independent and response variable.

Solution: The regression table and other pertinent information

	Estimate	Std. Error	t value	Pr(>|t|)
(Intercept)	0.4327	9.1203	0.05	0.9651
x	0.7318	2.0193	0.36	0.7411

Table 168: Linear regression model output.

	x	y	yhat	resid
1	6.0000	5.0000	4.8236	0.1764
2	1.0000	9.5000	1.1645	8.3355
3	5.0000	-4.0000	4.0918	-8.0918
4	2.0000	-6.5000	1.8964	-8.3964
5	6.0000	12.8000	4.8236	7.9764

$$\begin{aligned} MSE &= 89.70324 \\ \sum_{i=1}^{n}(x_i - \bar{x})^2 &= 22 \\ s(\hat{\beta}_1) &= \sqrt{MSE \div \sum_{i=1}^{n}(x_i - \bar{x})^2} = 2.01926 \end{aligned}$$

The t-value equals: $t^* = \frac{\hat{\beta}_1 - 0}{s(\hat{\beta}_1)} = 0.36242$

The t-values - lower and upper for .95; n-2: -3.1824 and 3.1824

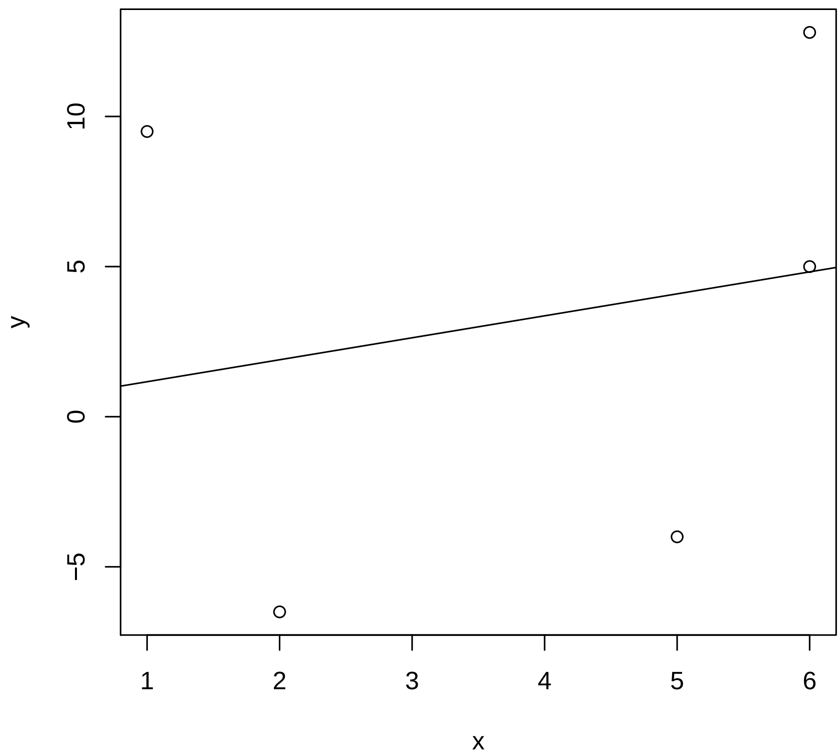

Figure 12: Scatter Plot with Regression Line

□

Problem 175. Test if the slope equals zero or not for the simple linear regression model.

	x	y
1	9.00	14.80
2	4.00	-35.10
3	3.00	21.90
4	2.00	8.10
5	3.00	9.90

Table 169: The independent and response variable.

Solution: The regression table and other pertinent information

	Estimate	Std. Error	t value	Pr(>|t|)
(Intercept)	0.9091	22.7061	0.04	0.9706
x	0.7169	4.6543	0.15	0.8874

Table 170: Linear regression model output.

	x	y	yhat	resid
1	9.0000	14.8000	7.3610	7.4390
2	4.0000	-35.1000	3.7766	-38.8766
3	3.0000	21.9000	3.0597	18.8403
4	2.0000	8.1000	2.3429	5.7571
5	3.0000	9.9000	3.0597	6.8403

$$
\begin{aligned}
MSE &= 667.20641 \\
\sum_{i=1}^{n}(x_i - \bar{x})^2 &= 30.8 \\
s(\hat{\beta}_1) &= \sqrt{MSE \div \sum_{i=1}^{n}(x_i - \bar{x})^2} = 4.6543
\end{aligned}
$$

The t-value equals: $t^* = \frac{\hat{\beta}_1 - 0}{s(\hat{\beta}_1)} = 0.15403$

The t-values - lower and upper for .95; n-2: -3.1824 and 3.1824

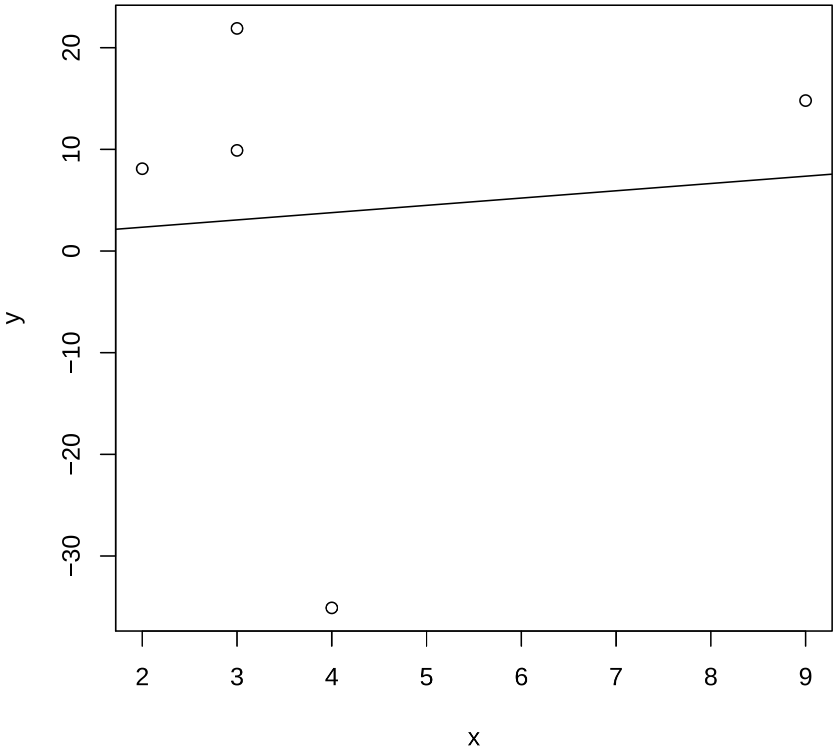

Figure 13: Scatter Plot with Regression Line

□

Problem 176. Test if the slope equals zero or not for the simple linear regression model.

	x	y
1	7.00	-27.40
2	0.00	-11.10
3	6.00	22.50
4	8.00	-14.90
5	4.00	-2.60

Table 171: The independent and response variable.

Solution: The regression table and other pertinent information

	Estimate	Std. Error	t value	Pr(>|t|)
(Intercept)	-4.3375	19.4497	-0.22	0.8378
x	-0.4725	3.3858	-0.14	0.8979

Table 172: Linear regression model output.

	x	y	yhat	resid
1	7.0000	-27.4000	-7.6450	-19.7550
2	0.0000	-11.1000	-4.3375	-6.7625
3	6.0000	22.5000	-7.1725	29.6725
4	8.0000	-14.9000	-8.1175	-6.7825
5	4.0000	-2.6000	-6.2275	3.6275

$$
\begin{aligned}
MSE &= 458.53658 \\
\sum_{i=1}^{n}(x_i - \bar{x})^2 &= 40 \\
s(\hat{\beta}_1) &= \sqrt{MSE \div \sum_{i=1}^{n}(x_i - \bar{x})^2} = 3.38577
\end{aligned}
$$

The t-value equals: $t^* = \frac{\hat{\beta}_1 - 0}{s(\hat{\beta}_1)} = -0.13955$

The t-values - lower and upper for .95; n-2: -3.1824 and 3.1824

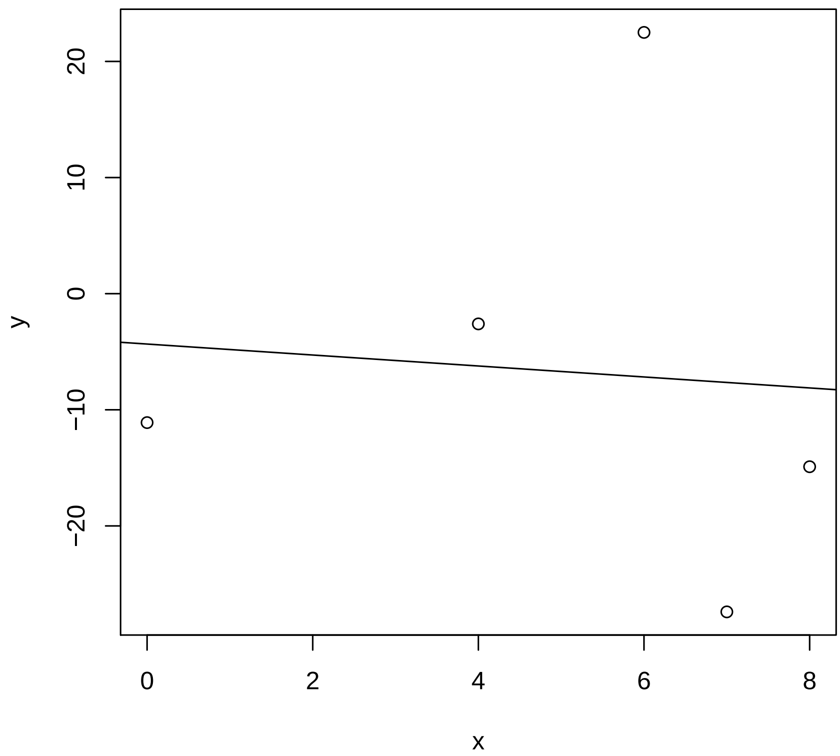

Figure 14: Scatter Plot with Regression Line

□

Problem 177. Test if the slope equals zero or not for the simple linear regression model.

	x	y
1	3.00	1.60
2	0.00	-11.70
3	2.00	22.10
4	5.00	-20.90
5	5.00	8.60

Table 173: The independent and response variable.

Solution: The regression table and other pertinent information

	Estimate	Std. Error	t value	Pr(>|t|)
(Intercept)	1.8733	16.2460	0.12	0.9155
x	-0.6444	4.5768	-0.14	0.8969

Table 174: Linear regression model output.

	x	y	yhat	resid
1	3.0000	1.6000	-0.0600	1.6600
2	0.0000	-11.7000	1.8733	-13.5733
3	2.0000	22.1000	0.5844	21.5156
4	5.0000	-20.9000	-1.3489	-19.5511
5	5.0000	8.6000	-1.3489	9.9489

$$
\begin{aligned}
MSE &= 377.04548 \\
\sum_{i=1}^{n}(x_i - \bar{x})^2 &= 18 \\
s(\hat{\beta}_1) &= \sqrt{MSE \div \sum_{i=1}^{n}(x_i - \bar{x})^2} = 4.57679
\end{aligned}
$$

The t-value equals: $t^* = \frac{\hat{\beta}_1 - 0}{s(\hat{\beta}_1)} = -0.14081$

The t-values - lower and upper for .95; n-2: -3.1824 and 3.1824

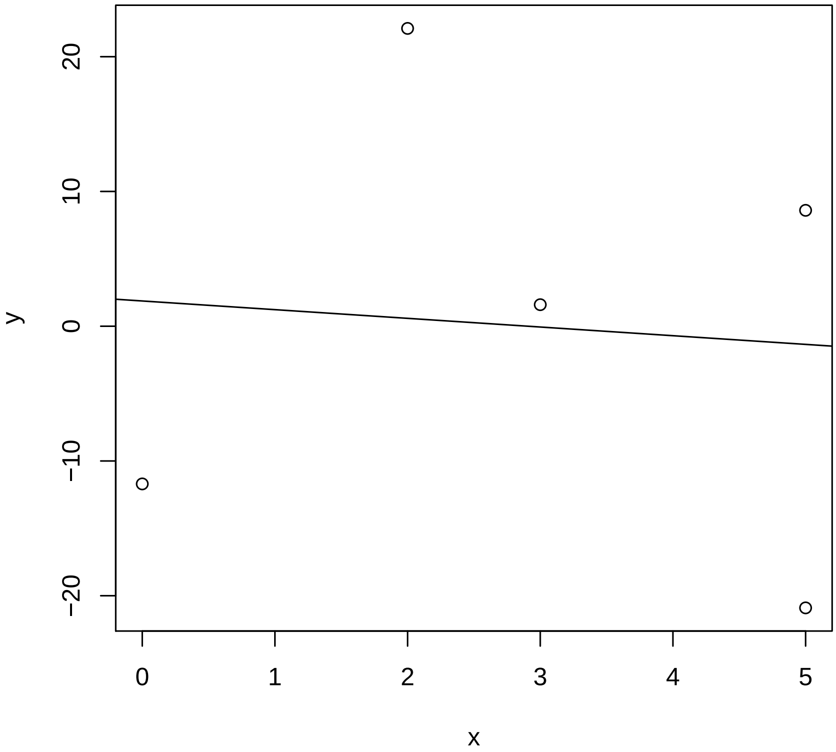

Figure 15: Scatter Plot with Regression Line

□

Problem 178. Test if the slope equals zero or not for the simple linear regression model.

	x	y
1	5.00	18.80
2	2.00	5.30
3	3.00	10.10
4	3.00	9.50
5	8.00	-13.30
6	2.00	-14.70

Table 175: The independent and response variable.

Solution: The regression table and other pertinent information

	Estimate	Std. Error	t value	Pr(>|t|)
(Intercept)	7.2714	12.5792	0.58	0.5942
x	-1.2143	2.8733	-0.42	0.6943

Table 176: Linear regression model output.

	x	y	yhat	resid
1	5.0000	18.8000	1.2000	17.6000
2	2.0000	5.3000	4.8429	0.4571
3	3.0000	10.1000	3.6286	6.4714
4	3.0000	9.5000	3.6286	5.8714
5	8.0000	-13.3000	-2.4429	-10.8571
6	2.0000	-14.7000	4.8429	-19.5429

$$\begin{aligned}
MSE &= 221.53071 \\
\sum_{i=1}^{n}(x_i - \bar{x})^2 &= 26.8333333333333 \\
s(\hat{\beta}_1) &= \sqrt{MSE \div \sum_{i=1}^{n}(x_i - \bar{x})^2} = 2.87329
\end{aligned}$$

The t-value equals: $t^* = \frac{\hat{\beta}_1 - 0}{s(\hat{\beta}_1)} = -0.42261$

The t-values - lower and upper for .95; n-2: -2.7764 and 2.7764

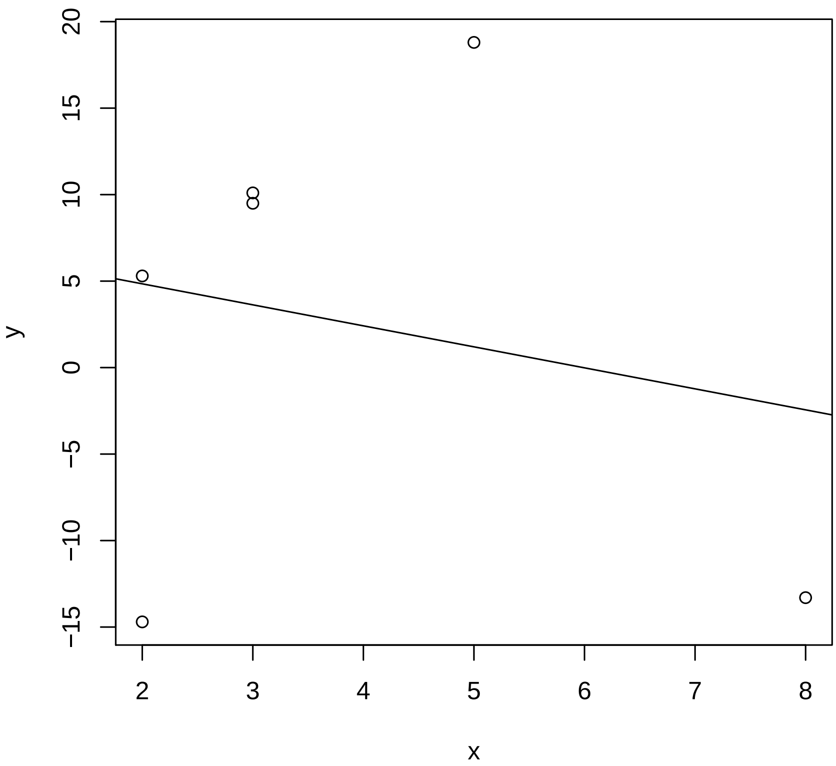

Figure 16: Scatter Plot with Regression Line

□

Problem 179. Test if the slope equals zero or not for the simple linear regression model.

	x	y
1	3.00	8.40
2	8.00	-60.10
3	7.00	-8.40
4	9.00	34.50
5	3.00	30.20
6	0.00	-17.50

Table 177: The independent and response variable.

Solution: The regression table and other pertinent information

	Estimate	Std. Error	t value	Pr(>|t|)
(Intercept)	1.7855	29.4694	0.06	0.9546
x	-0.7871	4.9577	-0.16	0.8815

Table 178: Linear regression model output.

	x	y	yhat	resid
1	3.0000	8.4000	-0.5758	8.9758
2	8.0000	-60.1000	-4.5113	-55.5887
3	7.0000	-8.4000	-3.7242	-4.6758
4	9.0000	34.5000	-5.2984	39.7984
5	3.0000	30.2000	-0.5758	30.7758
6	0.0000	-17.5000	1.7855	-19.2855

$$
\begin{aligned}
MSE &= 1523.88117 \\
\sum_{i=1}^{n}(x_i - \bar{x})^2 &= 62 \\
s(\hat{\beta}_1) &= \sqrt{MSE \div \sum_{i=1}^{n}(x_i - \bar{x})^2} = 4.95769
\end{aligned}
$$

The t-value equals: $t^* = \frac{\hat{\beta}_1 - 0}{s(\hat{\beta}_1)} = -0.15876$

The t-values - lower and upper for .95; n-2: -2.7764 and 2.7764

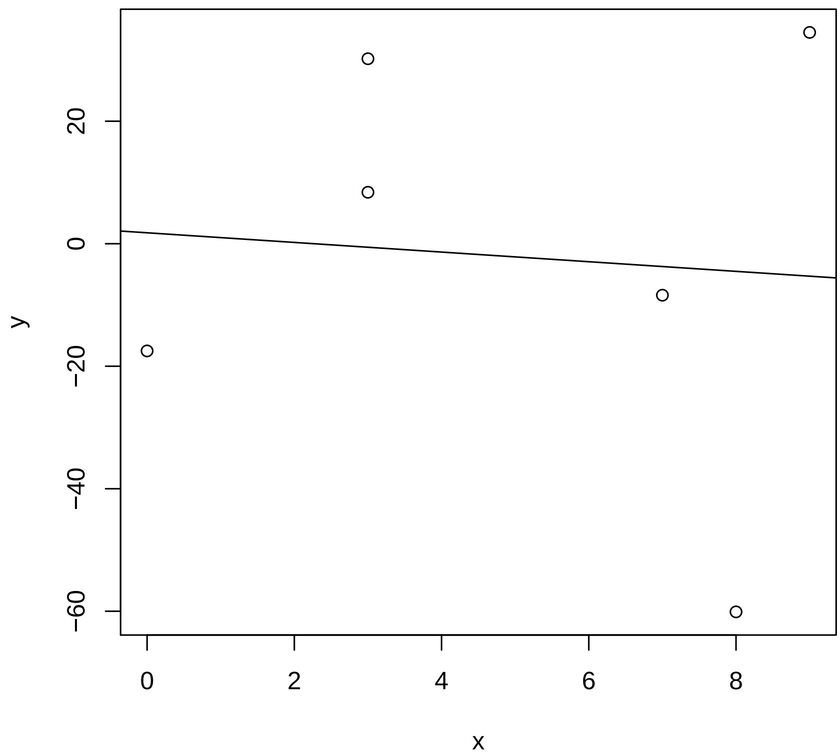

Figure 17: Scatter Plot with Regression Line

□

Problem 180. Test if the slope equals zero or not for the simple linear regression model.

	x	y
1	9.00	0.70
2	8.00	-1.00
3	9.00	-8.00
4	4.00	27.70
5	8.00	-25.50
6	5.00	-9.00
7	3.00	0.10

Table 179: The independent and response variable.

Solution: The regression table and other pertinent information

	Estimate	Std. Error	t value	Pr(>|t|)
(Intercept)	17.5515	17.5308	1.00	0.3627
x	-2.9970	2.5154	-1.19	0.2869

Table 180: Linear regression model output.

	x	y	yhat	resid
1	9.0000	0.7000	-9.4212	10.1212
2	8.0000	-1.0000	-6.4242	5.4242
3	9.0000	-8.0000	-9.4212	1.4212
4	4.0000	27.7000	5.5636	22.1364
5	8.0000	-25.5000	-6.4242	-19.0758
6	5.0000	-9.0000	2.5667	-11.5667
7	3.0000	0.1000	8.5606	-8.4606

$$\begin{aligned}
MSE &= 238.63079 \\
\sum_{i=1}^{n}(x_i - \bar{x})^2 &= 37.7142857142857 \\
s(\hat{\beta}_1) &= \sqrt{MSE \div \sum_{i=1}^{n}(x_i - \bar{x})^2} = 2.51542
\end{aligned}$$

The t-value equals: $t^* = \frac{\hat{\beta}_1 - 0}{s(\hat{\beta}_1)} = -1.19144$

The t-values - lower and upper for .95; n-2: -2.5706 and 2.5706

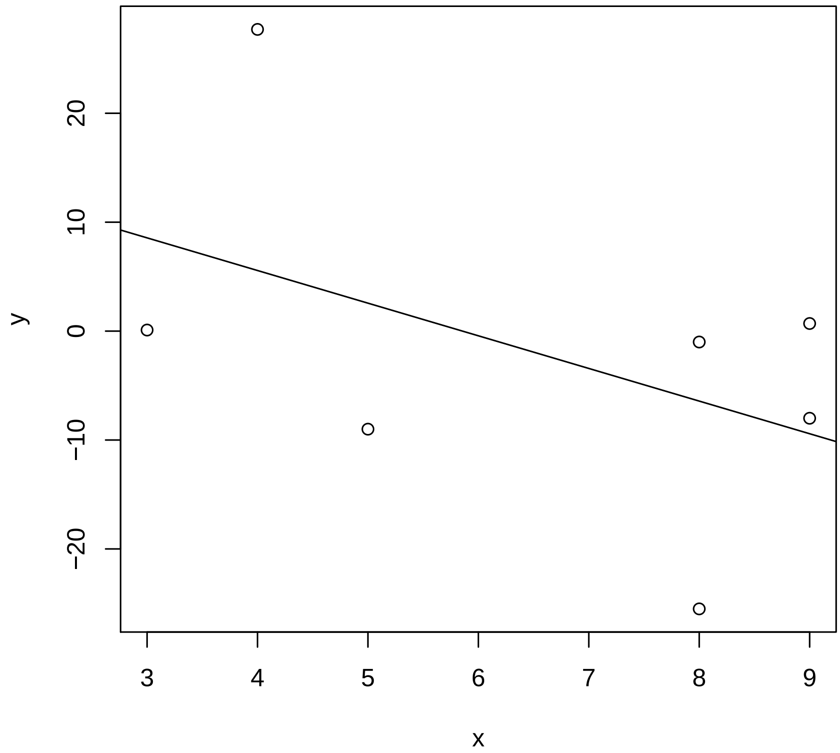

Figure 18: Scatter Plot with Regression Line

□

2 What To Do When - Select the most appropriate

Problem 181. Select the most appropriate test for each part. Write the number.

A. One Sample t-test (or confidence interval)

B. Two Sample t-test (or confidence interval)

C. Paired t-test (or confidence interval)

D. Chi-squared Test

E. One sample z-test for a proportion

F. Two sample z-test for proportions

G. ANOVA

H. Regression/General Linear Model

(a) It is desired to determine if men and women living in Bangkok make on average the same salary. The salaries for a random sample of 100 men and 100 women is taken. Data collected gender and income.

Solution: #2 □

(b) In families with exactly 2 children it is believed that the older child does better than the younger child in school. Better being defined as having a higher GPA (grade point average). Is the average GPA of the oldest child higher than that of the youngest child? Data collected the GPA of the oldest and youngest from 50 random selected families with two children. Thus 100 children in the survey from 50 families.

Solution: #3 □

(c) Is the response rate to mail marketing campaign type I higher than the response rate to mail marketing campaign type II? Data collected: For each marketing campaign the number of people mailed and the number of responses.

Solution: #6 □

(d) Understanding what might determine current salary. Data collected: Level of education (B.A., Masters, or Ph.D.), years of work experience, and current salary.

Solution: #8 □

(e) You believe the average height of people in management positions is greater than that of people in non-management. You take a simple random sample of working people of 1,000 with about 20% of the 1,000 people happen to have management positions. Test if the population average height of individuals in management is taller than that of non-management.

Solution: #2 □

(f) You believe the percent of men living in Bangkok is greater than the percent of women in Bangkok. You take a simple random sample of 1,000 people living in Bangkok.

Solution: #5 □