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1. Simple Linear Regression

1.1. Simple Linear Regression - Test $\beta_1 = 0$

Problem 1. Test if the slope equals zero or not for the simple linear regression model.

	x	y
1	5.00	12.70
2	0.00	-18.70
3	0.00	-2.90
4	1.00	2.50
5	3.00	16.00

Table 1: The independent and response variable.

Solution: The regression table and other pertinent information

	Estimate	Std. Error	t value	Pr(>|t|)
(Intercept)	-7.3404	5.7416	-1.28	0.2910
x	5.1447	2.1701	2.37	0.0984

Table 2: Linear regression model output.

	x	y	yhat	resid
1	5.0000	12.7000	18.3830	-5.6830
2	0.0000	-18.7000	-7.3404	-11.3596
3	0.0000	-2.9000	-7.3404	4.4404
4	1.0000	2.5000	-2.1957	4.6957
5	3.0000	16.0000	8.0936	7.9064

$$MSE = 88.53816$$

$$\sum_{i=1}^{n}(x_i - \bar{x})^2 = 18.8$$

$$s(\hat{\beta}_1) = \sqrt{MSE \div \sum_{i=1}^{n}(x_i - \bar{x})^2} = 2.17013$$

The t-value equals: $t^* = \frac{\hat{\beta}_1 - 0}{s(\hat{\beta}_1)} = 2.37068$

The t-values - lower and upper for .95; n-2: -3.1824 and 3.1824

□

Problem 2. Test if the slope equals zero or not for the simple linear regression model.

	x	y
1	3.00	-3.90
2	1.00	-0.40
3	9.00	-43.30
4	0.00	15.60
5	6.00	-20.10
6	2.00	-3.60

Table 3: The independent and response variable.

Solution: The regression table and other pertinent information

	Estimate	Std. Error	t value	Pr(>|t|)
(Intercept)	11.0867	2.8547	3.88	0.0178
x	-5.8200	0.6109	-9.53	0.0007

Table 4: Linear regression model output.

	x	y	yhat	resid
1	3.0000	-3.9000	-6.3733	2.4733
2	1.0000	-0.4000	5.2667	-5.6667
3	9.0000	-43.3000	-41.2933	-2.0067
4	0.0000	15.6000	11.0867	4.5133
5	6.0000	-20.1000	-23.8333	3.7333
6	2.0000	-3.6000	-0.5533	-3.0467

$$
\begin{aligned}
MSE &= 21.46133 \\
\sum_{i=1}^{n}(x_i-\bar{x})^2 &= 57.5 \\
s(\hat{\beta}_1) &= \sqrt{MSE \div \sum_{i=1}^{n}(x_i-\bar{x})^2} = 0.61093
\end{aligned}
$$

The t-value equals: $t^* = \frac{\hat{\beta}_1 - 0}{s(\hat{\beta}_1)} = -9.52639$

The t-values - lower and upper for .95; n-2: -2.7764 and 2.7764

□

Problem 3. Test if the slope equals zero or not for the simple linear regression model.

	x	y
1	2.00	26.10
2	2.00	14.30
3	8.00	-24.20
4	7.00	0.90
5	7.00	20.10

Table 5: The independent and response variable.

Solution: The regression table and other pertinent information

	Estimate	Std. Error	t value	Pr(>|t|)
(Intercept)	31.2345	16.8396	1.85	0.1607
x	-4.5759	2.8880	-1.58	0.2113

Table 6: Linear regression model output.

	x	y	yhat	resid
1	2.0000	26.1000	22.0828	4.0172
2	2.0000	14.3000	22.0828	-7.7828
3	8.0000	-24.2000	-5.3724	-18.8276
4	7.0000	0.9000	-0.7966	1.6966
5	7.0000	20.1000	-0.7966	20.8966

$$MSE = 290.24391$$

$$\sum_{i=1}^{n}(x_i - \bar{x})^2 = 34.8$$

$$s(\hat{\beta}_1) = \sqrt{MSE \div \sum_{i=1}^{n}(x_i - \bar{x})^2} = 2.88797$$

The t-value equals: $t^* = \frac{\hat{\beta}_1 - 0}{s(\hat{\beta}_1)} = -1.58446$

The t-values - lower and upper for .95; n-2: -3.1824 and 3.1824

□

Problem 4. Test if the slope equals zero or not for the simple linear regression model.

	x	y
1	6.00	-14.30
2	2.00	-7.20
3	4.00	-5.60
4	9.00	-25.60
5	8.00	36.60
6	2.00	7.00
7	2.00	-13.10

Table 7: The independent and response variable.

Solution: The regression table and other pertinent information

	Estimate	Std. Error	t value	Pr(>|t|)
(Intercept)	-6.0176	16.4233	-0.37	0.7291
x	0.6037	3.0056	0.20	0.8487

Table 8: Linear regression model output.

	x	y	yhat	resid
1	6.0000	-14.3000	-2.3952	-11.9048
2	2.0000	-7.2000	-4.8102	-2.3898
3	4.0000	-5.6000	-3.6027	-1.9973
4	9.0000	-25.6000	-0.5840	-25.0160
5	8.0000	36.6000	-1.1877	37.7877
6	2.0000	7.0000	-4.8102	11.8102
7	2.0000	-13.1000	-4.8102	-8.2898

$$MSE = 482.66785$$

$$\sum_{i=1}^{n}(x_i - \bar{x})^2 = 53.4285714285714$$

$$s(\hat{\beta}_1) = \sqrt{MSE \div \sum_{i=1}^{n}(x_i - \bar{x})^2} = 3.00564$$

The t-value equals: $t^* = \frac{\hat{\beta}_1 - 0}{s(\hat{\beta}_1)} = 0.20087$

The t-values - lower and upper for .95; n-2: -2.5706 and 2.5706

□

Problem 5. Test if the slope equals zero or not for the simple linear regression model.

	x	y
1	5.00	-26.30
2	7.00	-38.80
3	5.00	-35.00
4	1.00	2.50
5	5.00	2.40
6	9.00	8.30
7	8.00	-19.40

Table 9: The independent and response variable.

Solution: The regression table and other pertinent information

	Estimate	Std. Error	t value	Pr(>|t|)
(Intercept)	-10.1276	20.3898	-0.50	0.6405
x	-0.8852	3.2831	-0.27	0.7982

Table 10: Linear regression model output.

	x	y	yhat	resid
1	5.0000	-26.3000	-14.5534	-11.7466
2	7.0000	-38.8000	-16.3238	-22.4762
3	5.0000	-35.0000	-14.5534	-20.4466
4	1.0000	2.5000	-11.0128	13.5128
5	5.0000	2.4000	-14.5534	16.9534
6	9.0000	8.3000	-18.0941	26.3941
7	8.0000	-19.4000	-17.2090	-2.1910

$$MSE = 446.53761$$

$$\sum_{i=1}^{n}(x_i - \bar{x})^2 = 41.4285714285714$$

$$s(\hat{\beta}_1) = \sqrt{MSE \div \sum_{i=1}^{n}(x_i - \bar{x})^2} = 3.28306$$

The t-value equals: $t^* = \frac{\hat{\beta}_1 - 0}{s(\hat{\beta}_1)} = -0.26962$

The t-values - lower and upper for .95; n-2: -2.5706 and 2.5706

□

Problem 6. Test if the slope equals zero or not for the simple linear regression model.

	x	y
1	5.00	-12.70
2	5.00	11.00
3	1.00	-8.00
4	0.00	-6.90
5	9.00	28.20
6	9.00	2.10

Table 11: The independent and response variable.

Solution: The regression table and other pertinent information

	Estimate	Std. Error	t value	Pr(>|t|)
(Intercept)	-10.3243	9.0133	-1.15	0.3159
x	2.6085	1.5128	1.72	0.1597

Table 12: Linear regression model output.

	x	y	yhat	resid
1	5.0000	-12.7000	2.7181	-15.4181
2	5.0000	11.0000	2.7181	8.2819
3	1.0000	-8.0000	-7.7158	-0.2842
4	0.0000	-6.9000	-10.3243	3.4243
5	9.0000	28.2000	13.1519	15.0481
6	9.0000	2.1000	13.1519	-11.0519

$$
\begin{aligned}
MSE &= 166.67578 \\
\sum_{i=1}^{n}(x_i - \bar{x})^2 &= 72.8333333333333 \\
s(\hat{\beta}_1) &= \sqrt{MSE \div \sum_{i=1}^{n}(x_i - \bar{x})^2} = 1.51276
\end{aligned}
$$

The t-value equals: $t^* = \frac{\hat{\beta}_1 - 0}{s(\hat{\beta}_1)} = 1.72431$

The t-values - lower and upper for .95; n-2: -2.7764 and 2.7764

□

Problem 7. Test if the slope equals zero or not for the simple linear regression model.

	x	y
1	2.00	15.50
2	2.00	4.30
3	5.00	11.10
4	2.00	1.80
5	8.00	29.10
6	5.00	-37.80

Table 13: The independent and response variable.

Solution: The regression table and other pertinent information

	Estimate	Std. Error	t value	Pr(>|t|)
(Intercept)	-2.2000	20.8862	-0.11	0.9212
x	1.5500	4.5577	0.34	0.7509

Table 14: Linear regression model output.

	x	y	yhat	resid
1	2.0000	15.5000	0.9000	14.6000
2	2.0000	4.3000	0.9000	3.4000
3	5.0000	11.1000	5.5500	5.5500
4	2.0000	1.8000	0.9000	0.9000
5	8.0000	29.1000	10.2000	18.9000
6	5.0000	-37.8000	5.5500	-43.3500

$$MSE = 623.19125$$

$$\sum_{i=1}^{n}(x_i - \bar{x})^2 = 30$$

$$s(\hat{\beta}_1) = \sqrt{MSE \div \sum_{i=1}^{n}(x_i - \bar{x})^2 = 4.55775}$$

The t-value equals: $t^* = \frac{\hat{\beta}_1 - 0}{s(\hat{\beta}_1)} = 0.34008$

The t-values - lower and upper for .95; n-2: -2.7764 and 2.7764

□

Problem 8. Test if the slope equals zero or not for the simple linear regression model.

	x	y
1	3.00	20.80
2	6.00	-10.00
3	8.00	-20.30
4	4.00	-4.10
5	6.00	-15.20

Table 15: The independent and response variable.

Solution: The regression table and other pertinent information

	Estimate	Std. Error	t value	Pr(>|t|)
(Intercept)	34.0579	11.8914	2.86	0.0644
x	-7.3737	2.0956	-3.52	0.0390

Table 16: Linear regression model output.

	x	y	yhat	resid
1	3.0000	20.8000	11.9368	8.8632
2	6.0000	-10.0000	-10.1842	0.1842
3	8.0000	-20.3000	-24.9316	4.6316
4	4.0000	-4.1000	4.5632	-8.6632
5	6.0000	-15.2000	-10.1842	-5.0158

$$
\begin{aligned}
MSE &= 66.74982 \\
\sum_{i=1}^{n}(x_i-\bar{x})^2 &= 15.2 \\
s(\hat{\beta}_1) &= \sqrt{MSE \div \sum_{i=1}^{n}(x_i-\bar{x})^2} = 2.09558
\end{aligned}
$$

The t-value equals: $t^* = \frac{\hat{\beta}_1 - 0}{s(\hat{\beta}_1)} = -3.51869$

The t-values - lower and upper for .95; n-2: -3.1824 and 3.1824

□

Problem 9. Test if the slope equals zero or not for the simple linear regression model.

	x	y
1	7.00	26.60
2	4.00	32.90
3	3.00	7.80
4	3.00	7.60
5	3.00	-2.40
6	1.00	5.10
7	6.00	12.70

Table 17: The independent and response variable.

Solution: The regression table and other pertinent information

	Estimate	Std. Error	t value	Pr(>|t|)
(Intercept)	-1.0345	9.5287	-0.11	0.9178
x	3.6126	2.2197	1.63	0.1645

Table 18: Linear regression model output.

	x	y	yhat	resid
1	7.0000	26.6000	24.2540	2.3460
2	4.0000	32.9000	13.4161	19.4839
3	3.0000	7.8000	9.8034	-2.0034
4	3.0000	7.6000	9.8034	-2.2034
5	3.0000	-2.4000	9.8034	-12.2034
6	1.0000	5.1000	2.5782	2.5218
7	6.0000	12.7000	20.6414	-7.9414

$$MSE = 122.46892$$

$$\sum_{i=1}^{n}(x_i - \bar{x})^2 = 24.8571428571429$$

$$s(\hat{\beta}_1) = \sqrt{MSE \div \sum_{i=1}^{n}(x_i - \bar{x})^2 = 2.21966}$$

The t-value equals: $t^* = \frac{\hat{\beta}_1 - 0}{s(\hat{\beta}_1)} = 1.62756$

The t-values - lower and upper for .95; n-2: -2.5706 and 2.5706

□

Problem 10. Test if the slope equals zero or not for the simple linear regression model.

	x	y
1	5.00	-37.40
2	7.00	4.50
3	8.00	-43.40
4	2.00	0.70
5	9.00	-56.30

Table 19: The independent and response variable.

Solution: The regression table and other pertinent information

	Estimate	Std. Error	t value	Pr(>|t|)
(Intercept)	11.9110	29.6265	0.40	0.7146
x	-6.1760	4.4362	-1.39	0.2581

Table 20: Linear regression model output.

	x	y	yhat	resid
1	5.0000	-37.4000	-18.9688	-18.4312
2	7.0000	4.5000	-31.3208	35.8208
3	8.0000	-43.4000	-37.4968	-5.9032
4	2.0000	0.7000	-0.4409	1.1409
5	9.0000	-56.3000	-43.6727	-12.6273

$$
\begin{aligned}
MSE &= 606.14474 \\
\sum_{i=1}^{n}(x_i - \bar{x})^2 &= 30.8 \\
s(\hat{\beta}_1) &= \sqrt{MSE \div \sum_{i=1}^{n}(x_i - \bar{x})^2} = 4.43622
\end{aligned}
$$

The t-value equals: $t^* = \frac{\hat{\beta}_1 - 0}{s(\hat{\beta}_1)} = -1.39217$

The t-values - lower and upper for .95; n-2: -3.1824 and 3.1824

□

Problem 11. Create a 95% confidence interval for the slope.

	x	y
1	8.00	-21.50
2	2.00	-5.50
3	1.00	-7.60
4	2.00	-7.30

Table 21: The independent and response variable.

Solution: The regression table and other pertinent information

	Estimate	Std. Error	t value	Pr(>|t|)
(Intercept)	-3.1797	1.6971	-1.87	0.2019
x	-2.2447	0.3973	-5.65	0.0299

Table 22: Linear regression model output.

	x	y	yhat	resid
1	8.0000	-21.5000	-21.1374	-0.3626
2	2.0000	-5.5000	-7.6691	2.1691
3	1.0000	-7.6000	-5.4244	-2.1756
4	2.0000	-7.3000	-7.6691	0.3691

$$
\begin{aligned}
MSE &= 4.85301 \\
\sum_{i=1}^{n}(x_i - \bar{x})^2 &= 30.75 \\
s(\hat{\beta}_1) &= \sqrt{MSE \div \sum_{i=1}^{n}(x_i - \bar{x})^2} = 0.39727
\end{aligned}
$$

The t-values - lower and upper for .95; n-2: -4.3027 and 4.3027
The confidence interval for the slope:

The lower -3.954 and upper -0.5354 limit.

□

Problem 12. Create a 95% confidence interval for the slope.

	x	y
1	1.00	13.20
2	0.00	8.90
3	0.00	5.60
4	4.00	16.40

Table 23: The independent and response variable.

Solution: The regression table and other pertinent information

	Estimate	Std. Error	t value	Pr(>|t|)
(Intercept)	8.2721	1.7463	4.74	0.0418
x	2.2023	0.8471	2.60	0.1216

Table 24: Linear regression model output.

	x	y	yhat	resid
1	1.0000	13.2000	10.4744	2.7256
2	0.0000	8.9000	8.2721	0.6279
3	0.0000	5.6000	8.2721	-2.6721
4	4.0000	16.4000	17.0814	-0.6814

$$MSE = 7.71372$$

$$\sum_{i=1}^{n}(x_i - \bar{x})^2 = 10.75$$

$$s(\hat{\beta}_1) = \sqrt{MSE \div \sum_{i=1}^{n}(x_i - \bar{x})^2} = 0.84709$$

The t-values - lower and upper for .95; n-2: -4.3027 and 4.3027
The confidence interval for the slope:

The lower -1.4424 and upper 5.847 limit.

$\square$

Problem 13. Create a 95% confidence interval for the slope.

	x	y
1	6.00	-0.20
2	9.00	2.10
3	2.00	-3.20
4	1.00	-5.90

Table 25: The independent and response variable.

Solution: The regression table and other pertinent information

	Estimate	Std. Error	t value	Pr(>|t|)
(Intercept)	-5.9488	0.7929	-7.50	0.0173
x	0.9220	0.1436	6.42	0.0234

Table 26: Linear regression model output.

	x	y	yhat	resid
1	6.0000	-0.2000	-0.4171	0.2171
2	9.0000	2.1000	2.3488	-0.2488
3	2.0000	-3.2000	-4.1049	0.9049
4	1.0000	-5.9000	-5.0268	-0.8732

$$
\begin{aligned}
MSE &= 0.84512 \\
\sum_{i=1}^{n}(x_i - \bar{x})^2 &= 41 \\
s(\hat{\beta}_1) &= \sqrt{MSE \div \sum_{i=1}^{n}(x_i - \bar{x})^2} = 0.14357
\end{aligned}
$$

The t-values - lower and upper for .95; n-2: -4.3027 and 4.3027
The confidence interval for the slope:

The lower 0.3042 and upper 1.5397 limit.

□

Problem 14. Create a 95% confidence interval for the slope.

	x	y
1	4.00	32.00
2	9.00	55.10
3	5.00	36.90
4	3.00	26.30

Table 27: The independent and response variable.

Solution: The regression table and other pertinent information

	Estimate	Std. Error	t value	Pr(>|t|)
(Intercept)	12.7229	0.8027	15.85	0.0040
x	4.7337	0.1403	33.75	0.0009

Table 28: Linear regression model output.

	x	y	yhat	resid
1	4.0000	32.0000	31.6578	0.3422
2	9.0000	55.1000	55.3265	-0.2265
3	5.0000	36.9000	36.3916	0.5084
4	3.0000	26.3000	26.9241	-0.6241

$$MSE = 0.40819$$

$$\sum_{i=1}^{n}(x_i - \bar{x})^2 = 20.75$$

$$s(\hat{\beta}_1) = \sqrt{MSE \div \sum_{i=1}^{n}(x_i - \bar{x})^2} = 0.14026$$

The t-values - lower and upper for .95; n-2: -4.3027 and 4.3027
The confidence interval for the slope:

The lower 4.1303 and upper 5.3372 limit.

□

Problem 15. Create a 95% confidence interval for the slope.

	x	y
1	3.00	-21.80
2	5.00	-27.70
3	3.00	-20.30
4	8.00	-44.20

Table 29: The independent and response variable.

Solution: The regression table and other pertinent information

	Estimate	Std. Error	t value	Pr(>|t|)
(Intercept)	-6.6925	2.2266	-3.01	0.0952
x	-4.5910	0.4305	-10.66	0.0087

Table 30: Linear regression model output.

	x	y	yhat	resid
1	3.0000	-21.8000	-20.4657	-1.3343
2	5.0000	-27.7000	-29.6478	1.9478
3	3.0000	-20.3000	-20.4657	0.1657
4	8.0000	-44.2000	-43.4209	-0.7791

$$
\begin{aligned}
MSE &= 3.10433 \\
\sum_{i=1}^{n}(x_i - \bar{x})^2 &= 16.75 \\
s(\hat{\beta}_1) &= \sqrt{MSE \div \sum_{i=1}^{n}(x_i - \bar{x})^2} = 0.4305
\end{aligned}
$$

The t-values - lower and upper for .95; n-2: -4.3027 and 4.3027
The confidence interval for the slope:

The lower -6.4434 and upper -2.7387 limit.

□

Problem 16. Create a 95% confidence interval for the slope.

	x	y
1	4.00	-1.30
2	5.00	-2.00
3	6.00	-0.80
4	0.00	1.50

Table 31: The independent and response variable.

Solution: The regression table and other pertinent information

	Estimate	Std. Error	t value	Pr(>|t|)
(Intercept)	1.2024	0.9205	1.31	0.3215
x	-0.4940	0.2098	-2.35	0.1428

Table 32: Linear regression model output.

	x	y	yhat	resid
1	4.0000	-1.3000	-0.7735	-0.5265
2	5.0000	-2.0000	-1.2675	-0.7325
3	6.0000	-0.8000	-1.7614	0.9614
4	0.0000	1.5000	1.2024	0.2976

$$\begin{aligned} MSE &= 0.91337 \\ \sum_{i=1}^{n}(x_i - \bar{x})^2 &= 20.75 \\ s(\hat{\beta}_1) &= \sqrt{MSE \div \sum_{i=1}^{n}(x_i - \bar{x})^2 = 0.2098} \end{aligned}$$

The t-values - lower and upper for .95; n-2: -4.3027 and 4.3027
The confidence interval for the slope:

The lower -1.3967 and upper 0.4087 limit.

$\square$

Problem 17. Create a 95% confidence interval for the slope.

	x	y
1	0.00	-8.50
2	2.00	2.20
3	1.00	-4.30
4	5.00	13.30

Table 33: The independent and response variable.

Solution: The regression table and other pertinent information

	Estimate	Std. Error	t value	Pr(>|t|)
(Intercept)	-8.0679	0.9136	-8.83	0.0126
x	4.3714	0.3336	13.10	0.0058

Table 34: Linear regression model output.

	x	y	yhat	resid
1	0.0000	-8.5000	-8.0679	-0.4321
2	2.0000	2.2000	0.6750	1.5250
3	1.0000	-4.3000	-3.6964	-0.6036
4	5.0000	13.3000	13.7893	-0.4893

$$
\begin{aligned}
MSE &= 1.55804 \\
\sum_{i=1}^{n}(x_i - \bar{x})^2 &= 14 \\
s(\hat{\beta}_1) &= \sqrt{MSE \div \sum_{i=1}^{n}(x_i - \bar{x})^2} = 0.3336
\end{aligned}
$$

The t-values - lower and upper for .95; n-2: -4.3027 and 4.3027
The confidence interval for the slope:

The lower 2.9361 and upper 5.8068 limit.

□

Problem 18. Create a 95% confidence interval for the slope.

	x	y
1	1.00	7.40
2	7.00	12.10
3	0.00	9.70
4	6.00	7.30

Table 35: The independent and response variable.

Solution: The regression table and other pertinent information

	Estimate	Std. Error	t value	Pr(>|t|)
(Intercept)	8.3541	1.9945	4.19	0.0525
x	0.2203	0.4302	0.51	0.6595

Table 36: Linear regression model output.

	x	y	yhat	resid
1	1.0000	7.4000	8.5743	-1.1743
2	7.0000	12.1000	9.8959	2.2041
3	0.0000	9.7000	8.3541	1.3459
4	6.0000	7.3000	9.6757	-2.3757

$$
\begin{aligned}
MSE &= 6.84615 \\
\sum_{i=1}^{n}(x_i - \bar{x})^2 &= 37 \\
s(\hat{\beta}_1) &= \sqrt{MSE \div \sum_{i=1}^{n}(x_i - \bar{x})^2} = 0.43015
\end{aligned}
$$

The t-values - lower and upper for .95; n-2: -4.3027 and 4.3027
The confidence interval for the slope:

The lower -1.6305 and upper 2.0711 limit.

□

Problem 19. Create a 95% confidence interval for the slope.

	x	y
1	2.00	1.00
2	1.00	-0.50
3	8.00	2.80
4	2.00	2.30

Table 37: The independent and response variable.

Solution: The regression table and other pertinent information

	Estimate	Std. Error	t value	Pr(>|t|)
(Intercept)	0.3114	0.9576	0.33	0.7759
x	0.3350	0.2241	1.49	0.2737

Table 38: Linear regression model output.

	x	y	yhat	resid
1	2.0000	1.0000	0.9813	0.0187
2	1.0000	-0.5000	0.6463	-1.1463
3	8.0000	2.8000	2.9911	-0.1911
4	2.0000	2.3000	0.9813	1.3187

$$MSE = 1.54496$$

$$\sum_{i=1}^{n}(x_i - \bar{x})^2 = 30.75$$

$$s(\hat{\beta}_1) = \sqrt{MSE \div \sum_{i=1}^{n}(x_i - \bar{x})^2} = 0.22415$$

The t-values - lower and upper for .95; n-2: -4.3027 and 4.3027
The confidence interval for the slope:

The lower -0.6295 and upper 1.2994 limit.

□

Problem 20. Create a 95% confidence interval for the slope.

	x	y
1	2.00	11.80
2	7.00	23.20
3	9.00	32.90
4	4.00	15.50

Table 39: The independent and response variable.

Solution: The regression table and other pertinent information

	Estimate	Std. Error	t value	Pr(>|t|)
(Intercept)	4.6534	2.5123	1.85	0.2052
x	2.9448	0.4103	7.18	0.0189

Table 40: Linear regression model output.

	x	y	yhat	resid
1	2.0000	11.8000	10.5431	1.2569
2	7.0000	23.2000	25.2672	-2.0672
3	9.0000	32.9000	31.1569	1.7431
4	4.0000	15.5000	16.4328	-0.9328

$$MSE = 4.88086$$

$$\sum_{i=1}^{n}(x_i - \bar{x})^2 = 29$$

$$s(\hat{\beta}_1) = \sqrt{MSE \div \sum_{i=1}^{n}(x_i - \bar{x})^2} = 0.41025$$

The t-values - lower and upper for .95; n-2: -4.3027 and 4.3027
The confidence interval for the slope:

The lower 1.1797 and upper 4.71 limit.

□

Problem 21. Create a 95% confidence interval for $E[Y_h]$ and prediction interval for y_h at $x_h = 8$.

	x	y
1	4.00	-6.60
2	8.00	11.80
3	0.00	-18.00
4	2.00	-15.00

Table 41: The independent and response variable.

Solution: The regression table and other pertinent information

	Estimate	Std. Error	t value	Pr(>|t|)
(Intercept)	-20.4800	2.1605	-9.48	0.0109
x	3.8657	0.4715	8.20	0.0145

Table 42: Linear regression model output.

	x	y	yhat	resid
1	4.0000	-6.6000	-5.0171	-1.5829
2	8.0000	11.8000	10.4457	1.3543
3	0.0000	-18.0000	-20.4800	2.4800
4	2.0000	-15.0000	-12.7486	-2.2514

$$
\begin{aligned}
\hat{y}_h &= 10.44571 \\
MSE &= 7.77943 \\
\sum_{i=1}^{n}(x_i - \bar{x})^2 &= 35 \\
s(\hat{\beta}_1) &= \sqrt{MSE \div \sum_{i=1}^{n}(x_i - \bar{x})^2} = 0.47145 \\
s^2(\hat{Y}_h) &= MSE[\frac{1}{n} + \frac{(x_h - \bar{x})^2}{\sum_{i=1}^{n}(x_i - \bar{x})^2}] = 6.44581 \\
s^2(pred) &= MSE[1 + \frac{1}{n} + \frac{(x_h - \bar{x})^2}{\sum_{i=1}^{n}(x_i - \bar{x})^2}] = 14.22524
\end{aligned}
$$

	lower	upper
t-values for 95 percent	-4.3027	4.3027
confidence interval	-0.4781	21.3695
prediction interval	-5.7823	26.6738

□

Problem 22. Create a 95% confidence interval for $E[Y_h]$ and prediction interval for y_h at $x_h = 8$.

	x	y
1	4.00	-2.90
2	8.00	14.70
3	5.00	-1.50
4	8.00	15.10

Table 43: The independent and response variable.

Solution: The regression table and other pertinent information

	Estimate	Std. Error	t value	Pr(>|t|)
(Intercept)	-23.3314	3.1310	-7.45	0.0175
x	4.7490	0.4817	9.86	0.0101

Table 44: Linear regression model output.

	x	y	yhat	resid
1	4.0000	-2.9000	-4.3353	1.4353
2	8.0000	14.7000	14.6608	0.0392
3	5.0000	-1.5000	0.4137	-1.9137
4	8.0000	15.1000	14.6608	0.4392

$$\hat{y}_h = 14.66078$$

$$MSE = 2.95843$$

$$\sum_{i=1}^{n}(x_i - \bar{x})^2 = 12.75$$

$$s(\hat{\beta}_1) = \sqrt{MSE \div \sum_{i=1}^{n}(x_i - \bar{x})^2} = 0.4817$$

$$s^2(\hat{Y}_h) = MSE[\frac{1}{n} + \frac{(x_h - \bar{x})^2}{\sum_{i=1}^{n}(x_i - \bar{x})^2}] = 1.45021$$

$$s^2(pred) = MSE[1 + \frac{1}{n} + \frac{(x_h - \bar{x})^2}{\sum_{i=1}^{n}(x_i - \bar{x})^2}] = 4.40864$$

	lower	upper
t-values for 95 percent	-4.3027	4.3027
confidence interval	9.4793	19.8422
prediction interval	5.6266	23.6950

□

Problem 23. Create a 95% confidence interval for $E[Y_h]$ and prediction interval for y_h at $x_h = 8$.

	x	y
1	4.00	8.10
2	8.00	24.00
3	6.00	18.80
4	9.00	25.40

Table 45: The independent and response variable.

Solution: The regression table and other pertinent information

	Estimate	Std. Error	t value	Pr(>|t|)
(Intercept)	-4.1610	4.0861	-1.02	0.4157
x	3.4424	0.5823	5.91	0.0274

Table 46: Linear regression model output.

	x	y	yhat	resid
1	4.0000	8.1000	9.6085	-1.5085
2	8.0000	24.0000	23.3780	0.6220
3	6.0000	18.8000	16.4932	2.3068
4	9.0000	25.4000	26.8203	-1.4203

$$\hat{y}_h = 23.37797$$

$$MSE = 5.00051$$

$$\sum_{i=1}^{n}(x_i - \bar{x})^2 = 14.75$$

$$s(\hat{\beta}_1) = \sqrt{MSE \div \sum_{i=1}^{n}(x_i - \bar{x})^2} = 0.58225$$

$$s^2(\hat{Y}_h) = MSE[\frac{1}{n} + \frac{(x_h - \bar{x})^2}{\sum_{i=1}^{n}(x_i - \bar{x})^2}] = 1.77984$$

$$s^2(pred) = MSE[1 + \frac{1}{n} + \frac{(x_h - \bar{x})^2}{\sum_{i=1}^{n}(x_i - \bar{x})^2}] = 6.78035$$

	lower	upper
t-values for 95 percent	-4.3027	4.3027
confidence interval	17.6378	29.1182
prediction interval	12.1742	34.5817

□

Problem 24. Create a 95% confidence interval for $E[Y_h]$ and prediction interval for y_h at $x_h = 3$.

	x	y
1	0.00	14.70
2	3.00	18.00
3	7.00	11.20
4	0.00	16.00

Table 47: The independent and response variable.

Solution: The regression table and other pertinent information

	Estimate	Std. Error	t value	Pr(>|t|)
(Intercept)	16.2894	1.8397	8.85	0.0125
x	-0.5258	0.4831	-1.09	0.3902

Table 48: Linear regression model output.

	x	y	yhat	resid
1	0.0000	14.7000	16.2894	-1.5894
2	3.0000	18.0000	14.7121	3.2879
3	7.0000	11.2000	12.6091	-1.4091
4	0.0000	16.0000	16.2894	-0.2894

$$\hat{y}_h = 14.71212$$

$$MSE = 7.7028$$

$$\sum_{i=1}^{n}(x_i - \bar{x})^2 = 33$$

$$s(\hat{\beta}_1) = \sqrt{MSE \div \sum_{i=1}^{n}(x_i - \bar{x})^2} = 0.48313$$

$$s^2(\hat{Y}_h) = MSE[\frac{1}{n} + \frac{(x_h - \bar{x})^2}{\sum_{i=1}^{n}(x_i - \bar{x})^2}] = 1.98406$$

$$s^2(pred) = MSE[1 + \frac{1}{n} + \frac{(x_h - \bar{x})^2}{\sum_{i=1}^{n}(x_i - \bar{x})^2}] = 9.68686$$

	lower	upper
t-values for 95 percent	-4.3027	4.3027
confidence interval	8.6516	20.7727
prediction interval	1.3207	28.1036

□

Problem 25. Create a 95% confidence interval for $E[Y_h]$ and prediction interval for y_h at $x_h = 6$.

	x	y
1	5.00	1.60
2	6.00	-0.60
3	3.00	-1.50
4	2.00	0.50

Table 49: The independent and response variable.

Solution: The regression table and other pertinent information

	Estimate	Std. Error	t value	Pr(>|t|)
(Intercept)	-0.3600	2.2223	-0.16	0.8862
x	0.0900	0.5167	0.17	0.8778

Table 50: Linear regression model output.

	x	y	yhat	resid
1	5.0000	1.6000	0.0900	1.5100
2	6.0000	-0.6000	0.1800	-0.7800
3	3.0000	-1.5000	-0.0900	-1.4100
4	2.0000	0.5000	-0.1800	0.6800

$$\hat{y}_h = 0.18$$

$$MSE = 2.6695$$

$$\sum_{i=1}^{n}(x_i - \bar{x})^2 = 10$$

$$s(\hat{\beta}_1) = \sqrt{MSE \div \sum_{i=1}^{n}(x_i - \bar{x})^2} = 0.51667$$

$$s^2(\hat{Y}_h) = MSE[\frac{1}{n} + \frac{(x_h - \bar{x})^2}{\sum_{i=1}^{n}(x_i - \bar{x})^2}] = 1.73518$$

$$s^2(pred) = MSE[1 + \frac{1}{n} + \frac{(x_h - \bar{x})^2}{\sum_{i=1}^{n}(x_i - \bar{x})^2}] = 4.40468$$

	lower	upper
t-values for 95 percent	-4.3027	4.3027
confidence interval	-5.4877	5.8477
prediction interval	-8.8501	9.2101

□

Problem 26. Create a 95% confidence interval for $E[Y_h]$ and prediction interval for y_h at $x_h = 1$.

	x	y
1	4.00	-9.10
2	1.00	-12.00
3	0.00	-18.00
4	7.00	0.70

Table 51: The independent and response variable.

Solution: The regression table and other pertinent information

	Estimate	Std. Error	t value	Pr(>|t|)
(Intercept)	-16.7700	1.7626	-9.51	0.0109
x	2.3900	0.4339	5.51	0.0314

Table 52: Linear regression model output.

	x	y	yhat	resid
1	4.0000	-9.1000	-7.2100	-1.8900
2	1.0000	-12.0000	-14.3800	2.3800
3	0.0000	-18.0000	-16.7700	-1.2300
4	7.0000	0.7000	-0.0400	0.7400

$$\hat{y}_h = -14.38$$

$$MSE = 5.6485$$

$$\sum_{i=1}^{n}(x_i - \bar{x})^2 = 30$$

$$s(\hat{\beta}_1) = \sqrt{MSE \div \sum_{i=1}^{n}(x_i - \bar{x})^2} = 0.43392$$

$$s^2(\hat{Y}_h) = MSE[\frac{1}{n} + \frac{(x_h - \bar{x})^2}{\sum_{i=1}^{n}(x_i - \bar{x})^2}] = 2.16526$$

$$s^2(pred) = MSE[1 + \frac{1}{n} + \frac{(x_h - \bar{x})^2}{\sum_{i=1}^{n}(x_i - \bar{x})^2}] = 7.81376$$

	lower	upper
t-values for 95 percent	-4.3027	4.3027
confidence interval	-20.7113	-8.0487
prediction interval	-26.4072	-2.3528

□

Problem 27. Create a 95% confidence interval for $E[Y_h]$ and prediction interval for y_h at $x_h = 1$.

	x	y
1	9.00	4.50
2	1.00	0.30
3	9.00	7.20
4	4.00	2.60

Table 53: The independent and response variable.

Solution: The regression table and other pertinent information

	Estimate	Std. Error	t value	Pr(>|t|)
(Intercept)	-0.2920	1.3267	-0.22	0.8462
x	0.6856	0.1983	3.46	0.0745

Table 54: Linear regression model output.

	x	y	yhat	resid
1	9.0000	4.5000	5.8781	-1.3781
2	1.0000	0.3000	0.3936	-0.0936
3	9.0000	7.2000	5.8781	1.3219
4	4.0000	2.6000	2.4503	0.1497

$$\hat{y}_h = 0.39358$$

$$MSE = 1.83888$$

$$\sum_{i=1}^{n}(x_i - \bar{x})^2 = 46.75$$

$$s(\hat{\beta}_1) = \sqrt{MSE \div \sum_{i=1}^{n}(x_i - \bar{x})^2} = 0.19833$$

$$s^2(\hat{Y}_h) = MSE[\frac{1}{n} + \frac{(x_h - \bar{x})^2}{\sum_{i=1}^{n}(x_i - \bar{x})^2}] = 1.3472$$

$$s^2(pred) = MSE[1 + \frac{1}{n} + \frac{(x_h - \bar{x})^2}{\sum_{i=1}^{n}(x_i - \bar{x})^2}] = 3.18608$$

	lower	upper
t-values for 95 percent	-4.3027	4.3027
confidence interval	-4.6005	5.3876
prediction interval	-7.2865	8.0736

□

Problem 28. Create a 95% confidence interval for $E[Y_h]$ and prediction interval for y_h at $x_h = 2$.

	x	y
1	7.00	34.60
2	2.00	18.10
3	6.00	32.60
4	6.00	31.40

Table 55: The independent and response variable.

Solution: The regression table and other pertinent information

	Estimate	Std. Error	t value	Pr(>|t|)
(Intercept)	11.4763	1.0966	10.47	0.0090
x	3.3712	0.1962	17.18	0.0034

Table 56: Linear regression model output.

	x	y	yhat	resid
1	7.0000	34.6000	35.0746	-0.4746
2	2.0000	18.1000	18.2186	-0.1186
3	6.0000	32.6000	31.7034	0.8966
4	6.0000	31.4000	31.7034	-0.3034

$$\hat{y}_h = 18.21864$$

$$MSE = 0.56763$$

$$\sum_{i=1}^{n}(x_i - \bar{x})^2 = 14.75$$

$$s(\hat{\beta}_1) = \sqrt{MSE \div \sum_{i=1}^{n}(x_i - \bar{x})^2} = 0.19617$$

$$s^2(\hat{Y}_h) = MSE[\frac{1}{n} + \frac{(x_h - \bar{x})^2}{\sum_{i=1}^{n}(x_i - \bar{x})^2}] = 0.54839$$

$$s^2(pred) = MSE[1 + \frac{1}{n} + \frac{(x_h - \bar{x})^2}{\sum_{i=1}^{n}(x_i - \bar{x})^2}] = 1.11601$$

	lower	upper
t-values for 95 percent	-4.3027	4.3027
confidence interval	15.0324	21.4049
prediction interval	13.6733	22.7640

□

Problem 29. Create a 95% confidence interval for $E[Y_h]$ and prediction interval for y_h at $x_h = 8$.

	x	y
1	0.00	15.50
2	8.00	-24.40
3	4.00	-4.40
4	6.00	-15.20

Table 57: The independent and response variable.

Solution: The regression table and other pertinent information

	Estimate	Std. Error	t value	Pr(>|t|)
(Intercept)	15.4714	0.4228	36.60	0.0007
x	-5.0214	0.0785	-63.96	0.0002

Table 58: Linear regression model output.

	x	y	yhat	resid
1	0.0000	15.5000	15.4714	0.0286
2	8.0000	-24.4000	-24.7000	0.3000
3	4.0000	-4.4000	-4.6143	0.2143
4	6.0000	-15.2000	-14.6571	-0.5429

$$\hat{y}_h = -24.7$$

$$MSE = 0.21571$$

$$\sum_{i=1}^{n}(x_i - \bar{x})^2 = 35$$

$$s(\hat{\beta}_1) = \sqrt{MSE \div \sum_{i=1}^{n}(x_i - \bar{x})^2} = 0.07851$$

$$s^2(\hat{Y}_h) = MSE[\frac{1}{n} + \frac{(x_h - \bar{x})^2}{\sum_{i=1}^{n}(x_i - \bar{x})^2}] = 0.12943$$

$$s^2(pred) = MSE[1 + \frac{1}{n} + \frac{(x_h - \bar{x})^2}{\sum_{i=1}^{n}(x_i - \bar{x})^2}] = 0.34514$$

	lower	upper
t-values for 95 percent	-4.3027	4.3027
confidence interval	-26.2479	-23.1521
prediction interval	-27.2278	-22.1722

□

Problem 30. Create a 95% confidence interval for $E[Y_h]$ and prediction interval for y_h at $x_h = 3$.

	x	y
1	6.00	-26.30
2	3.00	-24.10
3	4.00	-23.90
4	7.00	-25.40

Table 59: The independent and response variable.

Solution: The regression table and other pertinent information

	Estimate	Std. Error	t value	Pr(>|t|)
(Intercept)	-22.4250	1.3612	-16.47	0.0037
x	-0.5000	0.2596	-1.93	0.1939

Table 60: Linear regression model output.

	x	y	yhat	resid
1	6.0000	-26.3000	-25.4250	-0.8750
2	3.0000	-24.1000	-23.9250	-0.1750
3	4.0000	-23.9000	-24.4250	0.5250
4	7.0000	-25.4000	-25.9250	0.5250

$$\hat{y}_h = -23.925$$

$$MSE = 0.67375$$

$$\sum_{i=1}^{n}(x_i - \bar{x})^2 = 10$$

$$s(\hat{\beta}_1) = \sqrt{MSE \div \sum_{i=1}^{n}(x_i - \bar{x})^2} = 0.25957$$

$$s^2(\hat{Y}_h) = MSE[\frac{1}{n} + \frac{(x_h - \bar{x})^2}{\sum_{i=1}^{n}(x_i - \bar{x})^2}] = 0.43794$$

$$s^2(pred) = MSE[1 + \frac{1}{n} + \frac{(x_h - \bar{x})^2}{\sum_{i=1}^{n}(x_i - \bar{x})^2}] = 1.11169$$

	lower	upper
t-values for 95 percent	-4.3027	4.3027
confidence interval	-26.7724	-21.0776
prediction interval	-28.4616	-19.3884

□

Problem 31. Test if the slope equals zero or not for the simple linear regression model. Also, create a 95% confidence interval for $E[Y_h]$ and prediction interval for y_h at $x_h = 9$.

	x	y
1	2.00	9.10
2	9.00	1.90
3	4.00	6.20
4	6.00	3.60

Table 61: The independent and response variable.

Solution: The regression table and other pertinent information

	Estimate	Std. Error	t value	Pr(>|t|)
(Intercept)	10.5972	0.9430	11.24	0.0078
x	-1.0280	0.1611	-6.38	0.0237

Table 62: Linear regression model output.

	x	y	yhat	resid
1	2.0000	9.1000	8.5411	0.5589
2	9.0000	1.9000	1.3449	0.5551
3	4.0000	6.2000	6.4850	-0.2850
4	6.0000	3.6000	4.4290	-0.8290

$$\hat{y}_h = 1.34486$$

$$MSE = 0.69449$$

$$\sum_{i=1}^{n}(x_i - \bar{x})^2 = 26.75$$

$$s(\hat{\beta}_1) = \sqrt{MSE \div \sum_{i=1}^{n}(x_i - \bar{x})^2} = 0.16113$$

$$s^2(\hat{Y}_h) = MSE[\frac{1}{n} + \frac{(x_h - \bar{x})^2}{\sum_{i=1}^{n}(x_i - \bar{x})^2}] = 0.53871$$

$$s^2(pred) = MSE[1 + \frac{1}{n} + \frac{(x_h - \bar{x})^2}{\sum_{i=1}^{n}(x_i - \bar{x})^2}] = 1.2332$$

The t-value equals: $t^* = \frac{\hat{\beta}_1 - 0}{s(\hat{\beta}_1)} = -6.38027$

	lower	upper
t-values for 95 percent	-4.3027	4.3027
confidence interval	-1.8132	4.5029
prediction interval	-3.4332	6.1229

□

Problem 32. Test if the slope equals zero or not for the simple linear regression model. Also, create a 95% confidence interval for $E[Y_h]$ and prediction interval for y_h at $x_h = 1$.

	x	y
1	1.00	-12.60
2	1.00	-12.00
3	6.00	-4.50
4	9.00	-6.00

Table 63: The independent and response variable.

Solution: The regression table and other pertinent information

	Estimate	Std. Error	t value	Pr(>|t|)
(Intercept)	-12.7364	1.8184	-7.00	0.0198
x	0.9321	0.3334	2.80	0.1077

Table 64: Linear regression model output.

	x	y	yhat	resid
1	1.0000	-12.6000	-11.8043	-0.7957
2	1.0000	-12.0000	-11.8043	-0.1957
3	6.0000	-4.5000	-7.1439	2.6439
4	9.0000	-6.0000	-4.3476	-1.6524

$$\hat{y}_h = -11.80428$$

$$MSE = 5.19594$$

$$\sum_{i=1}^{n}(x_i - \bar{x})^2 = 46.75$$

$$s(\hat{\beta}_1) = \sqrt{MSE \div \sum_{i=1}^{n}(x_i - \bar{x})^2} = 0.33338$$

$$s^2(\hat{Y}_h) = MSE[\frac{1}{n} + \frac{(x_h - \bar{x})^2}{\sum_{i=1}^{n}(x_i - \bar{x})^2}] = 2.47293$$

$$s^2(pred) = MSE[1 + \frac{1}{n} + \frac{(x_h - \bar{x})^2}{\sum_{i=1}^{n}(x_i - \bar{x})^2}] = 7.66887$$

The t-value equals: $t^* = \frac{\hat{\beta}_1 - 0}{s(\hat{\beta}_1)} = 2.79586$

	lower	upper
t-values for 95 percent	-4.3027	4.3027
confidence interval	-18.5704	-5.0381
prediction interval	-23.7195	0.1109

□

Problem 33. Test if the slope equals zero or not for the simple linear regression model. Also, create a 95% confidence interval for $E[Y_h]$ and prediction interval for y_h at $x_h = 9$.

	x	y
1	1.00	-2.00
2	9.00	-1.60
3	9.00	2.10
4	0.00	3.00

Table 65: The independent and response variable.

Solution: The regression table and other pertinent information

	Estimate	Std. Error	t value	Pr(>|t|)
(Intercept)	0.6770	2.3137	0.29	0.7974
x	-0.0636	0.3624	-0.18	0.8769

Table 66: Linear regression model output.

	x	y	yhat	resid
1	1.0000	-2.0000	0.6134	-2.6134
2	9.0000	-1.6000	0.1048	-1.7048
3	9.0000	2.1000	0.1048	1.9952
4	0.0000	3.0000	0.6770	2.3230

$$\hat{y}_h = 0.10481$$

$$MSE = 9.55674$$

$$\sum_{i=1}^{n}(x_i - \bar{x})^2 = 72.75$$

$$s(\hat{\beta}_1) = \sqrt{MSE \div \sum_{i=1}^{n}(x_i - \bar{x})^2} = 0.36244$$

$$s^2(\hat{Y}_h) = MSE[\frac{1}{n} + \frac{(x_h - \bar{x})^2}{\sum_{i=1}^{n}(x_i - \bar{x})^2}] = 4.76195$$

$$s^2(pred) = MSE[1 + \frac{1}{n} + \frac{(x_h - \bar{x})^2}{\sum_{i=1}^{n}(x_i - \bar{x})^2}] = 14.31868$$

The t-value equals: $t^* = \frac{\hat{\beta}_1 - 0}{s(\hat{\beta}_1)} = -0.1754$

	lower	upper
t-values for 95 percent	-4.3027	4.3027
confidence interval	-9.2844	9.4940
prediction interval	-16.1764	16.3861

□

Problem 34. Test if the slope equals zero or not for the simple linear regression model. Also, create a 95% confidence interval for $E[Y_h]$ and prediction interval for y_h at $x_h = 1$.

	x	y
1	8.00	15.50
2	1.00	-2.90
3	1.00	-6.90
4	6.00	12.00

Table 67: The independent and response variable.

Solution: The regression table and other pertinent information

	Estimate	Std. Error	t value	Pr(>|t|)
(Intercept)	-7.7224	1.9640	-3.93	0.0590
x	3.0368	0.3889	7.81	0.0160

Table 68: Linear regression model output.

	x	y	yhat	resid
1	8.0000	15.5000	16.5724	-1.0724
2	1.0000	-2.9000	-4.6855	1.7855
3	1.0000	-6.9000	-4.6855	-2.2145
4	6.0000	12.0000	10.4987	1.5013

$$\hat{y}_h = -4.68553$$

$$MSE = 5.74796$$

$$\sum_{i=1}^{n}(x_i - \bar{x})^2 = 38$$

$$s(\hat{\beta}_1) = \sqrt{MSE \div \sum_{i=1}^{n}(x_i - \bar{x})^2} = 0.38892$$

$$s^2(\hat{Y}_h) = MSE[\frac{1}{n} + \frac{(x_h - \bar{x})^2}{\sum_{i=1}^{n}(x_i - \bar{x})^2}] = 2.79835$$

$$s^2(pred) = MSE[1 + \frac{1}{n} + \frac{(x_h - \bar{x})^2}{\sum_{i=1}^{n}(x_i - \bar{x})^2}] = 8.54631$$

The t-value equals: $t^* = \frac{\hat{\beta}_1 - 0}{s(\hat{\beta}_1)} = 7.80831$

	lower	upper
t-values for 95 percent	-4.3027	4.3027
confidence interval	-11.8831	2.5121
prediction interval	-17.2639	7.8929

□

Problem 35. Test if the slope equals zero or not for the simple linear regression model. Also, create a 95% confidence interval for $E[Y_h]$ and prediction interval for y_h at $x_h = 6$.

	x	y
1	0.00	11.30
2	6.00	-6.30
3	6.00	-7.20
4	4.00	3.00

Table 69: The independent and response variable.

Solution: The regression table and other pertinent information

	Estimate	Std. Error	t value	Pr(>|t|)
(Intercept)	12.2333	2.2309	5.48	0.0317
x	-3.0083	0.4756	-6.33	0.0241

Table 70: Linear regression model output.

	x	y	yhat	resid
1	0.0000	11.3000	12.2333	-0.9333
2	6.0000	-6.3000	-5.8167	-0.4833
3	6.0000	-7.2000	-5.8167	-1.3833
4	4.0000	3.0000	0.2000	2.8000

$$\hat{y}_h = -5.81667$$

$$MSE = 5.42917$$

$$\sum_{i=1}^{n}(x_i - \bar{x})^2 = 24$$

$$s(\hat{\beta}_1) = \sqrt{MSE \div \sum_{i=1}^{n}(x_i - \bar{x})^2} = 0.47562$$

$$s^2(\hat{Y}_h) = MSE[\frac{1}{n} + \frac{(x_h - \bar{x})^2}{\sum_{i=1}^{n}(x_i - \bar{x})^2}] = 2.26215$$

$$s^2(pred) = MSE[1 + \frac{1}{n} + \frac{(x_h - \bar{x})^2}{\sum_{i=1}^{n}(x_i - \bar{x})^2}] = 7.69132$$

The t-value equals: $t^* = \frac{\hat{\beta}_1 - 0}{s(\hat{\beta}_1)} = -6.32506$

	lower	upper
t-values for 95 percent	-4.3027	4.3027
confidence interval	-12.2881	0.6547
prediction interval	-17.7493	6.1160

□

Problem 36. Test if the slope equals zero or not for the simple linear regression model. Also, create a 95% confidence interval for $E[Y_h]$ and prediction interval for y_h at $x_h = 2$.

	x	y
1	1.00	-17.30
2	2.00	-21.60
3	2.00	-23.40
4	4.00	-29.60

Table 71: The independent and response variable.

Solution: The regression table and other pertinent information

	Estimate	Std. Error	t value	Pr(>|t|)
(Intercept)	-14.0105	1.3491	-10.39	0.0091
x	-3.9842	0.5396	-7.38	0.0179

Table 72: Linear regression model output.

	x	y	yhat	resid
1	1.0000	-17.3000	-17.9947	0.6947
2	2.0000	-21.6000	-21.9789	0.3789
3	2.0000	-23.4000	-21.9789	-1.4211
4	4.0000	-29.6000	-29.9474	0.3474

$$\hat{y}_h = -21.97895$$

$$MSE = 1.38316$$

$$\sum_{i=1}^{n}(x_i - \bar{x})^2 = 4.75$$

$$s(\hat{\beta}_1) = \sqrt{MSE \div \sum_{i=1}^{n}(x_i - \bar{x})^2} = 0.53962$$

$$s^2(\hat{Y}_h) = MSE[\frac{1}{n} + \frac{(x_h - \bar{x})^2}{\sum_{i=1}^{n}(x_i - \bar{x})^2}] = 0.36399$$

$$s^2(pred) = MSE[1 + \frac{1}{n} + \frac{(x_h - \bar{x})^2}{\sum_{i=1}^{n}(x_i - \bar{x})^2}] = 1.74715$$

The t-value equals: $t^* = \frac{\hat{\beta}_1 - 0}{s(\hat{\beta}_1)} = -7.38335$

	lower	upper
t-values for 95 percent	-4.3027	4.3027
confidence interval	-24.5748	-19.3831
prediction interval	-27.6662	-16.2917

□

Problem 37. Test if the slope equals zero or not for the simple linear regression model. Also, create a 95% confidence interval for $E[Y_h]$ and prediction interval for y_h at $x_h = 1$.

	x	y
1	6.00	5.60
2	1.00	-7.00
3	1.00	-5.30
4	4.00	-0.40

Table 73: The independent and response variable.

Solution: The regression table and other pertinent information

	Estimate	Std. Error	t value	Pr(>|t|)
(Intercept)	-8.6083	0.9909	-8.69	0.0130
x	2.2778	0.2697	8.45	0.0137

Table 74: Linear regression model output.

	x	y	yhat	resid
1	6.0000	5.6000	5.0583	0.5417
2	1.0000	-7.0000	-6.3306	-0.6694
3	1.0000	-5.3000	-6.3306	1.0306
4	4.0000	-0.4000	0.5028	-0.9028

$$\begin{aligned}
\hat{y}_h &= -6.33056 \\
MSE &= 1.30931 \\
\sum_{i=1}^{n}(x_i-\bar{x})^2 &= 18 \\
s(\hat{\beta}_1) &= \sqrt{MSE \div \sum_{i=1}^{n}(x_i-\bar{x})^2} = 0.2697 \\
s^2(\hat{Y}_h) &= MSE[\frac{1}{n}+\frac{(x_h-\bar{x})^2}{\sum_{i=1}^{n}(x_i-\bar{x})^2}] = 0.61828 \\
s^2(pred) &= MSE[1+\frac{1}{n}+\frac{(x_h-\bar{x})^2}{\sum_{i=1}^{n}(x_i-\bar{x})^2}] = 1.92759
\end{aligned}$$

The t-value equals: $t^* = \frac{\hat{\beta}_1 - 0}{s(\hat{\beta}_1)} = 8.44553$

	lower	upper
t-values for 95 percent	-4.3027	4.3027
confidence interval	-9.7138	-2.9473
prediction interval	-12.3043	-0.3569

□

Problem 38. Test if the slope equals zero or not for the simple linear regression model. Also, create a 95% confidence interval for $E[Y_h]$ and prediction interval for y_h at $x_h = 0$.

	x	y
1	9.00	7.20
2	0.00	-18.00
3	6.00	-0.30
4	9.00	9.50

Table 75: The independent and response variable.

Solution: The regression table and other pertinent information

	Estimate	Std. Error	t value	Pr(>|t|)
(Intercept)	-17.9667	1.1038	-16.28	0.0038
x	2.9278	0.1569	18.66	0.0029

Table 76: Linear regression model output.

	x	y	yhat	resid
1	9.0000	7.2000	8.3833	-1.1833
2	0.0000	-18.0000	-17.9667	-0.0333
3	6.0000	-0.3000	-0.4000	0.1000
4	9.0000	9.5000	8.3833	1.1167

$$\hat{y}_h = -17.96667$$

$$MSE = 1.32917$$

$$\sum_{i=1}^{n}(x_i - \bar{x})^2 = 54$$

$$s(\hat{\beta}_1) = \sqrt{MSE \div \sum_{i=1}^{n}(x_i - \bar{x})^2} = 0.15689$$

$$s^2(\hat{Y}_h) = MSE[\frac{1}{n} + \frac{(x_h - \bar{x})^2}{\sum_{i=1}^{n}(x_i - \bar{x})^2}] = 1.2184$$

$$s^2(pred) = MSE[1 + \frac{1}{n} + \frac{(x_h - \bar{x})^2}{\sum_{i=1}^{n}(x_i - \bar{x})^2}] = 2.54757$$

The t-value equals: $t^* = \frac{\hat{\beta}_1 - 0}{s(\hat{\beta}_1)} = 18.66145$

	lower	upper
t-values for 95 percent	-4.3027	4.3027
confidence interval	-22.7160	-13.2173
prediction interval	-24.8342	-11.0992

□

Problem 39. Test if the slope equals zero or not for the simple linear regression model. Also, create a 95% confidence interval for $E[Y_h]$ and prediction interval for y_h at $x_h = 4$.

	x	y
1	8.00	37.80
2	4.00	28.10
3	0.00	5.80
4	0.00	7.20

Table 77: The independent and response variable.

Solution: The regression table and other pertinent information

	Estimate	Std. Error	t value	Pr(>|t|)
(Intercept)	7.5818	2.4646	3.08	0.0914
x	4.0477	0.5511	7.34	0.0180

Table 78: Linear regression model output.

	x	y	yhat	resid
1	8.0000	37.8000	39.9636	-2.1636
2	4.0000	28.1000	23.7727	4.3273
3	0.0000	5.8000	7.5818	-1.7818
4	0.0000	7.2000	7.5818	-0.3818

$$\hat{y}_h = 23.77273$$

$$MSE = 13.36364$$

$$\sum_{i=1}^{n}(x_i - \bar{x})^2 = 44$$

$$s(\hat{\beta}_1) = \sqrt{MSE \div \sum_{i=1}^{n}(x_i - \bar{x})^2} = 0.55111$$

$$s^2(\hat{Y}_h) = MSE[\frac{1}{n} + \frac{(x_h - \bar{x})^2}{\sum_{i=1}^{n}(x_i - \bar{x})^2}] = 3.64463$$

$$s^2(pred) = MSE[1 + \frac{1}{n} + \frac{(x_h - \bar{x})^2}{\sum_{i=1}^{n}(x_i - \bar{x})^2}] = 17.00826$$

The t-value equals: $t^* = \frac{\hat{\beta}_1 - 0}{s(\hat{\beta}_1)} = 7.34472$

	lower	upper
t-values for 95 percent	-4.3027	4.3027
confidence interval	15.5586	31.9869
prediction interval	6.0281	41.5173

□

Problem 40. Test if the slope equals zero or not for the simple linear regression model. Also, create a 95% confidence interval for $E[Y_h]$ and prediction interval for y_h at $x_h = 2$.

	x	y
1	4.00	12.30
2	2.00	13.50
3	3.00	10.20
4	3.00	13.40

Table 79: The independent and response variable.

Solution: The regression table and other pertinent information

	Estimate	Std. Error	t value	Pr(>|t|)
(Intercept)	14.1500	3.8773	3.65	0.0676
x	-0.6000	1.2580	-0.48	0.6804

Table 80: Linear regression model output.

	x	y	yhat	resid
1	4.0000	12.3000	11.7500	0.5500
2	2.0000	13.5000	12.9500	0.5500
3	3.0000	10.2000	12.3500	-2.1500
4	3.0000	13.4000	12.3500	1.0500

$$
\begin{aligned}
\hat{y}_h &= 12.95 \\
MSE &= 3.165 \\
\sum_{i=1}^{n}(x_i - \bar{x})^2 &= 2 \\
s(\hat{\beta}_1) &= \sqrt{MSE \div \sum_{i=1}^{n}(x_i - \bar{x})^2} = 1.25797 \\
s^2(\hat{Y}_h) &= MSE[\frac{1}{n} + \frac{(x_h - \bar{x})^2}{\sum_{i=1}^{n}(x_i - \bar{x})^2}] = 2.37375 \\
s^2(pred) &= MSE[1 + \frac{1}{n} + \frac{(x_h - \bar{x})^2}{\sum_{i=1}^{n}(x_i - \bar{x})^2}] = 5.53875
\end{aligned}
$$

The t-value equals: $t^* = \frac{\hat{\beta}_1 - 0}{s(\hat{\beta}_1)} = -0.47696$

	lower	upper
t-values for 95 percent	-4.3027	4.3027
confidence interval	6.3209	19.5791
prediction interval	2.8239	23.0761

□

Problem 41. Create a 95% confidence interval for $E[Y_h]$ and prediction interval for y_h at $x_h = 5$ and a another C.I. interval for at $x_h = 2$. Do a Bonferroni adjustment.

	x	y
1	2.00	-1.40
2	5.00	4.80
3	2.00	-2.70
4	4.00	2.70

Table 81: The independent and response variable.

Solution: The regression table and other pertinent information

	Estimate	Std. Error	t value	Pr(>|t|)
(Intercept)	-6.6370	0.8872	-7.48	0.0174
x	2.3037	0.2535	9.09	0.0119

Table 82: Linear regression model output.

	x	y	yhat	resid
1	2.0000	-1.4000	-2.0296	0.6296
2	5.0000	4.8000	4.8815	-0.0815
3	2.0000	-2.7000	-2.0296	-0.6704
4	4.0000	2.7000	2.5778	0.1222

$$
\begin{aligned}
\hat{y}_{h,(x=5)} &= 4.88148 \\
\hat{y}_{h,(x=2)} &= -2.02963 \\
MSE &= 0.4337 \\
\sum_{i=1}^{n}(x_i - \bar{x})^2 &= 6.75 \\
s^2(\hat{Y}_h) &= MSE[\frac{1}{n} + \frac{(5-\bar{x})^2}{\sum_{i=1}^{n}(x_i-\bar{x})^2}] = 0.3052 \\
s^2(\hat{Y}_h) &= MSE[\frac{1}{n} + \frac{(2-\bar{x})^2}{\sum_{i=1}^{n}(x_i-\bar{x})^2}] = 0.20882 \\
s^2(pred) &= MSE[1 + \frac{1}{n} + \frac{(5-\bar{x})^2}{\sum_{i=1}^{n}(x_i-\bar{x})^2}] = 0.7389 \\
s^2(pred) &= MSE[1 + \frac{1}{n} + \frac{(2-\bar{x})^2}{\sum_{i=1}^{n}(x_i-\bar{x})^2}] = 0.64252
\end{aligned}
$$

The Bonferoni adjusted t-values - lower and upper for .95; n-2: -6.2053 and 6.2053

□

Problem 42. Create a 95% confidence interval for $E[Y_h]$ and prediction interval for y_h at $x_h = 5$ and a another C.I. interval for at $x_h = 5$. Do a Bonferroni adjustment.

	x	y
1	5.00	-37.10
2	5.00	-33.30
3	5.00	-34.50
4	4.00	-32.20

Table 83: The independent and response variable.

Solution: The regression table and other pertinent information

	Estimate	Std. Error	t value	Pr(>|t|)
(Intercept)	-21.1333	10.6985	-1.98	0.1869
x	-2.7667	2.2430	-1.23	0.3427

Table 84: Linear regression model output.

	x	y	yhat	resid
1	5.0000	-37.1000	-34.9667	-2.1333
2	5.0000	-33.3000	-34.9667	1.6667
3	5.0000	-34.5000	-34.9667	0.4667
4	4.0000	-32.2000	-32.2000	0.0000

$$\hat{y}_{h,(x=5)} = -34.96667$$

$$\hat{y}_{h,(x=5)} = -34.96667$$

$$MSE = 3.77333$$

$$\sum_{i=1}^{n}(x_i - \bar{x})^2 = 0.75$$

$$s^2(\hat{Y}_h) = MSE[\frac{1}{n} + \frac{(5-\bar{x})^2}{\sum_{i=1}^{n}(x_i - \bar{x})^2}] = 1.25778$$

$$s^2(\hat{Y}_h) = MSE[\frac{1}{n} + \frac{(5-\bar{x})^2}{\sum_{i=1}^{n}(x_i - \bar{x})^2}] = 1.25778$$

$$s^2(pred) = MSE[1 + \frac{1}{n} + \frac{(5-\bar{x})^2}{\sum_{i=1}^{n}(x_i - \bar{x})^2}] = 5.03111$$

$$s^2(pred) = MSE[1 + \frac{1}{n} + \frac{(5-\bar{x})^2}{\sum_{i=1}^{n}(x_i - \bar{x})^2}] = 5.03111$$

The Bonferoni adjusted t-values - lower and upper for .95; n-2: -6.2053 and 6.2053

□

Problem 43. Create a 95% confidence interval for $E[Y_h]$ and prediction interval for y_h at $x_h = 0$ and a another C.I. interval for at $x_h = 1$. Do a Bonferroni adjustment.

	x	y
1	6.00	-15.60
2	0.00	-17.40
3	1.00	-15.10
4	0.00	-16.20

Table 85: The independent and response variable.

Solution: The regression table and other pertinent information

	Estimate	Std. Error	t value	Pr(>|t|)
(Intercept)	-16.3455	0.6636	-24.63	0.0016
x	0.1545	0.2182	0.71	0.5522

Table 86: Linear regression model output.

	x	y	yhat	resid
1	6.0000	-15.6000	-15.4182	-0.1818
2	0.0000	-17.4000	-16.3455	-1.0545
3	1.0000	-15.1000	-16.1909	1.0909
4	0.0000	-16.2000	-16.3455	0.1455

$$\hat{y}_{h,(x=0)} = -16.34545$$

$$\hat{y}_{h,(x=1)} = -16.19091$$

$$MSE = 1.17818$$

$$\sum_{i=1}^{n}(x_i - \bar{x})^2 = 24.75$$

$$s^2(\hat{Y}_h) = MSE[\frac{1}{n} + \frac{(0-\bar{x})^2}{\sum_{i=1}^{n}(x_i - \bar{x})^2}] = 0.44033$$

$$s^2(\hat{Y}_h) = MSE[\frac{1}{n} + \frac{(1-\bar{x})^2}{\sum_{i=1}^{n}(x_i - \bar{x})^2}] = 0.32132$$

$$s^2(pred) = MSE[1 + \frac{1}{n} + \frac{(0-\bar{x})^2}{\sum_{i=1}^{n}(x_i - \bar{x})^2}] = 1.61851$$

$$s^2(pred) = MSE[1 + \frac{1}{n} + \frac{(1-\bar{x})^2}{\sum_{i=1}^{n}(x_i - \bar{x})^2}] = 1.4995$$

The Bonferoni adjusted t-values - lower and upper for .95; n-2: -6.2053 and 6.2053

□

Problem 44. Create a 95% confidence interval for $E[Y_h]$ and prediction interval for y_h at $x_h = 5$ and a another C.I. interval for at $x_h = 7$. Do a Bonferroni adjustment.

	x	y
1	3.00	-14.10
2	5.00	-12.70
3	7.00	-8.30
4	2.00	-18.70

Table 87: The independent and response variable.

Solution: The regression table and other pertinent information

	Estimate	Std. Error	t value	Pr(>|t|)
(Intercept)	-21.3305	1.7922	-11.90	0.0070
x	1.8542	0.3843	4.83	0.0404

Table 88: Linear regression model output.

	x	y	yhat	resid
1	3.0000	-14.1000	-15.7678	1.6678
2	5.0000	-12.7000	-12.0593	-0.6407
3	7.0000	-8.3000	-8.3508	0.0508
4	2.0000	-18.7000	-17.6220	-1.0780

$$\hat{y}_{h,(x=5)} = -12.05932$$

$$\hat{y}_{h,(x=7)} = -8.35085$$

$$MSE = 2.17831$$

$$\sum_{i=1}^{n}(x_i - \bar{x})^2 = 14.75$$

$$s^2(\hat{Y}_h) = MSE[\frac{1}{n} + \frac{(5-\bar{x})^2}{\sum_{i=1}^{n}(x_i - \bar{x})^2}] = 0.62765$$

$$s^2(\hat{Y}_h) = MSE[\frac{1}{n} + \frac{(7-\bar{x})^2}{\sum_{i=1}^{n}(x_i - \bar{x})^2}] = 1.66142$$

$$s^2(pred) = MSE[1 + \frac{1}{n} + \frac{(5-\bar{x})^2}{\sum_{i=1}^{n}(x_i - \bar{x})^2}] = 2.80595$$

$$s^2(pred) = MSE[1 + \frac{1}{n} + \frac{(7-\bar{x})^2}{\sum_{i=1}^{n}(x_i - \bar{x})^2}] = 3.83972$$

The Bonferoni adjusted t-values - lower and upper for .95; n-2: -6.2053 and 6.2053

□

Problem 45. Create a 95% confidence interval for $E[Y_h]$ and prediction interval for y_h at $x_h = 7$ and a another C.I. interval for at $x_h = 0$. Do a Bonferroni adjustment.

	x	y
1	0.00	-10.20
2	7.00	10.80
3	0.00	-14.70
4	1.00	-5.40

Table 89: The independent and response variable.

Solution: The regression table and other pertinent information

	Estimate	Std. Error	t value	Pr(>|t|)
(Intercept)	-11.2985	1.9242	-5.87	0.0278
x	3.2118	0.5443	5.90	0.0275

Table 90: Linear regression model output.

	x	y	yhat	resid
1	0.0000	-10.2000	-11.2985	1.0985
2	7.0000	10.8000	11.1838	-0.3838
3	0.0000	-14.7000	-11.2985	-3.4015
4	1.0000	-5.4000	-8.0868	2.6868

$$
\begin{aligned}
\hat{y}_{h,(x=7)} &= 11.18382 \\
\hat{y}_{h,(x=0)} &= -11.29853 \\
MSE &= 10.0714 \\
\sum_{i=1}^{n}(x_i - \bar{x})^2 &= 34 \\
s^2(\hat{Y}_h) &= MSE[\frac{1}{n} + \frac{(7-\bar{x})^2}{\sum_{i=1}^{n}(x_i-\bar{x})^2}] = 9.92329 \\
s^2(\hat{Y}_h) &= MSE[\frac{1}{n} + \frac{(0-\bar{x})^2}{\sum_{i=1}^{n}(x_i-\bar{x})^2}] = 3.70272 \\
s^2(pred) &= MSE[1 + \frac{1}{n} + \frac{(7-\bar{x})^2}{\sum_{i=1}^{n}(x_i-\bar{x})^2}] = 19.99469 \\
s^2(pred) &= MSE[1 + \frac{1}{n} + \frac{(0-\bar{x})^2}{\sum_{i=1}^{n}(x_i-\bar{x})^2}] = 13.77412
\end{aligned}
$$

The Bonferoni adjusted t-values - lower and upper for .95; n-2: -6.2053 and 6.2053

□

Problem 46. Create a 95% confidence interval for $E[Y_h]$ and prediction interval for y_h at $x_h = 7$ and a another C.I. interval for at $x_h = 4$. Do a Bonferroni adjustment.

	x	y
1	6.00	-33.20
2	7.00	-33.60
3	4.00	-26.00
4	7.00	-36.30

Table 91: The independent and response variable.

Solution: The regression table and other pertinent information

	Estimate	Std. Error	t value	Pr(>|t|)
(Intercept)	-14.3750	3.8673	-3.72	0.0654
x	-2.9833	0.6315	-4.72	0.0420

Table 92: Linear regression model output.

	x	y	yhat	resid
1	6.0000	-33.2000	-32.2750	-0.9250
2	7.0000	-33.6000	-35.2583	1.6583
3	4.0000	-26.0000	-26.3083	0.3083
4	7.0000	-36.3000	-35.2583	-1.0417

$$
\begin{aligned}
\hat{y}_{h,(x=7)} &= -35.25833 \\
\hat{y}_{h,(x=4)} &= -26.30833 \\
MSE &= 2.39292 \\
\sum_{i=1}^{n}(x_i - \bar{x})^2 &= 6 \\
s^2(\hat{Y}_h) &= MSE[\frac{1}{n} + \frac{(7-\bar{x})^2}{\sum_{i=1}^{n}(x_i-\bar{x})^2}] = 0.99705 \\
s^2(\hat{Y}_h) &= MSE[\frac{1}{n} + \frac{(4-\bar{x})^2}{\sum_{i=1}^{n}(x_i-\bar{x})^2}] = 2.19351 \\
s^2(pred) &= MSE[1 + \frac{1}{n} + \frac{(7-\bar{x})^2}{\sum_{i=1}^{n}(x_i-\bar{x})^2}] = 3.38997 \\
s^2(pred) &= MSE[1 + \frac{1}{n} + \frac{(4-\bar{x})^2}{\sum_{i=1}^{n}(x_i-\bar{x})^2}] = 4.58642
\end{aligned}
$$

The Bonferoni adjusted t-values - lower and upper for .95; n-2: -6.2053 and 6.2053

□

Problem 47. Create a 95% confidence interval for $E[Y_h]$ and prediction interval for y_h at $x_h = 0$ and a another C.I. interval for at $x_h = 0$. Do a Bonferroni adjustment.

	x	y
1	3.00	15.30
2	0.00	9.60
3	0.00	4.70
4	8.00	31.50

Table 93: The independent and response variable.

Solution: The regression table and other pertinent information

	Estimate	Std. Error	t value	Pr(>|t|)
(Intercept)	6.9205	1.6481	4.20	0.0523
x	3.0380	0.3858	7.87	0.0157

Table 94: Linear regression model output.

	x	y	yhat	resid
1	3.0000	15.3000	16.0345	-0.7345
2	0.0000	9.6000	6.9205	2.6795
3	0.0000	4.7000	6.9205	-2.2205
4	8.0000	31.5000	31.2246	0.2754

$$\hat{y}_{h,(x=0)} = 6.92047$$

$$\hat{y}_{h,(x=0)} = 6.92047$$

$$MSE = 6.36287$$

$$\sum_{i=1}^{n}(x_i - \bar{x})^2 = 42.75$$

$$s^2(\hat{Y}_h) = MSE[\frac{1}{n} + \frac{(0-\bar{x})^2}{\sum_{i=1}^{n}(x_i - \bar{x})^2}] = 2.71631$$

$$s^2(\hat{Y}_h) = MSE[\frac{1}{n} + \frac{(0-\bar{x})^2}{\sum_{i=1}^{n}(x_i - \bar{x})^2}] = 2.71631$$

$$s^2(pred) = MSE[1 + \frac{1}{n} + \frac{(0-\bar{x})^2}{\sum_{i=1}^{n}(x_i - \bar{x})^2}] = 9.07918$$

$$s^2(pred) = MSE[1 + \frac{1}{n} + \frac{(0-\bar{x})^2}{\sum_{i=1}^{n}(x_i - \bar{x})^2}] = 9.07918$$

The Bonferoni adjusted t-values - lower and upper for .95; n-2: -6.2053 and 6.2053

□

Problem 48. Create a 95% confidence interval for $E[Y_h]$ and prediction interval for y_h at $x_h = 7$ and a another C.I. interval for at $x_h = 2$. Do a Bonferroni adjustment.

	x	y
1	1.00	-11.30
2	7.00	11.80
3	2.00	-8.80
4	7.00	2.90

Table 95: The independent and response variable.

Solution: The regression table and other pertinent information

	Estimate	Std. Error	t value	Pr(>|t|)
(Intercept)	-14.7496	4.0833	-3.61	0.0688
x	3.1528	0.8047	3.92	0.0594

Table 96: Linear regression model output.

	x	y	yhat	resid
1	1.0000	-11.3000	-11.5967	0.2967
2	7.0000	11.8000	7.3203	4.4797
3	2.0000	-8.8000	-8.4439	-0.3561
4	7.0000	2.9000	7.3203	-4.4203

$$
\begin{aligned}
\hat{y}_{h,(x=7)} &= 7.32033 \\
\hat{y}_{h,(x=2)} &= -8.4439 \\
MSE &= 19.91081 \\
\sum_{i=1}^{n}(x_i - \bar{x})^2 &= 30.75 \\
s^2(\hat{Y}_h) &= MSE[\frac{1}{n} + \frac{(7-\bar{x})^2}{\sum_{i=1}^{n}(x_i - \bar{x})^2}] = 9.87447 \\
s^2(\hat{Y}_h) &= MSE[\frac{1}{n} + \frac{(2-\bar{x})^2}{\sum_{i=1}^{n}(x_i - \bar{x})^2}] = 8.2557 \\
s^2(pred) &= MSE[1 + \frac{1}{n} + \frac{(7-\bar{x})^2}{\sum_{i=1}^{n}(x_i - \bar{x})^2}] = 29.78528 \\
s^2(pred) &= MSE[1 + \frac{1}{n} + \frac{(2-\bar{x})^2}{\sum_{i=1}^{n}(x_i - \bar{x})^2}] = 28.16652
\end{aligned}
$$

The Bonferoni adjusted t-values - lower and upper for .95; n-2: -6.2053 and 6.2053

□

Problem 49. Create a 95% confidence interval for $E[Y_h]$ and prediction interval for y_h at $x_h = 2$ and a another C.I. interval for at $x_h = 0$. Do a Bonferroni adjustment.

	x	y
1	0.00	9.40
2	2.00	3.70
3	0.00	10.10
4	2.00	3.40

Table 97: The independent and response variable.

Solution: The regression table and other pertinent information

	Estimate	Std. Error	t value	Pr(>|t|)
(Intercept)	9.7500	0.2693	36.21	0.0008
x	-3.1000	0.1904	-16.28	0.0038

Table 98: Linear regression model output.

	x	y	yhat	resid
1	0.0000	9.4000	9.7500	-0.3500
2	2.0000	3.7000	3.5500	0.1500
3	0.0000	10.1000	9.7500	0.3500
4	2.0000	3.4000	3.5500	-0.1500

$$\begin{aligned}
\hat{y}_{h,(x=2)} &= 3.55 \\
\hat{y}_{h,(x=0)} &= 9.75 \\
MSE &= 0.145 \\
\sum_{i=1}^{n}(x_i - \bar{x})^2 &= 4 \\
s^2(\hat{Y}_h) &= MSE[\frac{1}{n} + \frac{(2-\bar{x})^2}{\sum_{i=1}^{n}(x_i - \bar{x})^2}] = 0.0725 \\
s^2(\hat{Y}_h) &= MSE[\frac{1}{n} + \frac{(0-\bar{x})^2}{\sum_{i=1}^{n}(x_i - \bar{x})^2}] = 0.0725 \\
s^2(pred) &= MSE[1 + \frac{1}{n} + \frac{(2-\bar{x})^2}{\sum_{i=1}^{n}(x_i - \bar{x})^2}] = 0.2175 \\
s^2(pred) &= MSE[1 + \frac{1}{n} + \frac{(0-\bar{x})^2}{\sum_{i=1}^{n}(x_i - \bar{x})^2}] = 0.2175
\end{aligned}$$

The Bonferoni adjusted t-values - lower and upper for .95; n-2: -6.2053 and 6.2053

□

Problem 50. Create a 95% confidence interval for $E[Y_h]$ and prediction interval for y_h at $x_h = 6$ and a another C.I. interval for at $x_h = 0$. Do a Bonferroni adjustment.

	x	y
1	5.00	-18.90
2	6.00	-24.70
3	0.00	5.30
4	8.00	-36.90

Table 99: The independent and response variable.

Solution: The regression table and other pertinent information

	Estimate	Std. Error	t value	Pr(>|t|)
(Intercept)	5.9000	1.2723	4.64	0.0435
x	-5.2000	0.2276	-22.85	0.0019

Table 100: Linear regression model output.

	x	y	yhat	resid
1	5.0000	-18.9000	-20.1000	1.2000
2	6.0000	-24.7000	-25.3000	0.6000
3	0.0000	5.3000	5.9000	-0.6000
4	8.0000	-36.9000	-35.7000	-1.2000

$$
\begin{aligned}
\hat{y}_{h,(x=6)} &= -25.3 \\
\hat{y}_{h,(x=0)} &= 5.9 \\
MSE &= 1.8 \\
\sum_{i=1}^{n}(x_i - \bar{x})^2 &= 34.75 \\
s^2(\hat{Y}_h) &= MSE[\frac{1}{n} + \frac{(6-\bar{x})^2}{\sum_{i=1}^{n}(x_i - \bar{x})^2}] = 0.53094 \\
s^2(\hat{Y}_h) &= MSE[\frac{1}{n} + \frac{(0-\bar{x})^2}{\sum_{i=1}^{n}(x_i - \bar{x})^2}] = 1.61871 \\
s^2(pred) &= MSE[1 + \frac{1}{n} + \frac{(6-\bar{x})^2}{\sum_{i=1}^{n}(x_i - \bar{x})^2}] = 2.33094 \\
s^2(pred) &= MSE[1 + \frac{1}{n} + \frac{(0-\bar{x})^2}{\sum_{i=1}^{n}(x_i - \bar{x})^2}] = 3.41871
\end{aligned}
$$

The Bonferoni adjusted t-values - lower and upper for .95; n-2: -6.2053 and 6.2053

□

Problem 51. Create a 95% confidence limits for $X_{h(new)}$ given the $y_{new} = -27.5$

	x	y
1	0.00	-15.20
2	2.00	-22.80
3	6.00	-39.80
4	1.00	-17.70
5	3.00	-25.10

Table 101: The independent and response variable.

	Estimate	Std. Error	t value	Pr(>|t|)
(Intercept)	-14.1623	0.8315	-17.03	0.0004
x	-4.1491	0.2629	-15.78	0.0006

Table 102: Linear regression model output.

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
x	1	364.95	364.95	249.00	0.0006
Residuals	3	4.40	1.47		

Table 103: ANOVA format Linear regression model output.

Solution: Other pertinent information

	x	y	yhat	resid
1	0.0000	-15.2000	-14.1623	-1.0377
2	2.0000	-22.8000	-22.4604	-0.3396
3	6.0000	-39.8000	-39.0566	-0.7434
4	1.0000	-17.7000	-18.3113	0.6113
5	3.0000	-25.1000	-26.6094	1.5094

$$\hat{X}_{h,new} = 3.21464$$

$$MSE = 1.46566$$

$$\sum_{i=1}^{n}(x_i - \bar{x})^2 = 21.2$$

$$s^2(predX) = \frac{MSE}{\hat{\beta}_1^2}[1 + \frac{1}{n} + \frac{(\hat{X}_{h,new} - \bar{x})^2}{\sum_{i=1}^{n}(x_i - \bar{x})^2}] = 0.10483$$

□

	lower	upper
t-values for 95 percent	-3.1824	3.1824
95 percent interval	2.1842	4.2451

Problem 52. Create a 95% confidence limits for $X_{h(new)}$ given the $y_{new} = -10.5$

	x	y
1	7.00	-15.10
2	1.00	3.40
3	3.00	-5.90
4	0.00	5.20
5	0.00	4.10

Table 104: The independent and response variable.

	Estimate	Std. Error	t value	Pr(>|t|)
(Intercept)	4.7718	0.8900	5.36	0.0127
x	-2.9236	0.2591	-11.28	0.0015

Table 105: Linear regression model output.

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
x	1	297.44	297.44	127.32	0.0015
Residuals	3	7.01	2.34		

Table 106: ANOVA format Linear regression model output.

Solution: Other pertinent information

	x	y	yhat	resid
1	7.0000	-15.1000	-15.6931	0.5931
2	1.0000	3.4000	1.8483	1.5517
3	3.0000	-5.9000	-3.9989	-1.9011
4	0.0000	5.2000	4.7718	0.4282
5	0.0000	4.1000	4.7718	-0.6718

$$\hat{X}_{h,new} = 5.22371$$

$$MSE = 2.33623$$

$$\sum_{i=1}^{n}(x_i - \bar{x})^2 = 34.8$$

$$s^2(predX) = \frac{MSE}{\hat{\beta}_1^2}[1 + \frac{1}{n} + \frac{(\hat{X}_{h,new} - \bar{x})^2}{\sum_{i=1}^{n}(x_i - \bar{x})^2}] = 0.39981$$

□

	lower	upper
t-values for 95 percent	-3.1824	3.1824
95 percent interval	3.2114	7.2360

Problem 53. Create a 95% confidence limits for $X_{h(new)}$ given the $y_{new} = -3.05$

	x	y
1	3.00	-7.00
2	0.00	-16.00
3	6.00	0.90
4	8.00	8.20
5	5.00	-2.10

Table 107: The independent and response variable.

	Estimate	Std. Error	t value	Pr(>|t|)
(Intercept)	-16.1989	0.6164	-26.28	0.0001
x	2.9543	0.1191	24.81	0.0001

Table 108: Linear regression model output.

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
x	1	324.68	324.68	615.58	0.0001
Residuals	3	1.58	0.53		

Table 109: ANOVA format Linear regression model output.

Solution: Other pertinent information

	x	y	yhat	resid
1	3.0000	-7.0000	-7.3360	0.3360
2	0.0000	-16.0000	-16.1989	0.1989
3	6.0000	0.9000	1.5269	-0.6269
4	8.0000	8.2000	7.4355	0.7645
5	5.0000	-2.1000	-1.4274	-0.6726

$$\hat{X}_{h,new} = 4.45077$$

$$MSE = 0.52744$$

$$\sum_{i=1}^{n}(x_i - \bar{x})^2 = 37.2$$

$$s^2(predX) = \frac{MSE}{\hat{\beta}_1^2}[1 + \frac{1}{n} + \frac{(\hat{X}_{h,new} - \bar{x})^2}{\sum_{i=1}^{n}(x_i - \bar{x})^2}] = 0.07252$$

□

	lower	upper
t-values for 95 percent	-3.1824	3.1824
95 percent interval	3.5937	5.3078

Problem 54. Create a 95% confidence limits for $X_{h(new)}$ given the $y_{new} = -12.95$

	x	y
1	2.00	-1.20
2	9.00	-25.30
3	9.00	-24.70
4	6.00	-13.00
5	6.00	-18.10

Table 110: The independent and response variable.

	Estimate	Std. Error	t value	Pr(>|t|)
(Intercept)	5.1843	2.5727	2.02	0.1373
x	-3.3819	0.3729	-9.07	0.0028

Table 111: Linear regression model output.

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
x	1	379.72	379.72	82.26	0.0028
Residuals	3	13.85	4.62		

Table 112: ANOVA format Linear regression model output.

Solution: Other pertinent information

	x	y	yhat	resid
1	2.0000	-1.2000	-1.5795	0.3795
2	9.0000	-25.3000	-25.2530	-0.0470
3	9.0000	-24.7000	-25.2530	0.5530
4	6.0000	-13.0000	-15.1072	2.1072
5	6.0000	-18.1000	-15.1072	-2.9928

$$\hat{X}_{h,new} = 5.36213$$

$$MSE = 4.61639$$

$$\sum_{i=1}^{n}(x_i - \bar{x})^2 = 33.2$$

$$s^2(predX) = \frac{MSE}{\hat{\beta}_1^2}[1 + \frac{1}{n} + \frac{(\hat{X}_{h,new} - \bar{x})^2}{\sum_{i=1}^{n}(x_i - \bar{x})^2}] = 0.49744$$

□

	lower	upper
t-values for 95 percent	-3.1824	3.1824
95 percent interval	3.1176	7.6067

Problem 55. Create a 95% confidence limits for $X_{h(new)}$ given the $y_{new} = 27.95$

	x	y
1	9.00	28.00
2	1.00	12.90
3	9.00	27.90
4	0.00	4.60
5	8.00	24.20

Table 113: The independent and response variable.

	Estimate	Std. Error	t value	Pr(>|t|)
(Intercept)	7.3793	1.9318	3.82	0.0316
x	2.2483	0.2867	7.84	0.0043

Table 114: Linear regression model output.

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
x	1	410.45	410.45	61.50	0.0043
Residuals	3	20.02	6.67		

Table 115: ANOVA format Linear regression model output.

Solution: Other pertinent information

	x	y	yhat	resid
1	9.0000	28.0000	27.6138	0.3862
2	1.0000	12.9000	9.6276	3.2724
3	9.0000	27.9000	27.6138	0.2862
4	0.0000	4.6000	7.3793	-2.7793
5	8.0000	24.2000	25.3655	-1.1655

$$\hat{X}_{h,new} = 9.14954$$

$$MSE = 6.67425$$

$$\sum_{i=1}^{n}(x_i - \bar{x})^2 = 81.2$$

$$s^2(predX) = \frac{MSE}{\hat{\beta}_1^2}[1 + \frac{1}{n} + \frac{(\hat{X}_{h,new} - \bar{x})^2}{\sum_{i=1}^{n}(x_i - \bar{x})^2}] = 1.81309$$

□

	lower	upper
t-values for 95 percent	-3.1824	3.1824
95 percent interval	4.8643	13.4347

Problem 56. Create a 95% confidence limits for $X_{h(new)}$ given the $y_{new} = 16.8$

	x	y
1	2.00	7.60
2	8.00	29.00
3	8.00	26.00
4	5.00	16.30
5	7.00	23.80

Table 116: The independent and response variable.

	Estimate	Std. Error	t value	Pr(>|t|)
(Intercept)	0.4400	1.7271	0.25	0.8154
x	3.3500	0.2691	12.45	0.0011

Table 117: Linear regression model output.

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
x	1	291.78	291.78	155.01	0.0011
Residuals	3	5.65	1.88		

Table 118: ANOVA format Linear regression model output.

Solution: Other pertinent information

	x	y	yhat	resid
1	2.0000	7.6000	7.1400	0.4600
2	8.0000	29.0000	27.2400	1.7600
3	8.0000	26.0000	27.2400	-1.2400
4	5.0000	16.3000	17.1900	-0.8900
5	7.0000	23.8000	23.8900	-0.0900

$$\hat{X}_{h,new} = 4.88358$$

$$MSE = 1.88233$$

$$\sum_{i=1}^{n}(x_i - \bar{x})^2 = 26$$

$$s^2(predX) = \frac{MSE}{\hat{\beta}_1^2}[1 + \frac{1}{n} + \frac{(\hat{X}_{h,new} - \bar{x})^2}{\sum_{i=1}^{n}(x_i - \bar{x})^2}] = 0.20931$$

□

	lower	upper
t-values for 95 percent	-3.1824	3.1824
95 percent interval	3.4276	6.3396

Problem 57. Create a 95% confidence limits for $X_{h(new)}$ given the $y_{new} = 5.85$

	x	y
1	1.00	12.50
2	3.00	1.90
3	3.00	-0.80
4	2.00	4.30
5	5.00	-5.70

Table 119: The independent and response variable.

	Estimate	Std. Error	t value	Pr(>|t|)
(Intercept)	14.6136	2.3640	6.18	0.0085
x	-4.3477	0.7630	-5.70	0.0107

Table 120: Linear regression model output.

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
x	1	166.34	166.34	32.47	0.0107
Residuals	3	15.37	5.12		

Table 121: ANOVA format Linear regression model output.

Solution: Other pertinent information

	x	y	yhat	resid
1	1.0000	12.5000	10.2659	2.2341
2	3.0000	1.9000	1.5705	0.3295
3	3.0000	-0.8000	1.5705	-2.3705
4	2.0000	4.3000	5.9182	-1.6182
5	5.0000	-5.7000	-7.1250	1.4250

$$\hat{X}_{h,new} = 2.01568$$

$$MSE = 5.12265$$

$$\sum_{i=1}^{n}(x_i - \bar{x})^2 = 8.8$$

$$s^2(predX) = \frac{MSE}{\hat{\beta}_1^2}[1 + \frac{1}{n} + \frac{(\hat{X}_{h,new} - \bar{x})^2}{\sum_{i=1}^{n}(x_i - \bar{x})^2}] = 0.34414$$

□

	lower	upper
t-values for 95 percent	-3.1824	3.1824
95 percent interval	0.1487	3.8826

Problem 58. Create a 95% confidence limits for $X_{h(new)}$ given the $y_{new} = -5.6$

	x	y
1	2.00	1.00
2	6.00	-11.60
3	6.00	-12.20
4	2.00	-1.80
5	2.00	-0.10

Table 122: The independent and response variable.

	Estimate	Std. Error	t value	Pr(>|t|)
(Intercept)	5.5000	1.1015	4.99	0.0155
x	-2.9000	0.2687	-10.79	0.0017

Table 123: Linear regression model output.

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
x	1	161.47	161.47	116.45	0.0017
Residuals	3	4.16	1.39		

Table 124: ANOVA format Linear regression model output.

Solution: Other pertinent information

	x	y	yhat	resid
1	2.0000	1.0000	-0.3000	1.3000
2	6.0000	-11.6000	-11.9000	0.3000
3	6.0000	-12.2000	-11.9000	-0.3000
4	2.0000	-1.8000	-0.3000	-1.5000
5	2.0000	-0.1000	-0.3000	0.2000

$$\hat{X}_{h,new} = 3.82759$$

$$MSE = 1.38667$$

$$\sum_{i=1}^{n}(x_i - \bar{x})^2 = 19.2$$

$$s^2(predX) = \frac{MSE}{\hat{\beta}_1^2}[1 + \frac{1}{n} + \frac{(\hat{X}_{h,new} - \bar{x})^2}{\sum_{i=1}^{n}(x_i - \bar{x})^2}] = 0.1983$$

□

	lower	upper
t-values for 95 percent	-3.1824	3.1824
95 percent interval	2.4104	5.2448

Problem 59. Create a 95% confidence limits for $X_{h(new)}$ given the $y_{new} = 31.05$

	x	y
1	9.00	43.00
2	7.00	34.00
3	2.00	19.10
4	6.00	32.00
5	9.00	42.10

Table 125: The independent and response variable.

	Estimate	Std. Error	t value	Pr(>|t|)
(Intercept)	12.0175	1.2229	9.83	0.0022
x	3.3367	0.1726	19.33	0.0003

Table 126: Linear regression model output.

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
x	1	369.64	369.64	373.73	0.0003
Residuals	3	2.97	0.99		

Table 127: ANOVA format Linear regression model output.

Solution: Other pertinent information

	x	y	yhat	resid
1	9.0000	43.0000	42.0482	0.9518
2	7.0000	34.0000	35.3747	-1.3747
3	2.0000	19.1000	18.6910	0.4090
4	6.0000	32.0000	32.0380	-0.0380
5	9.0000	42.1000	42.0482	0.0518

$$\hat{X}_{h,new} = 5.70392$$

$$MSE = 0.98906$$

$$\sum_{i=1}^{n}(x_i - \bar{x})^2 = 33.2$$

$$s^2(predX) = \frac{MSE}{\hat{\beta}_1^2}[1 + \frac{1}{n} + \frac{(\hat{X}_{h,new} - \bar{x})^2}{\sum_{i=1}^{n}(x_i - \bar{x})^2}] = 0.10875$$

□

	lower	upper
t-values for 95 percent	-3.1824	3.1824
95 percent interval	4.6544	6.7534

Problem 60. Create a 95% confidence limits for $X_{h(new)}$ given the $y_{new} = -17.15$

	x	y
1	0.00	-19.20
2	8.00	-12.50
3	5.00	-15.10
4	8.00	-11.00
5	1.00	-16.00

Table 128: The independent and response variable.

	Estimate	Std. Error	t value	Pr(>|t|)
(Intercept)	-18.2385	0.9343	-19.52	0.0003
x	0.7906	0.1683	4.70	0.0183

Table 129: Linear regression model output.

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
x	1	35.75	35.75	22.05	0.0183
Residuals	3	4.86	1.62		

Table 130: ANOVA format Linear regression model output.

Solution: Other pertinent information

	x	y	yhat	resid
1	0.0000	-19.2000	-18.2385	-0.9615
2	8.0000	-12.5000	-11.9140	-0.5860
3	5.0000	-15.1000	-14.2857	-0.8143
4	8.0000	-11.0000	-11.9140	0.9140
5	1.0000	-16.0000	-17.4479	1.4479

$$\hat{X}_{h,new} = 1.37682$$

$$MSE = 1.62097$$

$$\sum_{i=1}^{n}(x_i - \bar{x})^2 = 57.2$$

$$s^2(predX) = \frac{MSE}{\hat{\beta}_1^2}[1 + \frac{1}{n} + \frac{(\hat{X}_{h,new} - \bar{x})^2}{\sum_{i=1}^{n}(x_i - \bar{x})^2}] = 3.52675$$

□

	lower	upper
t-values for 95 percent	-3.1824	3.1824
95 percent interval	-4.5997	7.3533

2. Matrix Algebra

2.1. Matrix Multiplication

Problem 61. Perform matrix multiplication $X \times Y$ and $Z \times X$

5.00	9.00	2.00
1.00	7.00	3.00

Table 131: X matrix

30.00
50.00
90.00

Table 132: Y matrix

20.00	80.00
60.00	60.00

Table 133: Z matrix

Solution:

780.00
650.00

Table 134: X times Y matrix

180.00	740.00	280.00
360.00	960.00	300.00

Table 135: Z times X matrix

□

Problem 62. Perform matrix multiplication $X \times Y$ and $Z \times X$

2.00	6.00	1.00
2.00	9.00	7.00

Table 136: X matrix

30.00
40.00
20.00

Table 137: Y matrix

20.00	30.00
50.00	80.00

Table 138: Z matrix

Solution:

320.00
560.00

Table 139: X times Y matrix

100.00	390.00	230.00
260.00	1020.00	610.00

Table 140: Z times X matrix

□

Problem 63. Perform matrix multiplication $X \times Y$ and $Z \times X$

6.00	2.00	3.00
0.00	4.00	3.00

Table 141: X matrix

20.00
40.00
80.00

Table 142: Y matrix

20.00	40.00
50.00	60.00

Table 143: Z matrix

Solution:

440.00
400.00

Table 144: X times Y matrix

120.00	200.00	180.00
300.00	340.00	330.00

Table 145: Z times X matrix

□

Problem 64. Perform matrix multiplication $X \times Y$ and $Z \times X$

4.00	5.00	6.00
1.00	7.00	8.00

Table 146: X matrix

50.00
30.00
70.00

Table 147: Y matrix

30.00	50.00
10.00	10.00

Table 148: Z matrix

Solution:

770.00
820.00

Table 149: X times Y matrix

170.00	500.00	580.00
50.00	120.00	140.00

Table 150: Z times X matrix

□

Problem 65. Perform matrix multiplication $X \times Y$ and $Z \times X$

7.00	6.00	0.00
0.00	5.00	4.00

Table 151: X matrix

40.00
50.00
40.00

Table 152: Y matrix

30.00	20.00
40.00	90.00

Table 153: Z matrix

Solution:

580.00
410.00

Table 154: X times Y matrix

210.00	280.00	80.00
280.00	690.00	360.00

Table 155: Z times X matrix

□

Problem 66. Perform matrix multiplication $X \times Y$ and $Z \times X$

8.00	5.00	2.00
2.00	0.00	2.00

Table 156: X matrix

10.00
90.00
70.00

Table 157: Y matrix

50.00	60.00
0.00	40.00

Table 158: Z matrix

Solution:

670.00
160.00

Table 159: X times Y matrix

520.00	250.00	220.00
80.00	0.00	80.00

Table 160: Z times X matrix

□

Problem 67. Perform matrix multiplication $X \times Y$ and $Z \times X$

6.00	4.00	6.00
5.00	7.00	6.00

Table 161: X matrix

20.00
60.00
20.00

Table 162: Y matrix

60.00	30.00
20.00	50.00

Table 163: Z matrix

Solution:

480.00
640.00

Table 164: X times Y matrix

510.00	450.00	540.00
370.00	430.00	420.00

Table 165: Z times X matrix

□

Problem 68. Perform matrix multiplication $X \times Y$ and $Z \times X$

0.00	0.00	1.00
4.00	3.00	7.00

Table 166: X matrix

60.00
0.00
20.00

Table 167: Y matrix

40.00	90.00
0.00	80.00

Table 168: Z matrix

Solution:

20.00
380.00

Table 169: X times Y matrix

360.00	270.00	670.00
320.00	240.00	560.00

Table 170: Z times X matrix

□

Problem 69. Perform matrix multiplication $X \times Y$ and $Z \times X$

4.00	2.00	2.00
7.00	4.00	4.00

Table 171: X matrix

30.00
90.00
40.00

Table 172: Y matrix

20.00	60.00
20.00	20.00

Table 173: Z matrix

Solution:

380.00
730.00

Table 174: X times Y matrix

500.00	280.00	280.00
220.00	120.00	120.00

Table 175: Z times X matrix

□

Problem 70. Perform matrix multiplication $X \times Y$ and $Z \times X$

5.00	9.00	7.00
1.00	9.00	1.00

Table 176: X matrix

70.00
0.00
90.00

Table 177: Y matrix

10.00	80.00
10.00	70.00

Table 178: Z matrix

Solution:

980.00
160.00

Table 179: X times Y matrix

130.00	810.00	150.00
120.00	720.00	140.00

Table 180: Z times X matrix

□

Problem 71. Solve for the determinant and inverse of the following matrix

15.00	6.00	17.00
7.00	-5.00	19.00
12.00	-20.00	-9.00

Table 181: X matrix

Solution: The determinant equals 6761.00000000001 and the inverse is:

0.0629	-0.0423	0.0294
0.0430	-0.0501	-0.0246
-0.0118	0.0550	-0.0173

Table 182: The inverse of the X matrix

□

Problem 72. Solve for the determinant and inverse of the following matrix

-2.00	0.00	-1.00
1.00	-5.00	-2.00
3.00	-2.00	9.00

Table 183: X matrix

Solution: The determinant equals 85 and the inverse is:

-0.5765	0.0235	-0.0588
-0.1765	-0.1765	-0.0588
0.1529	-0.0471	0.1176

Table 184: The inverse of the X matrix

□

Problem 73. Solve for the determinant and inverse of the following matrix

-15.00	-20.00	-9.00
18.00	14.00	-10.00
-20.00	-16.00	-16.00

Table 185: X matrix

Solution: The determinant equals -3928 and the inverse is:

0.0978	0.0448	-0.0830
-0.1242	-0.0153	0.0794
0.0020	-0.0407	-0.0382

Table 186: The inverse of the X matrix

□

Problem 74. Solve for the determinant and inverse of the following matrix

-8.00	-19.00	-19.00
-7.00	19.00	-9.00
1.00	3.00	7.00

Table 187: X matrix

Solution: The determinant equals -1280 and the inverse is:

-0.1250	-0.0594	-0.4156
-0.0312	0.0289	-0.0477
0.0312	-0.0039	0.2227

Table 188: The inverse of the X matrix

□

Problem 75. Solve for the determinant and inverse of the following matrix

19.00	19.00	6.00
-5.00	5.00	5.00
7.00	2.00	-17.00

Table 189: X matrix

Solution: The determinant equals -3025 and the inverse is:

0.0314	-0.1107	-0.0215
0.0165	0.1207	0.0413
0.0149	-0.0314	-0.0628

Table 190: The inverse of the X matrix

□

Problem 76. Solve for the determinant and inverse of the following matrix

-19.00	-1.00	-14.00
-8.00	-19.00	11.00
16.00	-7.00	11.00

Table 191: X matrix

Solution: The determinant equals -2796 and the inverse is:

0.0472	-0.0390	0.0991
-0.0944	-0.0054	-0.1148
-0.1288	0.0533	-0.1263

Table 192: The inverse of the X matrix

□

Problem 77. Solve for the determinant and inverse of the following matrix

4.00	18.00	17.00
-8.00	7.00	-19.00
-6.00	-10.00	1.00

Table 193: X matrix

Solution: The determinant equals 3538 and the inverse is:

-0.0517	-0.0531	-0.1303
0.0345	0.0300	-0.0170
0.0345	-0.0192	0.0486

Table 194: The inverse of the X matrix

□

Problem 78. Solve for the determinant and inverse of the following matrix

-14.00	-11.00	0.00
-2.00	-5.00	14.00
-20.00	-16.00	-1.00

Table 195: X matrix

Solution: The determinant equals -104 and the inverse is:

-2.2019	0.1058	1.4808
2.7115	-0.1346	-1.8846
0.6538	0.0385	-0.4615

Table 196: The inverse of the X matrix

□

Problem 79. Solve for the determinant and inverse of the following matrix

-8.00	8.00	8.00
-18.00	13.00	-15.00
5.00	13.00	1.00

Table 197: X matrix

Solution: The determinant equals -4511.99999999999 and the inverse is:

-0.0461	-0.0213	0.0496
0.0126	0.0106	0.0585
0.0663	-0.0319	-0.0089

Table 198: The inverse of the X matrix

□

Problem 80. Solve for the determinant and inverse of the following matrix

17.00	-14.00	-20.00
10.00	9.00	1.00
-16.00	-10.00	6.00

Table 199: X matrix

Solution: The determinant equals 1272 and the inverse is:

0.0503	0.2233	0.1305
-0.0597	-0.1714	-0.1706
0.0346	0.3097	0.2303

Table 200: The inverse of the X matrix

□

3. Multiple Linear Regression

3.1. Multiple Linear Regression - Calculate the Betas

Problem 81. Calculate the betas for the data below, intercept, $\hat{\beta}_1$ and $\hat{\beta}_2$

	y	x1	x2
1	-0.80	5.00	8.00
2	-0.60	2.00	8.00
3	-24.50	4.00	0.00
4	-6.60	1.00	2.00
5	-20.30	5.00	1.00

Solution: The regression table and other pertinent information

	Estimate	Std. Error	t value	Pr(>|t|)
(Intercept)	-12.9443	5.3071	-2.44	0.1349
x1	-2.1034	1.2507	-1.68	0.2346
x2	2.5095	0.5827	4.31	0.0499

Table 201: Linear regression model output.

	x0	x1	x2
1	1.00	5.00	8.00
2	1.00	2.00	8.00
3	1.00	4.00	0.00
4	1.00	1.00	2.00
5	1.00	5.00	1.00

Table 202: X matrix

	x0	x1	x2
x0	5.00	17.00	19.00
x1	17.00	71.00	63.00
x2	19.00	63.00	133.00

Table 203: $X^{'}X$

The determinant of $X^TX = 4000$

□

	x0	x1	x2
x0	5474.00	-1064.00	-278.00
x1	-1064.00	304.00	8.00
x2	-278.00	8.00	66.00

Table 204: $det(X^{'}X)(X^{'}X)^{-1}$

	1	2	3	4	5
x0	-2070.00	1122.00	1218.00	3854.00	-124.00
x1	520.00	-392.00	152.00	-744.00	464.00
x2	290.00	266.00	-246.00	-138.00	-172.00

Table 205: $det(X^{'}X)(X^{'}X)^{-1}X^{'}$

	x
intercept	-12.9443
$\hat{\beta}_1$	-2.1034
$\hat{\beta}_2$	2.5095

Table 206: The betas=$(X^{'}X)^{-1}X^{'}Y$

Problem 82. Calculate the betas for the data below, intercept, $\hat{\beta}_1$ and $\hat{\beta}_2$

	y	x1	x2
1	2.40	3.00	2.00
2	-4.40	7.00	8.00
3	-0.40	1.00	2.00
4	8.20	4.00	2.00
5	22.40	9.00	3.00

Solution: The regression table and other pertinent information

	Estimate	Std. Error	t value	Pr(>|t|)
(Intercept)	1.9749	1.9816	1.00	0.4240
x1	3.4667	0.3929	8.82	0.0126
x2	-3.8162	0.4812	-7.93	0.0155

Table 207: Linear regression model output.

	x0	x1	x2
1	1.00	3.00	2.00
2	1.00	7.00	8.00
3	1.00	1.00	2.00
4	1.00	4.00	2.00
5	1.00	9.00	3.00

Table 208: X matrix

	x0	x1	x2
x0	5.00	24.00	17.00
x1	24.00	156.00	99.00
x2	17.00	99.00	85.00

Table 209: $X^{'}X$

The determinant of $X^TX = 4035$

□

	x0	x1	x2
x0	3459.00	-357.00	-276.00
x1	-357.00	136.00	-87.00
x2	-276.00	-87.00	204.00

Table 210: $det(X^{'}X)(X^{'}X)^{-1}$

	1	2	3	4	5
x0	1836.00	-1248.00	2550.00	1479.00	-582.00
x1	-123.00	-101.00	-395.00	13.00	606.00
x2	-129.00	747.00	45.00	-216.00	-447.00

Table 211: $det(X^{'}X)(X^{'}X)^{-1}X^{'}$

	x
intercept	1.9749
$\hat{\beta}_1$	3.4667
$\hat{\beta}_2$	-3.8162

Table 212: The betas=$(X^{'}X)^{-1}X^{'}Y$

Problem 83. Calculate the betas for the data below, intercept, $\hat{\beta}_1$ and $\hat{\beta}_2$

	y	x1	x2
1	4.50	7.00	1.00
2	-15.40	3.00	3.00
3	-42.30	2.00	9.00
4	-2.00	4.00	1.00
5	-24.40	4.00	5.00

Solution: The regression table and other pertinent information

	Estimate	Std. Error	t value	Pr(>|t|)
(Intercept)	-7.8851	6.2043	-1.27	0.3316
x1	2.2897	1.0628	2.15	0.1640
x2	-4.5247	0.5941	-7.62	0.0168

Table 213: Linear regression model output.

	x0	x1	x2
1	1.00	7.00	1.00
2	1.00	3.00	3.00
3	1.00	2.00	9.00
4	1.00	4.00	1.00
5	1.00	4.00	5.00

Table 214: X matrix

	x0	x1	x2
x0	5.00	20.00	19.00
x1	20.00	94.00	58.00
x2	19.00	58.00	117.00

Table 215: $X^{'}X$

The determinant of $X^TX = 1516$

□

	x0	x1	x2
x0	7634.00	-1238.00	-626.00
x1	-1238.00	224.00	90.00
x2	-626.00	90.00	70.00

Table 216: $det(X^{'}X)(X^{'}X)^{-1}$

	1	2	3	4	5
x0	-1658.00	2042.00	-476.00	2056.00	-448.00
x1	420.00	-296.00	20.00	-252.00	108.00
x2	74.00	-146.00	184.00	-196.00	84.00

Table 217: $det(X^{'}X)(X^{'}X)^{-1}X^{'}$

	x
intercept	-7.8851
$\hat{\beta}_1$	2.2897
$\hat{\beta}_2$	-4.5247

Table 218: The betas=$(X^{'}X)^{-1}X^{'}Y$

Problem 84. Calculate the betas for the data below, intercept, $\hat{\beta}_1$ and $\hat{\beta}_2$

	y	x1	x2
1	27.90	9.00	3.00
2	4.90	2.00	7.00
3	21.20	4.00	8.00
4	35.20	9.00	6.00
5	5.20	1.00	8.00

Solution: The regression table and other pertinent information

	Estimate	Std. Error	t value	Pr(>|t|)
(Intercept)	-25.4155	7.4912	-3.39	0.0770
x1	4.7559	0.4642	10.25	0.0094
x2	3.2056	0.8523	3.76	0.0640

Table 219: Linear regression model output.

	x0	x1	x2
1	1.00	9.00	3.00
2	1.00	2.00	7.00
3	1.00	4.00	8.00
4	1.00	9.00	6.00
5	1.00	1.00	8.00

Table 220: X matrix

	x0	x1	x2
x0	5.00	25.00	32.00
x1	25.00	183.00	135.00
x2	32.00	135.00	222.00

Table 221: $X^{'}X$

The determinant of $X^TX = 1863$

□

	x0	x1	x2
x0	22401.00	-1230.00	-2481.00
x1	-1230.00	86.00	125.00
x2	-2481.00	125.00	290.00

Table 222: $det(X^{'}X)(X^{'}X)^{-1}$

	1	2	3	4	5
x0	3888.00	2574.00	-2367.00	-3555.00	1323.00
x1	-81.00	-183.00	114.00	294.00	-144.00
x2	-486.00	-201.00	339.00	384.00	-36.00

Table 223: $det(X^{'}X)(X^{'}X)^{-1}X^{'}$

	x
intercept	-25.4155
$\hat{\beta}_1$	4.7559
$\hat{\beta}_2$	3.2056

Table 224: The betas=$(X^{'}X)^{-1}X^{'}Y$

Problem 85. Calculate the betas for the data below, intercept, $\hat{\beta}_1$ and $\hat{\beta}_2$

	y	x1	x2
1	59.40	8.00	9.00
2	40.30	7.00	4.00
3	18.90	4.00	1.00
4	3.80	0.00	3.00
5	43.90	7.00	2.00

Solution: The regression table and other pertinent information

	Estimate	Std. Error	t value	Pr(>|t|)
(Intercept)	-3.0432	3.9527	-0.77	0.5219
x1	5.5677	0.7208	7.72	0.0163
x2	1.9346	0.7570	2.56	0.1250

Table 225: Linear regression model output.

	x0	x1	x2
1	1.00	8.00	9.00
2	1.00	7.00	4.00
3	1.00	4.00	1.00
4	1.00	0.00	3.00
5	1.00	7.00	2.00

Table 226: X matrix

	x0	x1	x2
x0	5.00	26.00	19.00
x1	26.00	178.00	118.00
x2	19.00	118.00	111.00

Table 227: $X^{'}X$

The determinant of $X^TX = 6460$

□

	x0	x1	x2
x0	5834.00	-644.00	-314.00
x1	-644.00	194.00	-96.00
x2	-314.00	-96.00	214.00

Table 228: $det(X^{'}X)(X^{'}X)^{-1}$

	1	2	3	4	5
x0	-2144.00	70.00	2944.00	4892.00	698.00
x1	44.00	330.00	36.00	-932.00	522.00
x2	844.00	-130.00	-484.00	328.00	-558.00

Table 229: $det(X^{'}X)(X^{'}X)^{-1}X^{'}$

	x
intercept	-3.0432
$\hat{\beta}_1$	5.5677
$\hat{\beta}_2$	1.9346

Table 230: The betas=$(X^{'}X)^{-1}X^{'}Y$

Problem 86. Calculate the betas for the data below, intercept, $\hat{\beta}_1$ and $\hat{\beta}_2$

	y	x1	x2
1	-26.90	8.00	3.00
2	-14.60	3.00	8.00
3	-3.40	2.00	6.00
4	-26.60	8.00	2.00
5	-16.20	6.00	2.00

Solution: The regression table and other pertinent information

	Estimate	Std. Error	t value	Pr(>|t|)
(Intercept)	19.6028	4.2838	4.58	0.0446
x1	-5.1184	0.4515	-11.34	0.0077
x2	-2.2627	0.4699	-4.82	0.0405

Table 231: Linear regression model output.

	x0	x1	x2
1	1.00	8.00	3.00
2	1.00	3.00	8.00
3	1.00	2.00	6.00
4	1.00	8.00	2.00
5	1.00	6.00	2.00

Table 232: X matrix

	x0	x1	x2
x0	5.00	27.00	21.00
x1	27.00	177.00	88.00
x2	21.00	88.00	117.00

Table 233: $X^{'}X$

The determinant of $X^T X = 1267$

□

	x0	x1	x2
x0	12965.00	-1311.00	-1341.00
x1	-1311.00	144.00	127.00
x2	-1341.00	127.00	156.00

Table 234: $det(X^{'}X)(X^{'}X)^{-1}$

	1	2	3	4	5
x0	-1546.00	-1696.00	2297.00	-205.00	2417.00
x1	222.00	137.00	-261.00	95.00	-193.00
x2	143.00	288.00	-151.00	-13.00	-267.00

Table 235: $det(X^{'}X)(X^{'}X)^{-1}X^{'}$

	x
intercept	19.6028
$\hat{\beta}_1$	-5.1184
$\hat{\beta}_2$	-2.2627

Table 236: The betas=$(X^{'}X)^{-1}X^{'}Y$

Problem 87. Calculate the betas for the data below, intercept, $\hat{\beta}_1$ and $\hat{\beta}_2$

	y	x1	x2
1	-8.90	7.00	9.00
2	-15.00	5.00	8.00
3	-17.10	0.00	0.00
4	-14.10	2.00	8.00
5	-12.60	7.00	8.00

Solution: The regression table and other pertinent information

	Estimate	Std. Error	t value	Pr(>|t|)
(Intercept)	-17.4095	2.4810	-7.02	0.0197
x1	0.5947	0.6480	0.92	0.4556
x2	0.2078	0.5433	0.38	0.7389

Table 237: Linear regression model output.

	x0	x1	x2
1	1.00	7.00	9.00
2	1.00	5.00	8.00
3	1.00	0.00	0.00
4	1.00	2.00	8.00
5	1.00	7.00	8.00

Table 238: X matrix

	x0	x1	x2
x0	5.00	21.00	33.00
x1	21.00	127.00	175.00
x2	33.00	175.00	273.00

Table 239: $X^{'}X$

The determinant of $X^TX = 4084$

□

	x0	x1	x2
x0	4046.00	42.00	-516.00
x1	42.00	276.00	-182.00
x2	-516.00	-182.00	194.00

Table 240: $det(X^{'}X)(X^{'}X)^{-1}$

	1	2	3	4	5
x0	-304.00	128.00	4046.00	2.00	212.00
x1	336.00	-34.00	42.00	-862.00	518.00
x2	-44.00	126.00	-516.00	672.00	-238.00

Table 241: $det(X^{'}X)(X^{'}X)^{-1}X^{'}$

	x
intercept	-17.4095
$\hat{\beta}_1$	0.5947
$\hat{\beta}_2$	0.2078

Table 242: The betas=$(X^{'}X)^{-1}X^{'}Y$

Problem 88. Calculate the betas for the data below, intercept, $\hat{\beta}_1$ and $\hat{\beta}_2$

	y	x1	x2
1	4.00	3.00	9.00
2	28.30	5.00	0.00
3	12.60	3.00	6.00
4	19.00	5.00	5.00
5	11.10	1.00	2.00

Solution: The regression table and other pertinent information

	Estimate	Std. Error	t value	Pr(>|t|)
(Intercept)	11.9358	2.1429	5.57	0.0308
x1	3.3187	0.4806	6.90	0.0203
x2	-1.8680	0.2293	-8.15	0.0147

Table 243: Linear regression model output.

	x0	x1	x2
1	1.00	3.00	9.00
2	1.00	5.00	0.00
3	1.00	3.00	6.00
4	1.00	5.00	5.00
5	1.00	1.00	2.00

Table 244: X matrix

	x0	x1	x2
x0	5.00	17.00	22.00
x1	17.00	69.00	72.00
x2	22.00	72.00	146.00

Table 245: $X^{'}X$

The determinant of $X^TX = 2716$

□

	x0	x1	x2
x0	4890.00	-898.00	-294.00
x1	-898.00	246.00	14.00
x2	-294.00	14.00	56.00

Table 246: $det(X^{'}X)(X^{'}X)^{-1}$

	1	2	3	4	5
x0	-450.00	400.00	432.00	-1070.00	3404.00
x1	-34.00	332.00	-76.00	402.00	-624.00
x2	252.00	-224.00	84.00	56.00	-168.00

Table 247: $det(X^{'}X)(X^{'}X)^{-1}X^{'}$

	x
intercept	11.9358
$\hat{\beta}_1$	3.3187
$\hat{\beta}_2$	-1.8680

Table 248: The betas=$(X^{'}X)^{-1}X^{'}Y$

Problem 89. Calculate the betas for the data below, intercept, $\hat{\beta}_1$ and $\hat{\beta}_2$

	y	x1	x2
1	-3.10	6.00	0.00
2	6.30	9.00	0.00
3	-8.10	6.00	9.00
4	-11.80	2.00	2.00
5	-15.10	2.00	3.00

Solution: The regression table and other pertinent information

	Estimate	Std. Error	t value	Pr(>|t|)
(Intercept)	-17.0156	1.7517	-9.71	0.0104
x1	2.5160	0.2760	9.12	0.0118
x2	-0.6873	0.2237	-3.07	0.0916

Table 249: Linear regression model output.

	x0	x1	x2
1	1.00	6.00	0.00
2	1.00	9.00	0.00
3	1.00	6.00	9.00
4	1.00	2.00	2.00
5	1.00	2.00	3.00

Table 250: X matrix

	x0	x1	x2
x0	5.00	25.00	14.00
x1	25.00	161.00	64.00
x2	14.00	64.00	94.00

Table 251: $X^{'}X$

The determinant of $X^TX = 9684$

□

	x0	x1	x2
x0	11038.00	-1454.00	-654.00
x1	-1454.00	274.00	30.00
x2	-654.00	30.00	180.00

Table 252: $det(X^{'}X)(X^{'}X)^{-1}$

	1	2	3	4	5
x0	2314.00	-2048.00	-3572.00	6822.00	6168.00
x1	190.00	1012.00	460.00	-846.00	-816.00
x2	-474.00	-384.00	1146.00	-234.00	-54.00

Table 253: $det(X^{'}X)(X^{'}X)^{-1}X^{'}$

	x
intercept	-17.0156
$\hat{\beta}_1$	2.5160
$\hat{\beta}_2$	-0.6873

Table 254: The betas=$(X^{'}X)^{-1}X^{'}Y$

Problem 90. Calculate the betas for the data below, intercept, $\hat{\beta}_1$ and $\hat{\beta}_2$

	y	x1	x2
1	14.10	0.00	7.00
2	13.40	0.00	6.00
3	-16.60	5.00	7.00
4	-40.70	8.00	1.00
5	-45.30	9.00	3.00

Solution: The regression table and other pertinent information

	Estimate	Std. Error	t value	Pr(>|t|)
(Intercept)	7.8483	0.4039	19.43	0.0026
x1	-6.1934	0.0346	-178.82	0.0000
x2	0.9130	0.0552	16.54	0.0036

Table 255: Linear regression model output.

	x0	x1	x2
1	1.00	0.00	7.00
2	1.00	0.00	6.00
3	1.00	5.00	7.00
4	1.00	8.00	1.00
5	1.00	9.00	3.00

Table 256: X matrix

	x0	x1	x2
x0	5.00	22.00	24.00
x1	22.00	170.00	70.00
x2	24.00	70.00	144.00

Table 257: $X^{'}X$

The determinant of $X^TX = 4204$

□

	x0	x1	x2
x0	19580.00	-1488.00	-2540.00
x1	-1488.00	144.00	178.00
x2	-2540.00	178.00	366.00

Table 258: $det(X^{'}X)(X^{'}X)^{-1}$

	1	2	3	4	5
x0	1800.00	4340.00	-5640.00	5136.00	-1432.00
x1	-242.00	-420.00	478.00	-158.00	342.00
x2	22.00	-344.00	912.00	-750.00	160.00

Table 259: $det(X^{'}X)(X^{'}X)^{-1}X^{'}$

	x
intercept	7.8483
$\hat{\beta}_1$	-6.1934
$\hat{\beta}_2$	0.9130

Table 260: The betas=$(X^{'}X)^{-1}X^{'}Y$

Problem 91. Solve for R-squared (R^2), coefficient of multiple determination and test:

$$H_0 : \beta_1 = \beta_2 \cdots \beta_{p-1} = 0 \text{ versus}$$

$$H_A : \beta_k \neq 0 \text{ for at least one } k \text{ where } 1 \leq k \leq (p-1)$$

	y	x1	x2
1	9.70	0.00	6.00
2	12.70	0.00	6.00
3	28.70	8.00	1.00
4	15.30	1.00	2.00
5	21.90	4.00	1.00

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Solution: The regression table and other pertinent information

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
x1	1	224.42	224.42	96.22	0.0102
x2	1	4.31	4.31	1.85	0.3070
Residuals	2	4.66	2.33		

Table 261: Linear regression model output in ANOVA table.

	Estimate	Std. Error	t value	Pr(>|t|)
(Intercept)	14.9917	2.3823	6.29	0.0243
x1	1.8087	0.3525	5.13	0.0359
x2	-0.6358	0.4677	-1.36	0.3070

Table 262: Linear regression model output.

The coefficient of multiple determination equals

$R^2 = \frac{SSR}{SSTO} = 1 - \frac{SSE}{SSTO} = 0.98001$,

and the F-statistic equals

$F^* = \frac{MSR}{MSE} = 49.03223$

The determinant of $X^T X = 2516$

$$SSTO = Y^{'}Y - \frac{1}{n}Y^{'}JY = Y^{'}[I - \frac{1}{n}J]Y = 233.392$$

	x0	x1	x2
1	1.00	0.00	6.00
2	1.00	0.00	6.00
3	1.00	8.00	1.00
4	1.00	1.00	2.00
5	1.00	4.00	1.00

Table 263: X matrix

	x0	x1	x2
x0	5.00	13.00	16.00
x1	13.00	81.00	14.00
x2	16.00	14.00	78.00

Table 264: $X^{'}X$

$$\begin{aligned} SSR &= Y^{'}[H - \frac{1}{n}J]Y = 228.72717 \\ SSE &= e^{'}e = Y^{'}[I-H]Y = 4.66483 \end{aligned}$$

where J is a $n \times n$ matrix of 1s and $H = X(X^{'}X)^{-1}X^{'}$ also size $n \times n$.

MSE=$\frac{SSE}{n-p} = 2.33242$ and MSR=$\frac{SSR}{p-1} = 114.36358$

□

	x0	x1	x2
x0	6122.00	-790.00	-1114.00
x1	-790.00	134.00	138.00
x2	-1114.00	138.00	236.00

Table 265: $det(X^{'}X)(X^{'}X)^{-1}$

	1	2	3	4	5
x0	-562.00	-562.00	-1312.00	3104.00	1848.00
x1	38.00	38.00	420.00	-380.00	-116.00
x2	302.00	302.00	226.00	-504.00	-326.00

Table 266: $det(X^{'}X)(X^{'}X)^{-1}X^{'}$

	x
intercept	14.9917
$\hat{\beta}_1$	1.8087
$\hat{\beta}_2$	-0.6358

Table 267: The betas=$(X^{'}X)^{-1}X^{'}Y$

	x1	x2	y	yhat	resid
1	0.00	6.00	9.70	11.18	-1.48
2	0.00	6.00	12.70	11.18	1.52
3	8.00	1.00	28.70	28.83	-0.13
4	1.00	2.00	15.30	15.53	-0.23
5	4.00	1.00	21.90	21.59	0.31

	1	2	3	4	5
1	0.4968	0.4968	0.0175	0.0318	-0.0429
2	0.4968	0.4968	0.0175	0.0318	-0.0429
3	0.0175	0.0175	0.9038	-0.1749	0.2361
4	0.0318	0.0318	-0.1749	0.6820	0.4293
5	-0.0429	-0.0429	0.2361	0.4293	0.4205

Table 268: H or the Hat matrix

Problem 92. Solve for R-squared (R^2), coefficient of multiple determination and test:

$H_0 : \beta_1 = \beta_2 \cdots \beta_{p-1} = 0$ versus

$H_A : \beta_k \neq 0$ for at least one k where $1 \leq k \leq (p-1)$

	y	x1	x2
1	-26.90	7.00	5.00
2	-10.70	0.00	5.00
3	-31.10	2.00	8.00
4	-18.10	5.00	2.00
5	-28.50	7.00	5.00

Solution: The regression table and other pertinent information

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
x1	1	81.52	81.52	4.71	0.1621
x2	1	170.23	170.23	9.84	0.0884
Residuals	2	34.61	17.30		

Table 269: Linear regression model output in ANOVA table.

	Estimate	Std. Error	t value	Pr(>|t|)
(Intercept)	2.5680	7.0808	0.36	0.7516
x1	-2.2082	0.7103	-3.11	0.0897
x2	-3.2707	1.0428	-3.14	0.0884

Table 270: Linear regression model output.

The coefficient of multiple determination equals

$R^2 = \frac{SSR}{SSTO} = 1 - \frac{SSE}{SSTO} = 0.87915,$

and the F-statistic equals

$F^* = \frac{MSR}{MSE} = 7.2747$

The determinant of $X^T X = 3087$

$$SSTO = Y^{'}Y - \frac{1}{n}Y^{'}JY = Y^{'}[I - \frac{1}{n}J]Y = 286.352$$

	x0	x1	x2
1	1.00	7.00	5.00
2	1.00	0.00	5.00
3	1.00	2.00	8.00
4	1.00	5.00	2.00
5	1.00	7.00	5.00

Table 271: X matrix

	x0	x1	x2
x0	5.00	21.00	25.00
x1	21.00	127.00	96.00
x2	25.00	96.00	143.00

Table 272: $X^{'}X$

$$SSR = Y^{'}[H - \frac{1}{n}J]Y = 251.74629$$
$$SSE = e^{'}e = Y^{'}[I - H]Y = 34.60571$$

where J is a $n \times n$ matrix of 1s and $H = X(X^{'}X)^{-1}X^{'}$ also size $n \times n$.

MSE=$\frac{SSE}{n-p}$ = 17.30286 and MSR=$\frac{SSR}{p-1}$ = 125.87314 □

	x0	x1	x2
x0	8945.00	-603.00	-1159.00
x1	-603.00	90.00	45.00
x2	-1159.00	45.00	194.00

Table 273: $det(X^{'}X)(X^{'}X)^{-1}$

	1	2	3	4	5
x0	-1071.00	3150.00	-1533.00	3612.00	-1071.00
x1	252.00	-378.00	-63.00	-63.00	252.00
x2	126.00	-189.00	483.00	-546.00	126.00

Table 274: $det(X^{'}X)(X^{'}X)^{-1}X^{'}$

	x
intercept	2.5680
$\hat{\beta}_1$	-2.2082
$\hat{\beta}_2$	-3.2707

Table 275: The betas=$(X^{'}X)^{-1}X^{'}Y$

	x1	x2	y	yhat	resid
1	7.00	5.00	-26.90	-29.24	2.34
2	0.00	5.00	-10.70	-13.79	3.09
3	2.00	8.00	-31.10	-28.01	-3.09
4	5.00	2.00	-18.10	-15.01	-3.09
5	7.00	5.00	-28.50	-29.24	0.74

	1	2	3	4	5
1	0.4286	-0.1429	0.1429	0.1429	0.4286
2	-0.1429	0.7143	0.2857	0.2857	-0.1429
3	0.1429	0.2857	0.7143	-0.2857	0.1429
4	0.1429	0.2857	-0.2857	0.7143	0.1429
5	0.4286	-0.1429	0.1429	0.1429	0.4286

Table 276: H or the Hat matrix

Problem 93. Solve for R-squared (R^2), coefficient of multiple determination and test:

$H_0 : \beta_1 = \beta_2 \cdots \beta_{p-1} = 0$ versus

$H_A : \beta_k \neq 0$ for at least one k where $1 \leq k \leq (p-1)$

	y	x1	x2
1	21.00	3.00	6.00
2	17.00	2.00	5.00
3	25.80	6.00	9.00
4	-1.60	2.00	0.00
5	37.50	1.00	8.00

Solution: The regression table and other pertinent information

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
x1	1	3.25	3.25	0.63	0.5114
x2	1	802.80	802.80	154.76	0.0064
Residuals	2	10.37	5.19		

Table 277: Linear regression model output in ANOVA table.

	Estimate	Std. Error	t value	Pr(>|t|)
(Intercept)	3.3915	2.2990	1.48	0.2781
x1	-3.0377	0.6557	-4.63	0.0436
x2	4.4739	0.3596	12.44	0.0064

Table 278: Linear regression model output.

The coefficient of multiple determination equals

$R^2 = \frac{SSR}{SSTO} = 1 - \frac{SSE}{SSTO} = 0.98729,$

and the F-statistic equals

$F^* = \frac{MSR}{MSE} = 77.69413$

The determinant of $X^T X = 2968$

$$SSTO \;=\; Y'Y - \frac{1}{n}Y'JY = Y'[I - \frac{1}{n}J]Y = 816.432$$

	x0	x1	x2
1	1.00	3.00	6.00
2	1.00	2.00	5.00
3	1.00	6.00	9.00
4	1.00	2.00	0.00
5	1.00	1.00	8.00

Table 279: X matrix

	x0	x1	x2
x0	5.00	14.00	28.00
x1	14.00	54.00	90.00
x2	28.00	90.00	206.00

Table 280: $X^{'}X$

$$\begin{aligned} SSR &= Y^{'}[H - \frac{1}{n}J]Y = 806.05725 \\ SSE &= e^{'}e = Y^{'}[I - H]Y = 10.37475 \end{aligned}$$

where J is a $n \times n$ matrix of 1s and $H = X(X^{'}X)^{-1}X^{'}$ also size $n \times n$.

MSE=$\frac{SSE}{n-p}$ $= 5.18738$ and MSR=$\frac{SSR}{p-1}$ $= 403.02862$

□

	x0	x1	x2
x0	3024.00	-364.00	-252.00
x1	-364.00	246.00	-58.00
x2	-252.00	-58.00	74.00

Table 281: $det(X^{'}X)(X^{'}X)^{-1}$

	1	2	3	4	5
x0	420.00	1036.00	-1428.00	2296.00	644.00
x1	26.00	-162.00	590.00	128.00	-582.00
x2	18.00	2.00	66.00	-368.00	282.00

Table 282: $det(X^{'}X)(X^{'}X)^{-1}X^{'}$

	x
intercept	3.3915
$\hat{\beta}_1$	-3.0377
$\hat{\beta}_2$	4.4739

Table 283: The betas=$(X^{'}X)^{-1}X^{'}Y$

	x1	x2	y	yhat	resid
1	3.00	6.00	21.00	21.12	-0.12
2	2.00	5.00	17.00	19.69	-2.69
3	6.00	9.00	25.80	25.43	0.37
4	2.00	0.00	-1.60	-2.68	1.08
5	1.00	8.00	37.50	36.15	1.35

	1	2	3	4	5
1	0.2042	0.1894	0.2487	0.1590	0.1988
2	0.1894	0.2433	0.0276	0.2399	0.2999
3	0.2487	0.0276	0.9117	-0.0836	-0.1044
4	0.1590	0.2399	-0.0836	0.8598	-0.1752
5	0.1988	0.2999	-0.1044	-0.1752	0.7810

Table 284: H or the Hat matrix

Problem 94. Solve for R-squared (R^2), coefficient of multiple determination and test:

$H_0 : \beta_1 = \beta_2 \cdots \beta_{p-1} = 0$ versus

$H_A : \beta_k \neq 0$ for at least one k where $1 \leq k \leq (p-1)$

	y	x1	x2
1	23.10	3.00	6.00
2	23.20	0.00	4.00
3	29.60	0.00	6.00
4	15.90	7.00	5.00
5	1.60	1.00	0.00

Solution: The regression table and other pertinent information

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
x1	1	15.57	15.57	2.98	0.2266
x2	1	432.63	432.63	82.69	0.0119
Residuals	2	10.46	5.23		

Table 285: Linear regression model output in ANOVA table.

	Estimate	Std. Error	t value	Pr(>|t|)
(Intercept)	3.9655	2.2232	1.78	0.2164
x1	-1.5079	0.3986	-3.78	0.0633
x2	4.2933	0.4721	9.09	0.0119

Table 286: Linear regression model output.

The coefficient of multiple determination equals

$R^2 = \frac{SSR}{SSTO} = 1 - \frac{SSE}{SSTO} = 0.97719,$

and the F-statistic equals

$F^* = \frac{MSR}{MSE} = 42.83295$

The determinant of $X^T X = 4084$

$$SSTO = Y^{'}Y - \frac{1}{n}Y^{'}JY = Y^{'}[I - \frac{1}{n}J]Y = 458.668$$

	x0	x1	x2
1	1.00	3.00	6.00
2	1.00	0.00	4.00
3	1.00	0.00	6.00
4	1.00	7.00	5.00
5	1.00	1.00	0.00

Table 287: X matrix

	x0	x1	x2
x0	5.00	11.00	21.00
x1	11.00	59.00	53.00
x2	21.00	53.00	113.00

Table 288: $X^{'}X$

$$SSR = Y^{'}[H - \frac{1}{n}J]Y = 448.204$$
$$SSE = e^{'}e = Y^{'}[I - H]Y = 10.464$$

where J is a $n \times n$ matrix of 1s and $H = X(X^{'}X)^{-1}X^{'}$ also size $n \times n$.

MSE=$\frac{SSE}{n-p} = 5.232$ and MSR=$\frac{SSR}{p-1} = 224.102$ $\square$

	x0	x1	x2
x0	3858.00	-130.00	-656.00
x1	-130.00	124.00	-34.00
x2	-656.00	-34.00	174.00

Table 289: $det(X^{'}X)(X^{'}X)^{-1}$

	1	2	3	4	5
x0	-468.00	1234.00	-78.00	-332.00	3728.00
x1	38.00	-266.00	-334.00	568.00	-6.00
x2	286.00	40.00	388.00	-24.00	-690.00

Table 290: $det(X^{'}X)(X^{'}X)^{-1}X^{'}$

	x
intercept	3.9655
$\hat{\beta}_1$	-1.5079
$\hat{\beta}_2$	4.2933

Table 291: The betas=$(X^{'}X)^{-1}X^{'}Y$

	x1	x2	y	yhat	resid
1	3.00	6.00	23.10	25.20	-2.10
2	0.00	4.00	23.20	21.14	2.06
3	0.00	6.00	29.60	29.73	-0.13
4	7.00	5.00	15.90	14.88	1.02
5	1.00	0.00	1.60	2.46	-0.86

	1	2	3	4	5
1	0.3335	0.1655	0.3056	0.3007	-0.1053
2	0.1655	0.3413	0.3609	-0.1048	0.2370
3	0.3056	0.3609	0.5509	-0.1166	-0.1009
4	0.3007	-0.1048	-0.1166	0.8629	0.0578
5	-0.1053	0.2370	-0.1009	0.0578	0.9114

Table 292: H or the Hat matrix

Problem 95. Solve for R-squared (R^2), coefficient of multiple determination and test:

$H_0 : \beta_1 = \beta_2 \cdots \beta_{p-1} = 0$ versus

$H_A : \beta_k \neq 0$ for at least one k where $1 \leq k \leq (p-1)$

	y	x1	x2
1	-13.20	8.00	3.00
2	-13.00	5.00	9.00
3	-12.60	4.00	5.00
4	-22.20	1.00	4.00
5	-7.60	6.00	1.00

Solution: The regression table and other pertinent information

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
x1	1	61.14	61.14	2.46	0.2574
x2	1	0.52	0.52	0.02	0.8980
Residuals	2	49.74	24.87		

Table 293: Linear regression model output in ANOVA table.

	Estimate	Std. Error	t value	Pr(>|t|)
(Intercept)	-20.2733	7.0312	-2.88	0.1022
x1	1.4797	0.9864	1.50	0.2724
x2	-0.1248	0.8607	-0.15	0.8980

Table 294: Linear regression model output.

The coefficient of multiple determination equals

$R^2 = \frac{SSR}{SSTO} = 1 - \frac{SSE}{SSTO} = 0.55352,$

and the F-statistic equals

$F^* = \frac{MSR}{MSE} = 1.23972$

The determinant of $X^T X = 4499$

$$SSTO \;=\; Y^{'}Y - \frac{1}{n}Y^{'}JY = Y^{'}[I - \frac{1}{n}J]Y = 111.408$$

	x0	x1	x2
1	1.00	8.00	3.00
2	1.00	5.00	9.00
3	1.00	4.00	5.00
4	1.00	1.00	4.00
5	1.00	6.00	1.00

Table 295: X matrix

	x0	x1	x2
x0	5.00	24.00	22.00
x1	24.00	142.00	99.00
x2	22.00	99.00	132.00

Table 296: $X^{'}X$

$$SSR = Y^{'}[H - \frac{1}{n}J]Y = 61.66608$$
$$SSE = e^{'}e = Y^{'}[I - H]Y = 49.74192$$

where J is a $n \times n$ matrix of 1s and $H = X(X^{'}X)^{-1}X^{'}$ also size $n \times n$.

MSE$=\frac{SSE}{n-p} = 24.87096$ and MSR$=\frac{SSR}{p-1} = 30.83304$

□

	x0	x1	x2
x0	8943.00	-990.00	-748.00
x1	-990.00	176.00	33.00
x2	-748.00	33.00	134.00

Table 297: $det(X^{'}X)(X^{'}X)^{-1}$

	1	2	3	4	5
x0	-1221.00	-2739.00	1243.00	4961.00	2255.00
x1	517.00	187.00	-121.00	-682.00	99.00
x2	-82.00	623.00	54.00	-179.00	-416.00

Table 298: $det(X^{'}X)(X^{'}X)^{-1}X^{'}$

	x
intercept	-20.2733
$\hat{\beta}_1$	1.4797
$\hat{\beta}_2$	-0.1248

Table 299: The betas=$(X^{'}X)^{-1}X^{'}Y$

	x1	x2	y	yhat	resid
1	8.00	3.00	-13.20	-8.81	-4.39
2	5.00	9.00	-13.00	-14.00	1.00
3	4.00	5.00	-12.60	-14.98	2.38
4	1.00	4.00	-22.20	-19.29	-2.91
5	6.00	1.00	-7.60	-11.52	3.92

	1	2	3	4	5
1	0.5932	0.1391	0.0971	-0.2294	0.3999
2	0.1391	0.8453	0.2498	-0.0133	-0.2209
3	0.0971	0.2498	0.2287	0.2974	0.1269
4	-0.2294	-0.0133	0.2974	0.7920	0.1534
5	0.3999	-0.2209	0.1269	0.1534	0.5408

Table 300: H or the Hat matrix

Problem 96. Solve for R-squared (R^2), coefficient of multiple determination and test:

$H_0 : \beta_1 = \beta_2 \cdots \beta_{p-1} = 0$ versus

$H_A : \beta_k \neq 0$ for at least one k where $1 \leq k \leq (p-1)$

	y	x1	x2
1	-25.50	4.00	3.00
2	-10.60	0.00	2.00
3	-31.40	5.00	7.00
4	-21.50	3.00	5.00
5	-33.90	5.00	3.00

Solution: The regression table and other pertinent information

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
x1	1	328.26	328.26	70.47	0.0139
x2	1	1.58	1.58	0.34	0.6197
Residuals	2	9.32	4.66		

Table 301: Linear regression model output in ANOVA table.

	Estimate	Std. Error	t value	Pr(>|t|)
(Intercept)	-10.5563	2.4692	-4.28	0.0506
x1	-4.5641	0.6195	-7.37	0.0179
x2	0.3735	0.6423	0.58	0.6197

Table 302: Linear regression model output.

The coefficient of multiple determination equals

$R^2 = \frac{SSR}{SSTO} = 1 - \frac{SSE}{SSTO} = 0.97253,$

and the F-statistic equals

$F^* = \frac{MSR}{MSE} = 35.40609$

The determinant of $X^T X = 971$

$$SSTO = Y^{'}Y - \frac{1}{n}Y^{'}JY = Y^{'}[I - \frac{1}{n}J]Y = 339.148$$

	x0	x1	x2
1	1.00	4.00	3.00
2	1.00	0.00	2.00
3	1.00	5.00	7.00
4	1.00	3.00	5.00
5	1.00	5.00	3.00

Table 303: X matrix

	x0	x1	x2
x0	5.00	17.00	20.00
x1	17.00	75.00	77.00
x2	20.00	77.00	96.00

Table 304: $X^{'}X$

$$SSR = Y^{'}[H - \frac{1}{n}J]Y = 329.8323$$
$$SSE = e^{'}e = Y^{'}[I - H]Y = 9.3157$$

where J is a $n \times n$ matrix of 1s and $H = X(X^{'}X)^{-1}X^{'}$ also size $n \times n$.

MSE=$\frac{SSE}{n-p} = 4.65785$ and MSR=$\frac{SSR}{p-1} = 164.91615$ □

	x0	x1	x2
x0	1271.00	-92.00	-191.00
x1	-92.00	80.00	-45.00
x2	-191.00	-45.00	86.00

Table 305: $det(X^{'}X)(X^{'}X)^{-1}$

	1	2	3	4	5
x0	330.00	889.00	-526.00	40.00	238.00
x1	93.00	-182.00	-7.00	-77.00	173.00
x2	-113.00	-19.00	186.00	104.00	-158.00

Table 306: $det(X^{'}X)(X^{'}X)^{-1}X^{'}$

	x
intercept	-10.5563
$\hat{\beta}_1$	-4.5641
$\hat{\beta}_2$	0.3735

Table 307: The betas=$(X^{'}X)^{-1}X^{'}Y$

	x1	x2	y	yhat	resid
1	4.00	3.00	-25.50	-27.69	2.19
2	0.00	2.00	-10.60	-9.81	-0.79
3	5.00	7.00	-31.40	-30.76	-0.64
4	3.00	5.00	-21.50	-22.38	0.88
5	5.00	3.00	-33.90	-32.26	-1.64

	1	2	3	4	5
1	0.3738	0.1071	0.0041	0.0453	0.4696
2	0.1071	0.8764	-0.1586	0.2554	-0.0803
3	0.0041	-0.1586	0.7631	0.3944	-0.0031
4	0.0453	0.2554	0.3944	0.3388	-0.0340
5	0.4696	-0.0803	-0.0031	-0.0340	0.6478

Table 308: H or the Hat matrix

Problem 97. Solve for R-squared (R^2), coefficient of multiple determination and test:

$H_0 : \beta_1 = \beta_2 \cdots \beta_{p-1} = 0$ versus

$H_A : \beta_k \neq 0$ for at least one k where $1 \leq k \leq (p-1)$

	y	x1	x2
1	13.30	1.00	8.00
2	-0.40	1.00	2.00
3	1.20	2.00	2.00
4	1.40	4.00	2.00
5	4.20	6.00	2.00

Solution: The regression table and other pertinent information

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
x1	1	4.37	4.37	7.29	0.1141
x2	1	114.91	114.91	191.95	0.0052
Residuals	2	1.20	0.60		

Table 309: Linear regression model output in ANOVA table.

	Estimate	Std. Error	t value	Pr(>|t|)
(Intercept)	-5.5542	0.9909	-5.61	0.0304
x1	0.8136	0.2015	4.04	0.0562
x2	2.2551	0.1628	13.85	0.0052

Table 310: Linear regression model output.

The coefficient of multiple determination equals

$R^2 = \frac{SSR}{SSTO} = 1 - \frac{SSE}{SSTO} = 0.99006$,

and the F-statistic equals

$F^* = \frac{MSR}{MSE} = 99.62072$

The determinant of $X^T X = 2124$

$$SSTO = Y^{'}Y - \frac{1}{n}Y^{'}JY = Y^{'}[I - \frac{1}{n}J]Y = 120.472$$

	x0	x1	x2
1	1.00	1.00	8.00
2	1.00	1.00	2.00
3	1.00	2.00	2.00
4	1.00	4.00	2.00
5	1.00	6.00	2.00

Table 311: X matrix

	x0	x1	x2
x0	5.00	14.00	16.00
x1	14.00	58.00	34.00
x2	16.00	34.00	80.00

Table 312: $X^{'}X$

$$SSR = Y^{'}[H - \frac{1}{n}J]Y = 119.27471$$
$$SSE = e^{'}e = Y^{'}[I - H]Y = 1.19729$$

where J is a $n \times n$ matrix of 1s and $H = X(X^{'}X)^{-1}X^{'}$ also size $n \times n$.

MSE=$\frac{SSE}{n-p} = 0.59864$ and MSR=$\frac{SSR}{p-1} = 59.63736$

□

	x0	x1	x2
x0	3484.00	-576.00	-452.00
x1	-576.00	144.00	54.00
x2	-452.00	54.00	94.00

Table 313: $det(X^{'}X)(X^{'}X)^{-1}$

	1	2	3	4	5
x0	-708.00	2004.00	1428.00	276.00	-876.00
x1	-0.00	-324.00	-180.00	108.00	396.00
x2	354.00	-210.00	-156.00	-48.00	60.00

Table 314: $det(X^{'}X)(X^{'}X)^{-1}X^{'}$

	x
intercept	-5.5542
$\hat{\beta}_1$	0.8136
$\hat{\beta}_2$	2.2551

Table 315: The betas=$(X^{'}X)^{-1}X^{'}Y$

	x1	x2	y	yhat	resid
1	1.00	8.00	13.30	13.30	0.00
2	1.00	2.00	-0.40	-0.23	-0.17
3	2.00	2.00	1.20	0.58	0.62
4	4.00	2.00	1.40	2.21	-0.81
5	6.00	2.00	4.20	3.84	0.36

	1	2	3	4	5
1	1.0000	0.0000	0.0000	0.0000	0.0000
2	0.0000	0.5932	0.4407	0.1356	-0.1695
3	0.0000	0.4407	0.3559	0.1864	0.0169
4	0.0000	0.1356	0.1864	0.2881	0.3898
5	-0.0000	-0.1695	0.0169	0.3898	0.7627

Table 316: H or the Hat matrix

Problem 98. Solve for R-squared (R^2), coefficient of multiple determination and test:

$H_0 : \beta_1 = \beta_2 \cdots \beta_{p-1} = 0$ versus

$H_A : \beta_k \neq 0$ for at least one k where $1 \leq k \leq (p-1)$

	y	x1	x2
1	-0.60	1.00	6.00
2	-0.40	1.00	1.00
3	-32.80	8.00	3.00
4	-13.90	4.00	0.00
5	1.50	1.00	9.00

Solution: The regression table and other pertinent information

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
x1	1	842.24	842.24	957.41	0.0010
x2	1	0.93	0.93	1.06	0.4119
Residuals	2	1.76	0.88		

Table 317: Linear regression model output in ANOVA table.

	Estimate	Std. Error	t value	Pr(>|t|)
(Intercept)	4.1618	0.9366	4.44	0.0471
x1	-4.6451	0.1640	-28.33	0.0012
x2	0.1404	0.1365	1.03	0.4119

Table 318: Linear regression model output.

The coefficient of multiple determination equals

$R^2 = \frac{SSR}{SSTO} = 1 - \frac{SSE}{SSTO} = 0.99792,$

and the F-statistic equals

$F^* = \frac{MSR}{MSE} = 479.23323$

The determinant of $X^TX = 8967$

$$SSTO = Y^{'}Y - \frac{1}{n}Y^{'}JY = Y^{'}[I - \frac{1}{n}J]Y = 844.932$$

	x0	x1	x2
1	1.00	1.00	6.00
2	1.00	1.00	1.00
3	1.00	8.00	3.00
4	1.00	4.00	0.00
5	1.00	1.00	9.00

Table 319: X matrix

	x0	x1	x2
x0	5.00	15.00	19.00
x1	15.00	83.00	40.00
x2	19.00	40.00	127.00

Table 320: $X^{'}X$

$$SSR = Y^{'}[H - \frac{1}{n}J]Y = 843.17258$$
$$SSE = e^{'}e = Y^{'}[I - H]Y = 1.75942$$

where J is a $n \times n$ matrix of 1s and $H = X(X^{'}X)^{-1}X^{'}$ also size $n \times n$.

MSE=$\frac{SSE}{n-p} = 0.87971$ and MSR=$\frac{SSR}{p-1} = 421.58629$

□

	x0	x1	x2
x0	8941.00	-1145.00	-977.00
x1	-1145.00	274.00	85.00
x2	-977.00	85.00	190.00

Table 321: $det(X^{'}X)(X^{'}X)^{-1}$

	1	2	3	4	5
x0	1934.00	6819.00	-3150.00	4361.00	-997.00
x1	-361.00	-786.00	1302.00	-49.00	-106.00
x2	248.00	-702.00	273.00	-637.00	818.00

Table 322: $det(X^{'}X)(X^{'}X)^{-1}X^{'}$

	x
intercept	4.1618
$\hat{\beta}_1$	-4.6451
$\hat{\beta}_2$	0.1404

Table 323: The betas=$(X^{'}X)^{-1}X^{'}Y$

	x1	x2	y	yhat	resid
1	1.00	6.00	-0.60	0.36	-0.96
2	1.00	1.00	-0.40	-0.34	-0.06
3	8.00	3.00	-32.80	-32.58	-0.22
4	4.00	0.00	-13.90	-14.42	0.52
5	1.00	9.00	1.50	0.78	0.72

	1	2	3	4	5
1	0.3414	0.2031	-0.0234	0.0546	0.4243
2	0.2031	0.5945	-0.1756	0.4098	-0.0318
3	-0.0234	-0.1756	0.9016	0.2295	0.0679
4	0.0546	0.4098	0.2295	0.4645	-0.1585
5	0.4243	-0.0318	0.0679	-0.1585	0.6980

Table 324: H or the Hat matrix

Problem 99. Solve for R-squared (R^2), coefficient of multiple determination and test:

$H_0 : \beta_1 = \beta_2 \cdots \beta_{p-1} = 0$ versus

$H_A : \beta_k \neq 0$ for at least one k where $1 \leq k \leq (p-1)$

	y	x1	x2
1	20.70	1.00	2.00
2	54.20	9.00	1.00
3	39.90	2.00	6.00
4	40.20	6.00	0.00
5	44.20	4.00	5.00

Solution: The regression table and other pertinent information

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
x1	1	409.06	409.06	116.46	0.0085
x2	1	175.61	175.61	50.00	0.0194
Residuals	2	7.02	3.51		

Table 325: Linear regression model output in ANOVA table.

	Estimate	Std. Error	t value	Pr(>|t|)
(Intercept)	11.1901	2.5934	4.31	0.0497
x1	4.5456	0.3524	12.90	0.0060
x2	3.0891	0.4369	7.07	0.0194

Table 326: Linear regression model output.

The coefficient of multiple determination equals

$R^2 = \frac{SSR}{SSTO} = 1 - \frac{SSE}{SSTO} = 0.98813,$

and the F-statistic equals

$F^* = \frac{MSR}{MSE} = 83.22907$

The determinant of $X^T X = 3791$

$$SSTO \;=\; Y^{'}Y - \frac{1}{n}Y^{'}JY = Y^{'}[I - \frac{1}{n}J]Y = 591.692$$

	x0	x1	x2
1	1.00	1.00	2.00
2	1.00	9.00	1.00
3	1.00	2.00	6.00
4	1.00	6.00	0.00
5	1.00	4.00	5.00

Table 327: X matrix

	x0	x1	x2
x0	5.00	22.00	14.00
x1	22.00	138.00	43.00
x2	14.00	43.00	66.00

Table 328: $X^{'}X$

$$
\begin{aligned}
SSR &= Y^{'}[H - \frac{1}{n}J]Y = 584.6672 \\
SSE &= e^{'}e = Y^{'}[I - H]Y = 7.0248
\end{aligned}
$$

where J is a $n \times n$ matrix of 1s and $H = X(X^{'}X)^{-1}X^{'}$ also size $n \times n$.

MSE=$\frac{SSE}{n-p} = 3.5124$ and MSR=$\frac{SSR}{p-1} = 292.3336$

□

	x0	x1	x2
x0	7259.00	-850.00	-986.00
x1	-850.00	134.00	93.00
x2	-986.00	93.00	206.00

Table 329: $det(X^{'}X)(X^{'}X)^{-1}$

	1	2	3	4	5
x0	4437.00	-1377.00	-357.00	2159.00	-1071.00
x1	-530.00	449.00	-24.00	-46.00	151.00
x2	-481.00	57.00	436.00	-428.00	416.00

Table 330: $det(X^{'}X)(X^{'}X)^{-1}X^{'}$

	x
intercept	11.1901
$\hat{\beta}_1$	4.5456
$\hat{\beta}_2$	3.0891

Table 331: The betas=$(X^{'}X)^{-1}X^{'}Y$

	x1	x2	y	yhat	resid
1	1.00	2.00	20.70	21.91	-1.21
2	9.00	1.00	54.20	55.19	-0.99
3	2.00	6.00	39.90	38.82	1.08
4	6.00	0.00	40.20	38.46	1.74
5	4.00	5.00	44.20	44.82	-0.62

	1	2	3	4	5
1	0.7768	-0.2147	0.1295	0.3316	-0.0232
2	-0.2147	0.7178	-0.0361	0.3474	0.1857
3	0.1295	-0.0361	0.5832	-0.1322	0.4556
4	0.3316	0.3474	-0.1322	0.4967	-0.0435
5	-0.0232	0.1857	0.4556	-0.0435	0.4255

Table 332: H or the Hat matrix

Problem 100. Solve for R-squared (R^2), coefficient of multiple determination and test:

$H_0 : \beta_1 = \beta_2 \cdots \beta_{p-1} = 0$ versus

$H_A : \beta_k \neq 0$ for at least one k where $1 \leq k \leq (p-1)$

	y	x1	x2
1	-0.20	7.00	7.00
2	18.10	9.00	3.00
3	5.40	1.00	2.00
4	-15.60	0.00	6.00
5	2.80	0.00	4.00

Solution: The regression table and other pertinent information

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
x1	1	235.16	235.16	8.06	0.1049
x2	1	292.46	292.46	10.03	0.0869
Residuals	2	58.35	29.17		

Table 333: Linear regression model output in ANOVA table.

	Estimate	Std. Error	t value	Pr(>|t|)
(Intercept)	13.6083	6.4302	2.12	0.1686
x1	1.9734	0.6339	3.11	0.0895
x2	-4.1404	1.3077	-3.17	0.0869

Table 334: Linear regression model output.

The coefficient of multiple determination equals

$R^2 = \frac{SSR}{SSTO} = 1 - \frac{SSE}{SSTO} = 0.90043,$

and the F-statistic equals

$F^* = \frac{MSR}{MSE} = 9.04301$

The determinant of $X^T X = 6244$

$$SSTO = Y^{'}Y - \frac{1}{n}Y^{'}JY = Y^{'}[I - \frac{1}{n}J]Y = 585.96$$

	x0	x1	x2
1	1.00	7.00	7.00
2	1.00	9.00	3.00
3	1.00	1.00	2.00
4	1.00	0.00	6.00
5	1.00	0.00	4.00

Table 335: X matrix

	x0	x1	x2
x0	5.00	17.00	22.00
x1	17.00	131.00	78.00
x2	22.00	78.00	114.00

Table 336: $X^{'}X$

$$\begin{aligned} SSR &= Y^{'}[H - \frac{1}{n}J]Y = 527.61494 \\ SSE &= e^{'}e = Y^{'}[I - H]Y = 58.34506 \end{aligned}$$

where J is a $n \times n$ matrix of 1s and $H = X(X^{'}X)^{-1}X^{'}$ also size $n \times n$.

MSE=$\frac{SSE}{n-p} = 29.17253$ and MSR=$\frac{SSR}{p-1} = 263.80747$

□

	x0	x1	x2
x0	8850.00	-222.00	-1556.00
x1	-222.00	86.00	-16.00
x2	-1556.00	-16.00	366.00

Table 337: $det(X^{'}X)(X^{'}X)^{-1}$

	1	2	3	4	5
x0	-3596.00	2184.00	5516.00	-486.00	2626.00
x1	268.00	504.00	-168.00	-318.00	-286.00
x2	894.00	-602.00	-840.00	640.00	-92.00

Table 338: $det(X^{'}X)(X^{'}X)^{-1}X^{'}$

	x
intercept	13.6083
$\hat{\beta}_1$	1.9734
$\hat{\beta}_2$	-4.1404

Table 339: The betas=$(X^{'}X)^{-1}X^{'}Y$

	x1	x2	y	yhat	resid
1	7.00	7.00	-0.20	-1.56	1.36
2	9.00	3.00	18.10	18.95	-0.85
3	1.00	2.00	5.40	7.30	-1.90
4	0.00	6.00	-15.60	-11.23	-4.37
5	0.00	4.00	2.80	-2.95	5.75

	1	2	3	4	5
1	0.7268	0.2399	-0.2466	0.2832	-0.0032
2	0.2399	0.7870	0.2377	-0.2287	-0.0359
3	-0.2466	0.2377	0.5874	0.0762	0.3453
4	0.2832	-0.2287	0.0762	0.5372	0.3322
5	-0.0032	-0.0359	0.3453	0.3322	0.3616

Table 340: H or the Hat matrix

3.3. Multiple Linear Regression - Test Individual Betas

Problem 101. Test the individual betas within the multiple regression model:

$H_0 : \beta_k = 0$ versus

$H_A : \beta_k \neq 0$ for $k = 1, 2$

	y	x1	x2
1	-26.10	8.00	5.00
2	-31.00	8.00	6.00
3	-33.10	2.00	4.00
4	-25.80	5.00	4.00
5	-2.20	8.00	0.00

Solution: The regression table and other pertinent information

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
x1	1	138.86	138.86	180.02	0.0055
x2	1	473.64	473.64	614.01	0.0016
Residuals	2	1.54	0.77		

Table 341: Linear regression model output in ANOVA table.

	Estimate	Std. Error	t value	Pr(>|t|)
(Intercept)	-17.2174	1.3556	-12.70	0.0061
x1	1.8968	0.1641	11.56	0.0074
x2	-4.7849	0.1931	-24.78	0.0016

Table 342: Linear regression model output.

The coefficient of multiple determination equals

$R^2 = \frac{SSR}{SSTO} = 1 - \frac{SSE}{SSTO} = 0.99749,$

and the F-statistic equals

$F^* = \frac{MSR}{MSE} = 397.01671$

The determinant of $X^T X = 2979$

$$
\begin{aligned}
SSTO &= Y^{'}Y - \frac{1}{n}Y^{'}JY = Y^{'}[I - \frac{1}{n}J]Y = 614.052 \\
SSR &= Y^{'}[H - \frac{1}{n}J]Y = 612.50922
\end{aligned}
$$

	x0	x1	x2
1	1.00	8.00	5.00
2	1.00	8.00	6.00
3	1.00	2.00	4.00
4	1.00	5.00	4.00
5	1.00	8.00	0.00

Table 343: X matrix

	x0	x1	x2
x0	5.00	31.00	19.00
x1	31.00	221.00	116.00
x2	19.00	116.00	93.00

Table 344: $X^{'}X$

$$SSE = e^{'}e = Y^{'}[I - H]Y = 1.54278$$

where J is a $n \times n$ matrix of 1s and $H = X(X^{'}X)^{-1}X^{'}$ also size $n \times n$.

MSE=$\frac{SSE}{n-p} = 0.77139$ and MSR=$\frac{SSR}{p-1} = 306.25461$

The standard errors for the betas are

$$s(\hat{\beta}_0) = \sqrt{MSE(X'X)^{-1}_{1,1}} = 1.355623$$
$$s(\hat{\beta}_1) = \sqrt{MSE(X'X)^{-1}_{2,2}} = 0.164104$$
$$s(\hat{\beta}_2) = \sqrt{MSE(X'X)^{-1}_{3,3}} = 0.1931$$

	x0	x1	x2
x0	7097.00	-679.00	-603.00
x1	-679.00	104.00	9.00
x2	-603.00	9.00	144.00

Table 345: $det(X^{'}X)(X^{'}X)^{-1}$

	1	2	3	4	5
x0	-1350.00	-1953.00	3327.00	1290.00	1665.00
x1	198.00	207.00	-435.00	-123.00	153.00
x2	189.00	333.00	-9.00	18.00	-531.00

Table 346: $det(X^{'}X)(X^{'}X)^{-1}X^{'}$

For testing an individual βk, $H_0 : \beta_k = 0$ versus $H_A : \beta_k \neq 0$ a t-test can be done where

$$t^* = \frac{\hat{\beta}_k - 0}{s(\hat{\beta}_k)} \tag{1}$$

and reject H_0, the null, if $|t^*|$ is greater than $t(1 - \frac{\alpha}{2}; n - p)$, otherwise fail to reject the null hypothesis.

The t-values - lower and upper for .95; n-p: -4.3027 and 4.3027 □

	x
intercept	-17.2174
$\hat{\beta}_1$	1.8968
$\hat{\beta}_2$	-4.7849

Table 347: The betas=$(X'X)^{-1}X'Y$

	x1	x2	y	yhat	resid
1	8.00	5.00	-26.10	-25.97	-0.13
2	8.00	6.00	-31.00	-30.75	-0.25
3	2.00	4.00	-33.10	-32.56	-0.54
4	5.00	4.00	-25.80	-26.87	1.07
5	8.00	0.00	-2.20	-2.04	-0.16

	1	2	3	4	5
1	1179.00	1368.00	-198.00	396.00	234.00
2	1368.00	1701.00	-207.00	414.00	-297.00
3	-198.00	-207.00	2421.00	1116.00	-153.00
4	396.00	414.00	1116.00	747.00	306.00
5	234.00	-297.00	-153.00	306.00	2889.00

Table 348: Det(X^TX) times the Hat matrix

	x0	x1	x2
x0	1.8377	-0.1758	-0.1561
x1	-0.1758	0.0269	0.0023
x2	-0.1561	0.0023	0.0373

Table 349: Variance Matrix

Problem 102. Test the individual betas within the multiple regression model:

$H_0 : \beta_k = 0$ versus

$H_A : \beta_k \neq 0$ for $k = 1, 2$

	y	x1	x2
1	-18.20	5.00	2.00
2	-14.30	2.00	2.00
3	-13.90	4.00	2.00
4	-12.80	1.00	4.00
5	-18.80	3.00	0.00

Solution: The regression table and other pertinent information

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
x1	1	10.82	10.82	2.40	0.2612
x2	1	9.60	9.60	2.13	0.2815
Residuals	2	9.00	4.50		

Table 350: Linear regression model output in ANOVA table.

	Estimate	Std. Error	t value	Pr(>|t|)
(Intercept)	-16.4000	3.4857	-4.70	0.0423
x1	-0.5500	0.7500	-0.73	0.5397
x2	1.2250	0.8385	1.46	0.2815

Table 351: Linear regression model output.

The coefficient of multiple determination equals

$R^2 = \frac{SSR}{SSTO} = 1 - \frac{SSE}{SSTO} = 0.69409,$

and the F-statistic equals

$F^* = \frac{MSR}{MSE} = 2.26889$

The determinant of $X^T X = 320$

$$\begin{aligned} SSTO &= Y'Y - \frac{1}{n}Y'JY = Y'[I - \frac{1}{n}J]Y = 29.42 \\ SSR &= Y'[H - \frac{1}{n}J]Y = 20.42 \end{aligned}$$

	x0	x1	x2
1	1.00	5.00	2.00
2	1.00	2.00	2.00
3	1.00	4.00	2.00
4	1.00	1.00	4.00
5	1.00	3.00	0.00

Table 352: X matrix

	x0	x1	x2
x0	5.00	15.00	10.00
x1	15.00	55.00	26.00
x2	10.00	26.00	28.00

Table 353: $X^{'}X$

$$SSE = e^{'}e = Y^{'}[I - H]Y = 9$$

where J is a $n \times n$ matrix of 1s and $H = X(X^{'}X)^{-1}X^{'}$ also size $n \times n$.

MSE=$\frac{SSE}{n-p} = 4.5$ and MSR=$\frac{SSR}{p-1} = 10.21$

The standard errors for the betas are

$$s(\hat{\beta}_0) = \sqrt{MSE(X'X)^{-1}_{1,1}} = 3.485685$$
$$s(\hat{\beta}_1) = \sqrt{MSE(X'X)^{-1}_{2,2}} = 0.75$$
$$s(\hat{\beta}_2) = \sqrt{MSE(X'X)^{-1}_{3,3}} = 0.838525$$

	x0	x1	x2
x0	864.00	-160.00	-160.00
x1	-160.00	40.00	20.00
x2	-160.00	20.00	50.00

Table 354: $det(X^{'}X)(X^{'}X)^{-1}$

	1	2	3	4	5
x0	-256.00	224.00	-96.00	64.00	384.00
x1	80.00	-40.00	40.00	-40.00	-40.00
x2	40.00	-20.00	20.00	60.00	-100.00

Table 355: $det(X^{'}X)(X^{'}X)^{-1}X^{'}$

For testing an individual βk, $H_0 : \beta_k = 0$ versus $H_A : \beta_k \neq 0$ a t-test can be done where

$$t^* = \frac{\hat{\beta}_k - 0}{s(\hat{\beta}_k)} \tag{2}$$

and reject H_0, the null, if $|t^*|$ is greater than $t(1 - \frac{\alpha}{2}; n - p)$, otherwise fail to reject the null hypothesis.

The t-values - lower and upper for .95; n-p: -4.3027 and 4.3027 □

	x
intercept	-16.4000
$\hat{\beta}_1$	-0.5500
$\hat{\beta}_2$	1.2250

Table 356: The betas=$(X^{'}X)^{-1}X^{'}Y$

	x1	x2	y	yhat	resid
1	5.00	2.00	-18.20	-16.70	-1.50
2	2.00	2.00	-14.30	-15.05	0.75
3	4.00	2.00	-13.90	-16.15	2.25
4	1.00	4.00	-12.80	-12.05	-0.75
5	3.00	0.00	-18.80	-18.05	-0.75

	1	2	3	4	5
1	224.00	-16.00	144.00	-16.00	-16.00
2	-16.00	104.00	24.00	104.00	104.00
3	144.00	24.00	104.00	24.00	24.00
4	-16.00	104.00	24.00	264.00	-56.00
5	-16.00	104.00	24.00	-56.00	264.00

Table 357: Det(X^TX) times the Hat matrix

	x0	x1	x2
x0	12.1500	-2.2500	-2.2500
x1	-2.2500	0.5625	0.2813
x2	-2.2500	0.2812	0.7031

Table 358: Variance Matrix

Problem 103. Test the individual betas within the multiple regression model:

$H_0 : \beta_k = 0$ versus

$H_A : \beta_k \neq 0$ for $k = 1, 2$

	y	x1	x2
1	-32.50	0.00	8.00
2	-7.20	6.00	2.00
3	-7.80	2.00	3.00
4	-33.90	9.00	7.00
5	-5.80	2.00	1.00

Solution: The regression table and other pertinent information

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
x1	1	36.90	36.90	2.64	0.2456
x2	1	766.17	766.17	54.84	0.0178
Residuals	2	27.94	13.97		

Table 359: Linear regression model output in ANOVA table.

	Estimate	Std. Error	t value	Pr(>|t|)
(Intercept)	3.4212	3.5097	0.97	0.4325
x1	-0.5660	0.5157	-1.10	0.3869
x2	-4.4549	0.6016	-7.41	0.0178

Table 360: Linear regression model output.

The coefficient of multiple determination equals

$R^2 = \frac{SSR}{SSTO} = 1 - \frac{SSE}{SSTO} = 0.96638,$

and the F-statistic equals

$F^* = \frac{MSR}{MSE} = 28.74052$

The determinant of $X^TX = 10192$

$$\begin{aligned} SSTO &= Y^{'}Y - \frac{1}{n}Y^{'}JY = Y^{'}[I - \frac{1}{n}J]Y = 831.012 \\ SSR &= Y^{'}[H - \frac{1}{n}J]Y = 803.06992 \end{aligned}$$

	x0	x1	x2
1	1.00	0.00	8.00
2	1.00	6.00	2.00
3	1.00	2.00	3.00
4	1.00	9.00	7.00
5	1.00	2.00	1.00

Table 361: X matrix

	x0	x1	x2
x0	5.00	19.00	21.00
x1	19.00	125.00	83.00
x2	21.00	83.00	127.00

Table 362: $X^{'}X$

$$SSE = e^{'}e = Y^{'}[I - H]Y = 27.94208$$

where J is a $n \times n$ matrix of 1s and $H = X(X^{'}X)^{-1}X^{'}$ also size $n \times n$.

MSE=$\frac{SSE}{n-p} = 13.97104$ and MSR=$\frac{SSR}{p-1} = 401.53496$

The standard errors for the betas are

$$s(\hat{\beta}_0) = \sqrt{MSE(X'X)^{-1}_{1,1}} = 3.509683$$
$$s(\hat{\beta}_1) = \sqrt{MSE(X'X)^{-1}_{2,2}} = 0.515686$$
$$s(\hat{\beta}_2) = \sqrt{MSE(X'X)^{-1}_{3,3}} = 0.601571$$

	x0	x1	x2
x0	8986.00	-670.00	-1048.00
x1	-670.00	194.00	-16.00
x2	-1048.00	-16.00	264.00

Table 363: $det(X^{'}X)(X^{'}X)^{-1}$

	1	2	3	4	5
x0	602.00	2870.00	4502.00	-4380.00	6598.00
x1	-798.00	462.00	-330.00	964.00	-298.00
x2	1064.00	-616.00	-288.00	656.00	-816.00

Table 364: $det(X^{'}X)(X^{'}X)^{-1}X^{'}$

For testing an individual βk, $H_0 : \beta_k = 0$ versus $H_A : \beta_k \neq 0$ a t-test can be done where

$$t^* = \frac{\hat{\beta}_k - 0}{s(\hat{\beta}_k)} \quad (3)$$

and reject H_0, the null, if $|t^*|$ is greater than $t(1 - \frac{\alpha}{2}; n - p)$, otherwise fail to reject the null hypothesis.

The t-values - lower and upper for .95; n-p: -4.3027 and 4.3027 □

	x
intercept	3.4212
$\hat{\beta}_1$	-0.5660
$\hat{\beta}_2$	-4.4549

Table 365: The betas=$(X^{'}X)^{-1}X^{'}Y$

	x1	x2	y	yhat	resid
1	0.00	8.00	-32.50	-32.22	-0.28
2	6.00	2.00	-7.20	-8.88	1.68
3	2.00	3.00	-7.80	-11.08	3.28
4	9.00	7.00	-33.90	-32.86	-1.04
5	2.00	1.00	-5.80	-2.17	-3.63

	1	2	3	4	5
1	9114.00	-2058.00	2198.00	868.00	70.00
2	-2058.00	4410.00	1946.00	2716.00	3178.00
3	2198.00	1946.00	2978.00	-484.00	3554.00
4	868.00	2716.00	-484.00	8888.00	-1796.00
5	70.00	3178.00	3554.00	-1796.00	5186.00

Table 366: Det(X^TX) times the Hat matrix

	x0	x1	x2
x0	12.3179	-0.9184	-1.4366
x1	-0.9184	0.2659	-0.0219
x2	-1.4366	-0.0219	0.3619

Table 367: Variance Matrix

Problem 104. Test the individual betas within the multiple regression model:

$H_0 : \beta_k = 0$ versus

$H_A : \beta_k \neq 0$ for $k = 1, 2$

	y	x1	x2
1	-58.70	4.00	9.00
2	-56.90	7.00	9.00
3	-37.20	0.00	4.00
4	-20.50	7.00	0.00
5	-43.40	2.00	5.00

Solution: The regression table and other pertinent information

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
x1	1	0.30	0.30	0.16	0.7281
x2	1	975.06	975.06	511.93	0.0019
Residuals	2	3.81	1.90		

Table 368: Linear regression model output in ANOVA table.

	Estimate	Std. Error	t value	Pr(>|t|)
(Intercept)	-21.8323	1.4549	-15.01	0.0044
x1	0.1981	0.2239	0.88	0.4696
x2	-4.1297	0.1825	-22.63	0.0019

Table 369: Linear regression model output.

The coefficient of multiple determination equals

$R^2 = \frac{SSR}{SSTO} = 1 - \frac{SSE}{SSTO} = 0.99611,$

and the F-statistic equals

$F^* = \frac{MSR}{MSE} = 256.04468$

The determinant of $X^T X = 10863$

$$\begin{aligned} SSTO &= Y^{'}Y - \frac{1}{n}Y^{'}JY = Y^{'}[I - \frac{1}{n}J]Y = 979.172 \\ SSR &= Y^{'}[H - \frac{1}{n}J]Y = 975.36265 \end{aligned}$$

	x0	x1	x2
1	1.00	4.00	9.00
2	1.00	7.00	9.00
3	1.00	0.00	4.00
4	1.00	7.00	0.00
5	1.00	2.00	5.00

Table 370: X matrix

	x0	x1	x2
x0	5.00	20.00	27.00
x1	20.00	118.00	109.00
x2	27.00	109.00	203.00

Table 371: $X^{'}X$

$$SSE \quad = \quad e^{'}e = Y^{'}[I - H]Y = 3.80935$$

where J is a $n \times n$ matrix of 1s and $H = X(X^{'}X)^{-1}X^{'}$ also size $n \times n$.

MSE=$\frac{SSE}{n-p} = 1.90467$ and MSR=$\frac{SSR}{p-1} = 487.68133$

The standard errors for the betas are

$$s(\hat{\beta}_0) \quad = \quad \sqrt{MSE(X'X)^{-1}_{1,1}} = 1.454933$$
$$s(\hat{\beta}_1) \quad = \quad \sqrt{MSE(X'X)^{-1}_{2,2}} = 0.223933$$
$$s(\hat{\beta}_2) \quad = \quad \sqrt{MSE(X'X)^{-1}_{3,3}} = 0.182521$$

	x0	x1	x2
x0	12073.00	-1117.00	-1006.00
x1	-1117.00	286.00	-5.00
x2	-1006.00	-5.00	190.00

Table 372: $det(X^{'}X)(X^{'}X)^{-1}$

	1	2	3	4	5
x0	-1449.00	-4800.00	8049.00	4254.00	4809.00
x1	-18.00	840.00	-1137.00	885.00	-570.00
x2	684.00	669.00	-246.00	-1041.00	-66.00

Table 373: $det(X^{'}X)(X^{'}X)^{-1}X^{'}$

For testing an individual βk, $H_0 : \beta_k = 0$ versus $H_A : \beta_k \neq 0$ a t-test can be done where

$$t^* = \frac{\hat{\beta}_k - 0}{s(\hat{\beta}_k)} \tag{4}$$

and reject H_0, the null, if $|t^*|$ is greater than $t(1 - \frac{\alpha}{2}; n - p)$, otherwise fail to reject the null hypothesis.

The t-values - lower and upper for .95; n-p: -4.3027 and 4.3027 □

	x
intercept	-21.8323
$\hat{\beta}_1$	0.1981
$\hat{\beta}_2$	-4.1297

Table 374: The betas=$(X^{'}X)^{-1}X^{'}Y$

	x1	x2	y	yhat	resid
1	4.00	9.00	-58.70	-58.21	-0.49
2	7.00	9.00	-56.90	-57.61	0.71
3	0.00	4.00	-37.20	-38.35	1.15
4	7.00	0.00	-20.50	-20.45	-0.05
5	2.00	5.00	-43.40	-42.08	-1.32

	1	2	3	4	5
1	4635.00	4581.00	1287.00	-1575.00	1935.00
2	4581.00	7101.00	-2124.00	1080.00	225.00
3	1287.00	-2124.00	7065.00	90.00	4545.00
4	-1575.00	1080.00	90.00	10449.00	819.00
5	1935.00	225.00	4545.00	819.00	3339.00

Table 375: Det(X^TX) times the Hat matrix

	x0	x1	x2
x0	2.1168	-0.1959	-0.1764
x1	-0.1959	0.0501	-0.0009
x2	-0.1764	-0.0009	0.0333

Table 376: Variance Matrix

Problem 105. Test the individual betas within the multiple regression model:

$H_0 : \beta_k = 0$ versus

$H_A : \beta_k \neq 0$ for $k = 1, 2$

	y	x1	x2
1	8.80	1.00	7.00
2	4.10	6.00	8.00
3	-2.60	7.00	8.00
4	8.40	3.00	1.00
5	13.60	0.00	7.00

Solution: The regression table and other pertinent information

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
x1	1	129.55	129.55	15.40	0.0592
x2	1	1.50	1.50	0.18	0.7141
Residuals	2	16.83	8.41		

Table 377: Linear regression model output in ANOVA table.

	Estimate	Std. Error	t value	Pr(>|t|)
(Intercept)	13.9617	3.4370	4.06	0.0556
x1	-1.8167	0.4898	-3.71	0.0656
x2	-0.2137	0.5064	-0.42	0.7141

Table 378: Linear regression model output.

The coefficient of multiple determination equals

$R^2 = \frac{SSR}{SSTO} = 1 - \frac{SSE}{SSTO} = 0.8862,$

and the F-statistic equals

$F^* = \frac{MSR}{MSE} = 7.78771$

The determinant of $X^T X = 6103$

$$\begin{aligned} SSTO &= Y'Y - \frac{1}{n}Y'JY = Y'[I - \frac{1}{n}J]Y = 147.872 \\ SSR &= Y'[H - \frac{1}{n}J]Y = 131.04486 \end{aligned}$$

	x0	x1	x2
1	1.00	1.00	7.00
2	1.00	6.00	8.00
3	1.00	7.00	8.00
4	1.00	3.00	1.00
5	1.00	0.00	7.00

Table 379: X matrix

	x0	x1	x2
x0	5.00	17.00	31.00
x1	17.00	95.00	114.00
x2	31.00	114.00	227.00

Table 380: $X^{'}X$

$$SSE = e^{'}e = Y^{'}[I - H]Y = 16.82714$$

where J is a $n \times n$ matrix of 1s and $H = X(X^{'}X)^{-1}X^{'}$ also size $n \times n$.

MSE=$\frac{SSE}{n-p} = 8.41357$ and MSR=$\frac{SSR}{p-1} = 65.52243$

The standard errors for the betas are

$$s(\hat{\beta}_0) = \sqrt{MSE(X'X)^{-1}_{1,1}} = 3.437031$$
$$s(\hat{\beta}_1) = \sqrt{MSE(X'X)^{-1}_{2,2}} = 0.489771$$
$$s(\hat{\beta}_2) = \sqrt{MSE(X'X)^{-1}_{3,3}} = 0.506378$$

	x0	x1	x2
x0	8569.00	-325.00	-1007.00
x1	-325.00	174.00	-43.00
x2	-1007.00	-43.00	186.00

Table 381: $det(X^{'}X)(X^{'}X)^{-1}$

	1	2	3	4	5
x0	1195.00	-1437.00	-1762.00	6587.00	1520.00
x1	-452.00	375.00	549.00	154.00	-626.00
x2	252.00	223.00	180.00	-950.00	295.00

Table 382: $det(X^{'}X)(X^{'}X)^{-1}X^{'}$

For testing an individual βk, $H_0 : \beta_k = 0$ versus $H_A : \beta_k \neq 0$ a t-test can be done where

$$t^* = \frac{\hat{\beta}_k - 0}{s(\hat{\beta}_k)} \tag{5}$$

and reject H_0, the null, if $|t^*|$ is greater than $t(1 - \frac{\alpha}{2}; n - p)$, otherwise fail to reject the null hypothesis.

The t-values - lower and upper for .95; n-p: -4.3027 and 4.3027 □

	x
intercept	13.9617
$\hat{\beta}_1$	-1.8167
$\hat{\beta}_2$	-0.2137

Table 383: The betas=$(X^{'}X)^{-1}X^{'}Y$

	x1	x2	y	yhat	resid
1	1.00	7.00	8.80	10.65	-1.85
2	6.00	8.00	4.10	1.35	2.75
3	7.00	8.00	-2.60	-0.46	-2.14
4	3.00	1.00	8.40	8.30	0.10
5	0.00	7.00	13.60	12.47	1.13

	1	2	3	4	5
1	2507.00	499.00	47.00	91.00	2959.00
2	499.00	2597.00	2972.00	-89.00	124.00
3	47.00	2972.00	3521.00	65.00	-502.00
4	91.00	-89.00	65.00	6099.00	-63.00
5	2959.00	124.00	-502.00	-63.00	3585.00

Table 384: Det(X^TX) times the Hat matrix

	x0	x1	x2
x0	11.8132	-0.4480	-1.3882
x1	-0.4480	0.2399	-0.0593
x2	-1.3882	-0.0593	0.2564

Table 385: Variance Matrix

Problem 106. Test the individual betas within the multiple regression model:

$H_0 : \beta_k = 0$ versus

$H_A : \beta_k \neq 0$ for $k = 1, 2$

	y	x1	x2
1	46.50	9.00	0.00
2	14.30	4.00	3.00
3	9.40	6.00	6.00
4	24.50	7.00	3.00
5	-3.60	5.00	8.00

Solution: The regression table and other pertinent information

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
x1	1	985.66	985.66	1208.05	0.0008
x2	1	421.17	421.17	516.20	0.0019
Residuals	2	1.63	0.82		

Table 386: Linear regression model output in ANOVA table.

	Estimate	Std. Error	t value	Pr(>|t|)
(Intercept)	11.8250	2.4632	4.80	0.0408
x1	3.8045	0.3031	12.55	0.0063
x2	-4.2982	0.1892	-22.72	0.0019

Table 387: Linear regression model output.

The coefficient of multiple determination equals

$R^2 = \frac{SSR}{SSTO} = 1 - \frac{SSE}{SSTO} = 0.99884,$

and the F-statistic equals

$F^* = \frac{MSR}{MSE} = 862.12717$
The determinant of $X^T X = 1687$

$$\begin{aligned} SSTO &= Y'Y - \frac{1}{n}Y'JY = Y'[I - \frac{1}{n}J]Y = 1408.468 \\ SSR &= Y'[H - \frac{1}{n}J]Y = 1406.83618 \end{aligned}$$

	x0	x1	x2
1	1.00	9.00	0.00
2	1.00	4.00	3.00
3	1.00	6.00	6.00
4	1.00	7.00	3.00
5	1.00	5.00	8.00

Table 388: X matrix

	x0	x1	x2
x0	5.00	31.00	20.00
x1	31.00	207.00	109.00
x2	20.00	109.00	118.00

Table 389: $X^{'}X$

$$SSE = e^{'}e = Y^{'}[I - H]Y = 1.63182$$

where J is a $n \times n$ matrix of 1s and $H = X(X^{'}X)^{-1}X^{'}$ also size $n \times n$.

MSE=$\frac{SSE}{n-p} = 0.81591$ and MSR=$\frac{SSR}{p-1} = 703.41809$

The standard errors for the betas are

$$\begin{aligned} s(\hat{\beta}_0) &= \sqrt{MSE(X'X)^{-1}_{1,1}} = 2.463196 \\ s(\hat{\beta}_1) &= \sqrt{MSE(X'X)^{-1}_{2,2}} = 0.303138 \\ s(\hat{\beta}_2) &= \sqrt{MSE(X'X)^{-1}_{3,3}} = 0.189182 \end{aligned}$$

	x0	x1	x2
x0	12545.00	-1478.00	-761.00
x1	-1478.00	190.00	75.00
x2	-761.00	75.00	74.00

Table 390: $det(X^{'}X)(X^{'}X)^{-1}$

	1	2	3	4	5
x0	-757.00	4350.00	-889.00	-84.00	-933.00
x1	232.00	-493.00	112.00	77.00	72.00
x2	-86.00	-239.00	133.00	-14.00	206.00

Table 391: $det(X^{'}X)(X^{'}X)^{-1}X^{'}$

For testing an individual βk, $H_0 : \beta_k = 0$ versus $H_A : \beta_k \neq 0$ a t-test can be done where

$$t^* = \frac{\hat{\beta}_k - 0}{s(\hat{\beta}_k)} \tag{6}$$

and reject H_0, the null, if $|t^*|$ is greater than $t(1 - \frac{\alpha}{2}; n - p)$, otherwise fail to reject the null hypothesis.

The t-values - lower and upper for .95; n-p: -4.3027 and 4.3027 □

	x
intercept	11.8250
$\hat{\beta}_1$	3.8045
$\hat{\beta}_2$	-4.2982

Table 392: The betas=$(X^{'}X)^{-1}X^{'}Y$

	x1	x2	y	yhat	resid
1	9.00	0.00	46.50	46.07	0.43
2	4.00	3.00	14.30	14.15	0.15
3	6.00	6.00	9.40	8.86	0.54
4	7.00	3.00	24.50	25.56	-1.06
5	5.00	8.00	-3.60	-3.54	-0.06

	1	2	3	4	5
1	1331.00	-87.00	119.00	609.00	-285.00
2	-87.00	1661.00	-42.00	182.00	-27.00
3	119.00	-42.00	581.00	294.00	735.00
4	609.00	182.00	294.00	413.00	189.00
5	-285.00	-27.00	735.00	189.00	1075.00

Table 393: Det(X^TX) times the Hat matrix

	x0	x1	x2
x0	6.0673	-0.7148	-0.3681
x1	-0.7148	0.0919	0.0363
x2	-0.3681	0.0363	0.0358

Table 394: Variance Matrix

Problem 107. Test the individual betas within the multiple regression model:

$H_0 : \beta_k = 0$ versus

$H_A : \beta_k \neq 0$ for $k = 1, 2$

	y	x1	x2
1	11.60	8.00	1.00
2	-15.20	9.00	7.00
3	-3.50	7.00	5.00
4	12.40	7.00	1.00
5	-21.90	2.00	8.00

Solution: The regression table and other pertinent information

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
x1	1	241.99	241.99	60.20	0.0162
x2	1	706.08	706.08	175.65	0.0056
Residuals	2	8.04	4.02		

Table 395: Linear regression model output in ANOVA table.

	Estimate	Std. Error	t value	Pr(>|t|)
(Intercept)	14.3036	3.7996	3.76	0.0639
x1	0.3584	0.4169	0.86	0.4806
x2	-4.5429	0.3428	-13.25	0.0056

Table 396: Linear regression model output.

The coefficient of multiple determination equals

$R^2 = \frac{SSR}{SSTO} = 1 - \frac{SSE}{SSTO} = 0.99159,$

and the F-statistic equals

$F^* = \frac{MSR}{MSE} = 117.92323$

The determinant of $X^T X = 4995$

$$\begin{aligned} SSTO &= Y^{'}Y - \frac{1}{n}Y^{'}JY = Y^{'}[I - \frac{1}{n}J]Y = 956.108 \\ SSR &= Y^{'}[H - \frac{1}{n}J]Y = 948.06829 \end{aligned}$$

	x0	x1	x2
1	1.00	8.00	1.00
2	1.00	9.00	7.00
3	1.00	7.00	5.00
4	1.00	7.00	1.00
5	1.00	2.00	8.00

Table 397: X matrix

	x0	x1	x2
x0	5.00	33.00	22.00
x1	33.00	247.00	129.00
x2	22.00	129.00	140.00

Table 398: $X^{'}X$

$$SSE = e^{'}e = Y^{'}[I-H]Y = 8.03971$$

where J is a $n \times n$ matrix of 1s and $H = X(X^{'}X)^{-1}X^{'}$ also size $n \times n$.

MSE=$\frac{SSE}{n-p} = 4.01985$ and MSR=$\frac{SSR}{p-1} = 474.03415$

The standard errors for the betas are

$$s(\hat{\beta}_0) = \sqrt{MSE(X'X)^{-1}_{1,1}} = 3.799588$$
$$s(\hat{\beta}_1) = \sqrt{MSE(X'X)^{-1}_{2,2}} = 0.416931$$
$$s(\hat{\beta}_2) = \sqrt{MSE(X'X)^{-1}_{3,3}} = 0.342779$$

	x0	x1	x2
x0	17939.00	-1782.00	-1177.00
x1	-1782.00	216.00	81.00
x2	-1177.00	81.00	146.00

Table 399: $det(X^{'}X)(X^{'}X)^{-1}$

	1	2	3	4	5
x0	2506.00	-6338.00	-420.00	4288.00	4959.00
x1	27.00	729.00	135.00	-189.00	-702.00
x2	-383.00	574.00	120.00	-464.00	153.00

Table 400: $det(X^{'}X)(X^{'}X)^{-1}X^{'}$

For testing an individual βk, $H_0 : \beta_k = 0$ versus $H_A : \beta_k \neq 0$ a t-test can be done where

$$t^* = \frac{\hat{\beta}_k - 0}{s(\hat{\beta}_k)} \tag{7}$$

and reject H_0, the null, if $|t^*|$ is greater than $t(1 - \frac{\alpha}{2}; n - p)$, otherwise fail to reject the null hypothesis.

The t-values - lower and upper for .95; n-p: -4.3027 and 4.3027 □

	x
intercept	14.3036
$\hat{\beta}_1$	0.3584
$\hat{\beta}_2$	-4.5429

Table 401: The betas=$(X^{'}X)^{-1}X^{'}Y$

	x1	x2	y	yhat	resid
1	8.00	1.00	11.60	12.63	-1.03
2	9.00	7.00	-15.20	-14.27	-0.93
3	7.00	5.00	-3.50	-5.90	2.40
4	7.00	1.00	12.40	12.27	0.13
5	2.00	8.00	-21.90	-21.32	-0.58

	1	2	3	4	5
1	2339.00	68.00	780.00	2312.00	-504.00
2	68.00	4241.00	1635.00	-661.00	-288.00
3	780.00	1635.00	1125.00	645.00	810.00
4	2312.00	-661.00	645.00	2501.00	198.00
5	-504.00	-288.00	810.00	198.00	4779.00

Table 402: Det(X^TX) times the Hat matrix

	x0	x1	x2
x0	14.4369	-1.4341	-0.9472
x1	-1.4341	0.1738	0.0652
x2	-0.9472	0.0652	0.1175

Table 403: Variance Matrix

Problem 108. Test the individual betas within the multiple regression model:

$H_0 : \beta_k = 0$ versus

$H_A : \beta_k \neq 0$ for $k = 1, 2$

	y	x1	x2
1	1.60	1.00	0.00
2	-25.30	6.00	6.00
3	3.40	3.00	0.00
4	-33.10	7.00	9.00
5	-23.00	4.00	7.00

Solution: The regression table and other pertinent information

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
x1	1	894.63	894.63	107.57	0.0092
x2	1	200.16	200.16	24.07	0.0391
Residuals	2	16.63	8.32		

Table 404: Linear regression model output in ANOVA table.

	Estimate	Std. Error	t value	Pr(>|t|)
(Intercept)	4.1945	3.1684	1.32	0.3166
x1	-1.0091	1.2297	-0.82	0.4981
x2	-3.4628	0.7059	-4.91	0.0391

Table 405: Linear regression model output.

The coefficient of multiple determination equals

$R^2 = \frac{SSR}{SSTO} = 1 - \frac{SSE}{SSTO} = 0.98503,$

and the F-statistic equals

$F^* = \frac{MSR}{MSE} = 65.8177$

The determinant of $X^T X = 1903$

$$\begin{aligned} SSTO &= Y^{'}Y - \frac{1}{n}Y^{'}JY = Y^{'}[I - \frac{1}{n}J]Y = 1111.428 \\ SSR &= Y^{'}[H - \frac{1}{n}J]Y = 1094.79426 \end{aligned}$$

	x0	x1	x2
1	1.00	1.00	0.00
2	1.00	6.00	6.00
3	1.00	3.00	0.00
4	1.00	7.00	9.00
5	1.00	4.00	7.00

Table 406: X matrix

	x0	x1	x2
x0	5.00	21.00	22.00
x1	21.00	111.00	127.00
x2	22.00	127.00	166.00

Table 407: $X^{'}X$

$$SSE = e^{'}e = Y^{'}[I - H]Y = 16.63374$$

where J is a $n \times n$ matrix of 1s and $H = X(X^{'}X)^{-1}X^{'}$ also size $n \times n$.

MSE=$\frac{SSE}{n-p} = 8.31687$ and MSR=$\frac{SSR}{p-1} = 547.39713$

The standard errors for the betas are

$$s(\hat{\beta}_0) = \sqrt{MSE(X'X)^{-1}_{1,1}} = 3.168407$$
$$s(\hat{\beta}_1) = \sqrt{MSE(X'X)^{-1}_{2,2}} = 1.229698$$
$$s(\hat{\beta}_2) = \sqrt{MSE(X'X)^{-1}_{3,3}} = 0.705851$$

	x0	x1	x2
x0	2297.00	-692.00	225.00
x1	-692.00	346.00	-173.00
x2	225.00	-173.00	114.00

Table 408: $det(X^{'}X)(X^{'}X)^{-1}$

	1	2	3	4	5
x0	1605.00	-505.00	221.00	-522.00	1104.00
x1	-346.00	346.00	346.00	173.00	-519.00
x2	52.00	-129.00	-294.00	40.00	331.00

Table 409: $det(X^{'}X)(X^{'}X)^{-1}X^{'}$

For testing an individual βk, $H_0 : \beta_k = 0$ versus $H_A : \beta_k \neq 0$ a t-test can be done where

$$t^* = \frac{\hat{\beta}_k - 0}{s(\hat{\beta}_k)} \tag{8}$$

and reject H_0, the null, if $|t^*|$ is greater than $t(1 - \frac{\alpha}{2}; n - p)$, otherwise fail to reject the null hypothesis.

The t-values - lower and upper for .95; n-p: -4.3027 and 4.3027 □

	x
intercept	4.1945
$\hat{\beta}_1$	-1.0091
$\hat{\beta}_2$	-3.4628

Table 410: The betas=$(X^{'}X)^{-1}X^{'}Y$

	x1	x2	y	yhat	resid
1	1.00	0.00	1.60	3.19	-1.59
2	6.00	6.00	-25.30	-22.64	-2.66
3	3.00	0.00	3.40	1.17	2.23
4	7.00	9.00	-33.10	-34.03	0.93
5	4.00	7.00	-23.00	-24.08	1.08

	1	2	3	4	5
1	1259.00	-159.00	567.00	-349.00	585.00
2	-159.00	797.00	533.00	756.00	-24.00
3	567.00	533.00	1259.00	-3.00	-453.00
4	-349.00	756.00	-3.00	1049.00	450.00
5	585.00	-24.00	-453.00	450.00	1345.00

Table 411: Det(X^TX) times the Hat matrix

	x0	x1	x2
x0	10.0388	-3.0243	0.9833
x1	-3.0243	1.5122	-0.7561
x2	0.9833	-0.7561	0.4982

Table 412: Variance Matrix

Problem 109. Test the individual betas within the multiple regression model:

$H_0 : \beta_k = 0$ versus

$H_A : \beta_k \neq 0$ for $k = 1, 2$

	y	x1	x2
1	7.10	1.00	0.00
2	-21.80	5.00	9.00
3	-24.40	6.00	9.00
4	-16.10	6.00	5.00
5	-13.40	6.00	1.00

Solution: The regression table and other pertinent information

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
x1	1	490.42	490.42	137.42	0.0072
x2	1	121.03	121.03	33.92	0.0282
Residuals	2	7.14	3.57		

Table 413: Linear regression model output in ANOVA table.

	Estimate	Std. Error	t value	Pr(>|t|)
(Intercept)	10.4060	2.2565	4.61	0.0439
x1	-3.4998	0.5158	-6.79	0.0210
x2	-1.5264	0.2621	-5.82	0.0282

Table 414: Linear regression model output.

The coefficient of multiple determination equals

$R^2 = \frac{SSR}{SSTO} = 1 - \frac{SSE}{SSTO} = 0.98846,$

and the F-statistic equals

$F^* = \frac{MSR}{MSE} = 85.66899$

The determinant of $X^TX = 4883$

$$\begin{aligned} SSTO &= Y^{'}Y - \frac{1}{n}Y^{'}JY = Y^{'}[I - \frac{1}{n}J]Y = 618.588 \\ SSR &= Y^{'}[H - \frac{1}{n}J]Y = 611.45064 \end{aligned}$$

	x0	x1	x2
1	1.00	1.00	0.00
2	1.00	5.00	9.00
3	1.00	6.00	9.00
4	1.00	6.00	5.00
5	1.00	6.00	1.00

Table 415: X matrix

	x0	x1	x2
x0	5.00	24.00	24.00
x1	24.00	134.00	135.00
x2	24.00	135.00	188.00

Table 416: $X^{'}X$

$$SSE = e^{'}e = Y^{'}[I-H]Y = 7.13736$$

where J is a $n \times n$ matrix of 1s and $H = X(X^{'}X)^{-1}X^{'}$ also size $n \times n$.

MSE=$\frac{SSE}{n-p} = 3.56868$ and MSR=$\frac{SSR}{p-1} = 305.72532$

The standard errors for the betas are

$$s(\hat{\beta}_0) = \sqrt{MSE(X'X)^{-1}_{1,1}} = 2.25649$$
$$s(\hat{\beta}_1) = \sqrt{MSE(X'X)^{-1}_{2,2}} = 0.515776$$
$$s(\hat{\beta}_2) = \sqrt{MSE(X'X)^{-1}_{3,3}} = 0.262104$$

	x0	x1	x2
x0	6967.00	-1272.00	24.00
x1	-1272.00	364.00	-99.00
x2	24.00	-99.00	94.00

Table 417: $det(X^{'}X)(X^{'}X)^{-1}$

	1	2	3	4	5
x0	5695.00	823.00	-449.00	-545.00	-641.00
x1	-908.00	-343.00	21.00	417.00	813.00
x2	-75.00	375.00	276.00	-100.00	-476.00

Table 418: $det(X^{'}X)(X^{'}X)^{-1}X^{'}$

For testing an individual βk, $H_0 : \beta_k = 0$ versus $H_A : \beta_k \neq 0$ a t-test can be done where

$$t^* = \frac{\hat{\beta}_k - 0}{s(\hat{\beta}_k)} \tag{9}$$

and reject H_0, the null, if $|t^*|$ is greater than $t(1 - \frac{\alpha}{2}; n - p)$, otherwise fail to reject the null hypothesis.

The t-values - lower and upper for .95; n-p: -4.3027 and 4.3027 □

	x
intercept	10.4060
$\hat{\beta}_1$	-3.4998
$\hat{\beta}_2$	-1.5264

Table 419: The betas=$(X^{'}X)^{-1}X^{'}Y$

	x1	x2	y	yhat	resid
1	1.00	0.00	7.10	6.91	0.19
2	5.00	9.00	-21.80	-20.83	-0.97
3	6.00	9.00	-24.40	-24.33	-0.07
4	6.00	5.00	-16.10	-18.23	2.13
5	6.00	1.00	-13.40	-12.12	-1.28

	1	2	3	4	5
1	4787.00	480.00	-428.00	-128.00	172.00
2	480.00	2483.00	2140.00	640.00	-860.00
3	-428.00	2140.00	2161.00	1057.00	-47.00
4	-128.00	640.00	1057.00	1457.00	1857.00
5	172.00	-860.00	-47.00	1857.00	3761.00

Table 420: Det(X^TX) times the Hat matrix

	x0	x1	x2
x0	5.0917	-0.9296	0.0175
x1	-0.9296	0.2660	-0.0724
x2	0.0175	-0.0724	0.0687

Table 421: Variance Matrix

Problem 110. Test the individual betas within the multiple regression model:

$H_0 : \beta_k = 0$ versus

$H_A : \beta_k \neq 0$ for $k = 1, 2$

	y	x1	x2
1	7.60	6.00	7.00
2	32.40	6.00	0.00
3	3.20	0.00	0.00
4	27.00	9.00	7.00
5	7.10	2.00	9.00

Solution: The regression table and other pertinent information

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
x1	1	391.72	391.72	4.59	0.1654
x2	1	139.71	139.71	1.64	0.3291
Residuals	2	170.68	85.34		

Table 422: Linear regression model output in ANOVA table.

	Estimate	Std. Error	t value	Pr(>|t|)
(Intercept)	7.1336	8.0092	0.89	0.4671
x1	3.2496	1.3452	2.42	0.1370
x2	-1.4395	1.1250	-1.28	0.3291

Table 423: Linear regression model output.

The coefficient of multiple determination equals

$R^2 = \frac{SSR}{SSTO} = 1 - \frac{SSE}{SSTO} = 0.75691,$

and the F-statistic equals

$F^* = \frac{MSR}{MSE} = 3.11373$

The determinant of $X^T X = 17260$

$$\begin{aligned} SSTO &= Y^{'}Y - \frac{1}{n}Y^{'}JY = Y^{'}[I - \frac{1}{n}J]Y = 702.112 \\ SSR &= Y^{'}[H - \frac{1}{n}J]Y = 531.4369 \end{aligned}$$

	x0	x1	x2
1	1.00	6.00	7.00
2	1.00	6.00	0.00
3	1.00	0.00	0.00
4	1.00	9.00	7.00
5	1.00	2.00	9.00

Table 424: X matrix

	x0	x1	x2
x0	5.00	23.00	23.00
x1	23.00	157.00	123.00
x2	23.00	123.00	179.00

Table 425: $X^{'}X$

$$SSE = e^{'}e = Y^{'}[I - H]Y = 170.6751$$

where J is a $n \times n$ matrix of 1s and $H = X(X^{'}X)^{-1}X^{'}$ also size $n \times n$.

MSE=$\frac{SSE}{n-p} = 85.33755$ and MSR=$\frac{SSR}{p-1} = 265.71845$

The standard errors for the betas are

$$s(\hat{\beta}_0) = \sqrt{MSE(X'X)^{-1}_{1,1}} = 8.009154$$

$$s(\hat{\beta}_1) = \sqrt{MSE(X'X)^{-1}_{2,2}} = 1.34521$$

$$s(\hat{\beta}_2) = \sqrt{MSE(X'X)^{-1}_{3,3}} = 1.125044$$

	x0	x1	x2
x0	12974.00	-1288.00	-782.00
x1	-1288.00	366.00	-86.00
x2	-782.00	-86.00	256.00

Table 426: $det(X^{'}X)(X^{'}X)^{-1}$

	1	2	3	4	5
x0	-228.00	5246.00	12974.00	-4092.00	3360.00
x1	306.00	908.00	-1288.00	1404.00	-1330.00
x2	494.00	-1298.00	-782.00	236.00	1350.00

Table 427: $det(X^{'}X)(X^{'}X)^{-1}X^{'}$

For testing an individual βk, $H_0 : \beta_k = 0$ versus $H_A : \beta_k \neq 0$ a t-test can be done where

$$t^* = \frac{\hat{\beta}_k - 0}{s(\hat{\beta}_k)} \tag{10}$$

and reject H_0, the null, if $|t^*|$ is greater than $t(1 - \frac{\alpha}{2}; n - p)$, otherwise fail to reject the null hypothesis.

The t-values - lower and upper for .95; n-p: -4.3027 and 4.3027 □

	x
intercept	7.1336
$\hat{\beta}_1$	3.2496
$\hat{\beta}_2$	-1.4395

Table 428: The betas=$(X^{'}X)^{-1}X^{'}Y$

	x1	x2	y	yhat	resid
1	6.00	7.00	7.60	16.55	-8.95
2	6.00	0.00	32.40	26.63	5.77
3	0.00	0.00	3.20	7.13	-3.93
4	9.00	7.00	27.00	26.30	0.70
5	2.00	9.00	7.10	0.68	6.42

	1	2	3	4	5
1	5066.00	1608.00	-228.00	5984.00	4830.00
2	1608.00	10694.00	5246.00	4332.00	-4620.00
3	-228.00	5246.00	12974.00	-4092.00	3360.00
4	5984.00	4332.00	-4092.00	10196.00	840.00
5	4830.00	-4620.00	3360.00	840.00	12850.00

Table 429: Det(X^TX) times the Hat matrix

	x0	x1	x2
x0	64.1465	-6.3682	-3.8664
x1	-6.3682	1.8096	-0.4252
x2	-3.8664	-0.4252	1.2657

Table 430: Variance Matrix

3.4. Multiple Linear Regression - Test Betas etc but given info

Problem 111. Solve for R-squared (R^2), coefficient of multiple determination and test:

$H_0 : \beta_1 = \beta_2 \cdots \beta_{p-1} = 0$ versus

$H_A : \beta_k \neq 0$ for at least one k where $1 \leq k \leq (p-1)$

Also, test the individual betas:

$H_0 : \beta_k = 0$ versus

$H_A : \beta_k \neq 0$ for at each k where $1 \leq k \leq 2$

The determinant of $X^T X = 13494$

	y	x1	x2
1	-15.80	0.00	3.00
2	-34.40	8.00	2.00
3	-23.90	6.00	9.00
4	-35.70	8.00	7.00
5	-37.20	8.00	2.00
6	-29.50	5.00	2.00

	x0	x1	x2
x0	6.00	35.00	25.00
x1	35.00	253.00	152.00
x2	25.00	152.00	151.00

Table 431: $X^{'}X$

	x0	x1	x2
x0	15099.00	-1485.00	-1005.00
x1	-1485.00	281.00	-37.00
x2	-1005.00	-37.00	293.00

Table 432: $det(X^{'}X)(X^{'}X)^{-1}$

	1	2	3	4	5	6
x0	12084.00	1209.00	-2856.00	-3816.00	1209.00	5664.00
x1	-1596.00	689.00	-132.00	504.00	689.00	-154.00
x2	-126.00	-715.00	1410.00	750.00	-715.00	-604.00

Table 433: $det(X^{'}X)(X^{'}X)^{-1}X^{'}$

	1	2	3	4	5	6
1	11706.00	-936.00	1374.00	-1566.00	-936.00	3852.00
2	-936.00	5291.00	-1092.00	1716.00	5291.00	3224.00
3	1374.00	-1092.00	9042.00	5958.00	-1092.00	-696.00
4	-1566.00	1716.00	5958.00	5466.00	1716.00	204.00
5	-936.00	5291.00	-1092.00	1716.00	5291.00	3224.00
6	3852.00	3224.00	-696.00	204.00	3224.00	3686.00

Table 434: Det($X^T X$) times the Hat matrix

Solution: The regression table and other pertinent information

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
x1	1	293.49	293.49	45.81	0.0066
x2	1	28.04	28.04	4.38	0.1275
Residuals	3	19.22	6.41		

Table 435: Linear regression model output in ANOVA table.

	Estimate	Std. Error	t value	Pr(>|t|)
(Intercept)	-17.7924	2.6775	-6.65	0.0069
x1	-2.5501	0.3653	-6.98	0.0060
x2	0.7803	0.3730	2.09	0.1275

Table 436: Linear regression model output.

The coefficient of multiple determination equals

$$R^2 = \frac{SSR}{SSTO} = 1 - \frac{SSE}{SSTO} = 0.94359,$$

and the F-statistic equals

$$F^* = \frac{MSR}{MSE} = 25.0926$$

	x
intercept	-17.7924
$\hat{\beta}_1$	-2.5501
$\hat{\beta}_2$	0.7803

Table 437: The betas=$(X^{'}X)^{-1}X^{'}Y$

	x1	x2	y	yhat	resid
1	0.00	3.00	-15.80	-15.45	-0.35
2	8.00	2.00	-34.40	-36.63	2.23
3	6.00	9.00	-23.90	-26.07	2.17
4	8.00	7.00	-35.70	-32.73	-2.97
5	8.00	2.00	-37.20	-36.63	-0.57
6	5.00	2.00	-29.50	-28.98	-0.52

$$\begin{aligned} SSTO &= Y^{'}Y - \frac{1}{n}Y^{'}JY = Y^{'}[I - \frac{1}{n}J]Y = 340.74833 \\ SSR &= Y^{'}[H - \frac{1}{n}J]Y = 321.52785 \\ SSE &= e^{'}e = Y^{'}[I - H]Y = 19.22048 \end{aligned}$$

where J is a $n \times n$ matrix of 1s and $H = X(X^{'}X)^{-1}X^{'}$ also size $n \times n$.

MSE=$\frac{SSE}{n-p} = 6.40683$ and MSR=$\frac{SSR}{p-1} = 160.76393$

The standard errors for the betas are

	x0	x1	x2
x0	7.1689	-0.7051	-0.4772
x1	-0.7051	0.1334	-0.0176
x2	-0.4772	-0.0176	0.1391

Table 438: Variance Matrix

$$
\begin{aligned}
s(\hat{\beta}_0) &= \sqrt{MSE(X'X)^{-1}_{1,1}} = 2.677474 \\
s(\hat{\beta}_1) &= \sqrt{MSE(X'X)^{-1}_{2,2}} = 0.365262 \\
s(\hat{\beta}_2) &= \sqrt{MSE(X'X)^{-1}_{3,3}} = 0.372979
\end{aligned}
$$

For testing an individual βk, $H_0 : \beta_k = 0$ versus $H_A : \beta_k \neq 0$ a t-test can be done where

$$t^* = \frac{\hat{\beta}_k - 0}{s(\hat{\beta}_k)} \tag{11}$$

and reject H_0, the null, if $|t^*|$ is greater than $t(1 - \frac{\alpha}{2}; n - p)$, otherwise fail to reject the null hypothesis.

The t-values - lower and upper for .95; n-p: -3.1824 and 3.1824 □

Problem 112. Solve for R-squared (R^2), coefficient of multiple determination and test:

$H_0 : \beta_1 = \beta_2 \cdots \beta_{p-1} = 0$ versus

$H_A : \beta_k \neq 0$ for at least one k where $1 \leq k \leq (p-1)$

Also, test the individual betas:

$H_0 : \beta_k = 0$ versus

$H_A : \beta_k \neq 0$ for at each k where $1 \leq k \leq 2$

The determinant of $X^T X = 1098$

	y	x1	x2
1	-22.10	5.00	3.00
2	-7.60	1.00	3.00
3	2.30	1.00	5.00
4	6.20	2.00	6.00
5	-14.50	3.00	2.00
6	-16.70	2.00	1.00

	x0	x1	x2
x0	6.00	14.00	20.00
x1	14.00	44.00	43.00
x2	20.00	43.00	84.00

Table 439: $X^{'}X$

	x0	x1	x2
x0	1847.00	-316.00	-278.00
x1	-316.00	104.00	22.00
x2	-278.00	22.00	68.00

Table 440: $det(X^{'}X)(X^{'}X)^{-1}$

	1	2	3	4	5	6
x0	-567.00	697.00	141.00	-453.00	343.00	937.00
x1	270.00	-146.00	-102.00	24.00	40.00	-86.00
x2	36.00	-52.00	84.00	174.00	-76.00	-166.00

Table 441: $det(X^{'}X)(X^{'}X)^{-1}X^{'}$

	1	2	3	4	5	6
1	891.00	-189.00	-117.00	189.00	315.00	9.00
2	-189.00	395.00	291.00	93.00	155.00	353.00
3	-117.00	291.00	459.00	441.00	3.00	21.00
4	189.00	93.00	441.00	639.00	-33.00	-231.00
5	315.00	155.00	3.00	-33.00	311.00	347.00
6	9.00	353.00	21.00	-231.00	347.00	599.00

Table 442: Det(X^TX) times the Hat matrix

Solution: The regression table and other pertinent information

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
x1	1	297.16	297.16	39.45	0.0081
x2	1	301.65	301.65	40.05	0.0080
Residuals	3	22.60	7.53		

Table 443: Linear regression model output in ANOVA table.

	Estimate	Std. Error	t value	Pr(>|t|)
(Intercept)	-14.4556	3.5595	-4.06	0.0269
x1	-3.7222	0.8446	-4.41	0.0217
x2	4.3222	0.6830	6.33	0.0080

Table 444: Linear regression model output.

The coefficient of multiple determination equals

$R^2 = \frac{SSR}{SSTO} = 1 - \frac{SSE}{SSTO} = 0.96364,$

and the F-statistic equals

$F^* = \frac{MSR}{MSE} = 39.75236$

	x
intercept	-14.4556
$\hat{\beta}_1$	-3.7222
$\hat{\beta}_2$	4.3222

Table 445: The betas=$(X^{'}X)^{-1}X^{'}Y$

	x1	x2	y	yhat	resid
1	5.00	3.00	-22.10	-20.10	-2.00
2	1.00	3.00	-7.60	-5.21	-2.39
3	1.00	5.00	2.30	3.43	-1.13
4	2.00	6.00	6.20	4.03	2.17
5	3.00	2.00	-14.50	-16.98	2.48
6	2.00	1.00	-16.70	-17.58	0.88

$$\begin{aligned} SSTO &= Y^{'}Y - \frac{1}{n}Y^{'}JY = Y^{'}[I - \frac{1}{n}J]Y = 621.41333 \\ SSR &= Y^{'}[H - \frac{1}{n}J]Y = 598.81778 \\ SSE &= e^{'}e = Y^{'}[I - H]Y = 22.59556 \end{aligned}$$

where J is a $n \times n$ matrix of 1s and $H = X(X^{'}X)^{-1}X^{'}$ also size $n \times n$.

MSE=$\frac{SSE}{n-p} = 7.53185$ and MSR=$\frac{SSR}{p-1} = 299.40889$

The standard errors for the betas are

	x0	x1	x2
x0	12.6697	-2.1676	-1.9070
x1	-2.1676	0.7134	0.1509
x2	-1.9070	0.1509	0.4665

Table 446: Variance Matrix

$$\begin{aligned} s(\hat{\beta}_0) &= \sqrt{MSE(X'X)^{-1}_{1,1}} = 3.559452 \\ s(\hat{\beta}_1) &= \sqrt{MSE(X'X)^{-1}_{2,2}} = 0.84463 \\ s(\hat{\beta}_2) &= \sqrt{MSE(X'X)^{-1}_{3,3}} = 0.682974 \end{aligned}$$

For testing an individual βk, $H_0 : \beta_k = 0$ versus $H_A : \beta_k \neq 0$ a t-test can be done where

$$t^* = \frac{\hat{\beta}_k - 0}{s(\hat{\beta}_k)} \tag{12}$$

and reject H_0, the null, if $|t^*|$ is greater than $t(1 - \frac{\alpha}{2}; n - p)$, otherwise fail to reject the null hypothesis.

The t-values - lower and upper for .95; n-p: -3.1824 and 3.1824 □

Problem 113. Solve for R-squared (R^2), coefficient of multiple determination and test:

$H_0 : \beta_1 = \beta_2 \cdots \beta_{p-1} = 0$ versus

$H_A : \beta_k \neq 0$ for at least one k where $1 \leq k \leq (p-1)$

Also, test the individual betas:

$H_0 : \beta_k = 0$ versus

$H_A : \beta_k \neq 0$ for at each k where $1 \leq k \leq 2$

The determinant of $X^T X = 11808$

	y	x1	x2
1	-8.60	9.00	5.00
2	-11.00	3.00	7.00
3	11.80	2.00	1.00
4	-12.60	9.00	5.00
5	-26.70	6.00	8.00
6	14.40	1.00	1.00

	x0	x1	x2
x0	6.00	30.00	27.00
x1	30.00	212.00	162.00
x2	27.00	162.00	165.00

Table 447: $X^{'}X$

	x0	x1	x2
x0	8736.00	-576.00	-864.00
x1	-576.00	261.00	-162.00
x2	-864.00	-162.00	372.00

Table 448: $det(X^{'}X)(X^{'}X)^{-1}$

	1	2	3	4	5	6
x0	-768.00	960.00	6720.00	-768.00	-1632.00	7296.00
x1	963.00	-927.00	-216.00	963.00	-306.00	-477.00
x2	-462.00	1254.00	-816.00	-462.00	1140.00	-654.00

Table 449: $det(X^{'}X)(X^{'}X)^{-1}X^{'}$

	1	2	3	4	5	6
1	5589.00	-1113.00	696.00	5589.00	1314.00	-267.00
2	-1113.00	6957.00	360.00	-1113.00	5430.00	1287.00
3	696.00	360.00	5472.00	696.00	-1104.00	5688.00
4	5589.00	-1113.00	696.00	5589.00	1314.00	-267.00
5	1314.00	5430.00	-1104.00	1314.00	5652.00	-798.00
6	-267.00	1287.00	5688.00	-267.00	-798.00	6165.00

Table 450: Det(X^TX) times the Hat matrix

Solution: The regression table and other pertinent information

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
x1	1	537.20	537.20	34.60	0.0098
x2	1	651.22	651.22	41.95	0.0075
Residuals	3	46.58	15.53		

Table 451: Linear regression model output in ANOVA table.

	Estimate	Std. Error	t value	Pr(>|t|)
(Intercept)	19.7878	3.3891	5.84	0.0100
x1	-0.9710	0.5858	-1.66	0.1960
x2	-4.5295	0.6994	-6.48	0.0075

Table 452: Linear regression model output.

The coefficient of multiple determination equals

$$R^2 = \frac{SSR}{SSTO} = 1 - \frac{SSE}{SSTO} = 0.96229,$$

and the F-statistic equals

$$F^* = \frac{MSR}{MSE} = 38.27329$$

	x
intercept	19.7878
$\hat{\beta}_1$	-0.9710
$\hat{\beta}_2$	-4.5295

Table 453: The betas=$(X^{'}X)^{-1}X^{'}Y$

	x1	x2	y	yhat	resid
1	9.00	5.00	-8.60	-11.60	3.00
2	3.00	7.00	-11.00	-14.83	3.83
3	2.00	1.00	11.80	13.32	-1.52
4	9.00	5.00	-12.60	-11.60	-1.00
5	6.00	8.00	-26.70	-22.27	-4.43
6	1.00	1.00	14.40	14.29	0.11

$$
\begin{aligned}
SSTO &= Y^{'}Y - \frac{1}{n}Y^{'}JY = Y^{'}[I - \frac{1}{n}J]Y = 1234.995 \\
SSR &= Y^{'}[H - \frac{1}{n}J]Y = 1188.4187 \\
SSE &= e^{'}e = Y^{'}[I - H]Y = 46.5763
\end{aligned}
$$

where J is a $n \times n$ matrix of 1s and $H = X(X^{'}X)^{-1}X^{'}$ also size $n \times n$.

MSE=$\frac{SSE}{n-p} = 15.52543$ and MSR=$\frac{SSR}{p-1} = 594.20935$

The standard errors for the betas are

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	x0	x1	x2
x0	11.4863	-0.7573	-1.1360
x1	-0.7573	0.3432	-0.2130
x2	-1.1360	-0.2130	0.4891

Table 454: Variance Matrix

$$s(\hat{\beta}_0) = \sqrt{MSE(X'X)^{-1}_{1,1}} = 3.389144$$
$$s(\hat{\beta}_1) = \sqrt{MSE(X'X)^{-1}_{2,2}} = 0.585806$$
$$s(\hat{\beta}_2) = \sqrt{MSE(X'X)^{-1}_{3,3}} = 0.699367$$

For testing an individual βk, $H_0 : \beta_k = 0$ versus $H_A : \beta_k \neq 0$ a t-test can be done where

$$t^* = \frac{\hat{\beta}_k - 0}{s(\hat{\beta}_k)} \quad (13)$$

and reject H_0, the null, if $|t^*|$ is greater than $t(1 - \frac{\alpha}{2}; n - p)$, otherwise fail to reject the null hypothesis.

The t-values - lower and upper for .95; n-p: -3.1824 and 3.1824 □

Problem 114. Solve for R-squared (R^2), coefficient of multiple determination and test:

$H_0 : \beta_1 = \beta_2 \cdots \beta_{p-1} = 0$ versus

$H_A : \beta_k \neq 0$ for at least one k where $1 \leq k \leq (p-1)$

Also, test the individual betas:

$H_0 : \beta_k = 0$ versus

$H_A : \beta_k \neq 0$ for at each k where $1 \leq k \leq 2$

The determinant of $X^T X = 11764$

	y	x1	x2
1	16.00	4.00	0.00
2	-9.80	9.00	7.00
3	18.00	8.00	1.00
4	-5.60	3.00	4.00
5	-19.50	0.00	5.00
6	5.20	7.00	3.00

	x0	x1	x2
x0	6.00	31.00	20.00
x1	31.00	219.00	104.00
x2	20.00	104.00	100.00

Table 455: $X^{'}X$

	x0	x1	x2
x0	11084.00	-1020.00	-1156.00
x1	-1020.00	200.00	-4.00
x2	-1156.00	-4.00	353.00

Table 456: $det(X^{'}X)(X^{'}X)^{-1}$

	1	2	3	4	5	6
x0	7004.00	-6188.00	1768.00	3400.00	5304.00	476.00
x1	-220.00	752.00	576.00	-436.00	-1040.00	368.00
x2	-1172.00	1279.00	-835.00	244.00	609.00	-125.00

Table 457: $det(X^{'}X)(X^{'}X)^{-1}X^{'}$

	1	2	3	4	5	6
1	6124.00	-3180.00	4072.00	1656.00	1144.00	1948.00
2	-3180.00	9533.00	1107.00	1184.00	207.00	2913.00
3	4072.00	1107.00	5541.00	156.00	-2407.00	3295.00
4	1656.00	1184.00	156.00	3068.00	4620.00	1080.00
5	1144.00	207.00	-2407.00	4620.00	8349.00	-149.00
6	1948.00	2913.00	3295.00	1080.00	-149.00	2677.00

Table 458: Det(X^TX) times the Hat matrix

Solution: The regression table and other pertinent information

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
x1	1	233.40	233.40	132.79	0.0014
x2	1	872.93	872.93	496.65	0.0002
Residuals	3	5.27	1.76		

Table 459: Linear regression model output in ANOVA table.

	Estimate	Std. Error	t value	Pr(>|t|)
(Intercept)	7.1861	1.2869	5.58	0.0113
x1	2.0498	0.1729	11.86	0.0013
x2	-5.1180	0.2297	-22.29	0.0002

Table 460: Linear regression model output.

The coefficient of multiple determination equals

$R^2 = \frac{SSR}{SSTO} = 1 - \frac{SSE}{SSTO} = 0.99526,$

and the F-statistic equals

$F^* = \frac{MSR}{MSE} = 314.72022$

	x
intercept	7.1861
$\hat{\beta}_1$	2.0498
$\hat{\beta}_2$	-5.1180

Table 461: The betas=$(X^{'}X)^{-1}X^{'}Y$

	x1	x2	y	yhat	resid
1	4.00	0.00	16.00	15.39	0.61
2	9.00	7.00	-9.80	-10.19	0.39
3	8.00	1.00	18.00	18.47	-0.47
4	3.00	4.00	-5.60	-7.14	1.54
5	0.00	5.00	-19.50	-18.40	-1.10
6	7.00	3.00	5.20	6.18	-0.98

$$\begin{aligned} SSTO &= Y^{'}Y - \frac{1}{n}Y^{'}JY = Y^{'}[I - \frac{1}{n}J]Y = 1111.60833 \\ SSR &= Y^{'}[H - \frac{1}{n}J]Y = 1106.33539 \\ SSE &= e^{'}e = Y^{'}[I - H]Y = 5.27295 \end{aligned}$$

where J is a $n \times n$ matrix of 1s and $H = X(X^{'}X)^{-1}X^{'}$ also size $n \times n$.

MSE=$\frac{SSE}{n-p} = 1.75765$ and MSR=$\frac{SSR}{p-1} = 553.16769$

The standard errors for the betas are

	x0	x1	x2
x0	1.6561	-0.1524	-0.1727
x1	-0.1524	0.0299	-0.0006
x2	-0.1727	-0.0006	0.0527

Table 462: Variance Matrix

$$s(\hat{\beta}_0) = \sqrt{MSE(X'X)^{-1}_{1,1}} = 1.286876$$
$$s(\hat{\beta}_1) = \sqrt{MSE(X'X)^{-1}_{2,2}} = 0.172864$$
$$s(\hat{\beta}_2) = \sqrt{MSE(X'X)^{-1}_{3,3}} = 0.229655$$

For testing an individual βk, $H_0 : \beta_k = 0$ versus $H_A : \beta_k \neq 0$ a t-test can be done where

$$t^* = \frac{\hat{\beta}_k - 0}{s(\hat{\beta}_k)} \tag{14}$$

and reject H_0, the null, if $|t^*|$ is greater than $t(1 - \frac{\alpha}{2}; n - p)$, otherwise fail to reject the null hypothesis.

The t-values - lower and upper for .95; n-p: -3.1824 and 3.1824 □

Problem 115. Solve for R-squared (R^2), coefficient of multiple determination and test:

$H_0 : \beta_1 = \beta_2 \cdots \beta_{p-1} = 0$ versus

$H_A : \beta_k \neq 0$ for at least one k where $1 \leq k \leq (p-1)$

Also, test the individual betas:

$H_0 : \beta_k = 0$ versus

$H_A : \beta_k \neq 0$ for at each k where $1 \leq k \leq 2$

The determinant of $X^T X = 4650$

	y	x1	x2
1	-43.20	7.00	5.00
2	-35.30	5.00	5.00
3	-36.00	7.00	0.00
4	-3.00	0.00	0.00
5	-35.60	6.00	3.00
6	-4.10	3.00	1.00

	x0	x1	x2
x0	6.00	28.00	14.00
x1	28.00	168.00	81.00
x2	14.00	81.00	60.00

Table 463: $X^{'}X$

	x0	x1	x2
x0	3519.00	-546.00	-84.00
x1	-546.00	164.00	-94.00
x2	-84.00	-94.00	224.00

Table 464: $det(X^{'}X)(X^{'}X)^{-1}$

	1	2	3	4	5	6
x0	-723.00	369.00	-303.00	3519.00	-9.00	1797.00
x1	132.00	-196.00	602.00	-546.00	156.00	-148.00
x2	378.00	566.00	-742.00	-84.00	24.00	-142.00

Table 465: $det(X^{'}X)(X^{'}X)^{-1}X^{'}$

	1	2	3	4	5	6
1	2091.00	1827.00	201.00	-723.00	1203.00	51.00
2	1827.00	2219.00	-1003.00	369.00	891.00	347.00
3	201.00	-1003.00	3911.00	-303.00	1083.00	761.00
4	-723.00	369.00	-303.00	3519.00	-9.00	1797.00
5	1203.00	891.00	1083.00	-9.00	999.00	483.00
6	51.00	347.00	761.00	1797.00	483.00	1211.00

Table 466: Det(X^TX) times the Hat matrix

Solution: The regression table and other pertinent information

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
x1	1	1334.42	1334.42	25.08	0.0153
x2	1	88.80	88.80	1.67	0.2869
Residuals	3	159.64	53.21		

Table 467: Linear regression model output in ANOVA table.

	Estimate	Std. Error	t value	Pr(>|t|)
(Intercept)	2.4756	6.3459	0.39	0.7225
x1	-5.1106	1.3699	-3.73	0.0336
x2	-2.0683	1.6011	-1.29	0.2869

Table 468: Linear regression model output.

The coefficient of multiple determination equals

$$R^2 = \frac{SSR}{SSTO} = 1 - \frac{SSE}{SSTO} = 0.89915,$$

and the F-statistic equals

$$F^* = \frac{MSR}{MSE} = 13.37288$$

	x
intercept	2.4756
$\hat{\beta}_1$	-5.1106
$\hat{\beta}_2$	-2.0683

Table 469: The betas=$(X^{'}X)^{-1}X^{'}Y$

	x1	x2	y	yhat	resid
1	7.00	5.00	-43.20	-43.64	0.44
2	5.00	5.00	-35.30	-33.42	-1.88
3	7.00	0.00	-36.00	-33.30	-2.70
4	0.00	0.00	-3.00	2.48	-5.48
5	6.00	3.00	-35.60	-34.39	-1.21
6	3.00	1.00	-4.10	-14.92	10.82

$$
\begin{aligned}
SSTO &= Y^{'}Y - \frac{1}{n}Y^{'}JY = Y^{'}[I - \frac{1}{n}J]Y = 1582.86 \\
SSR &= Y^{'}[H - \frac{1}{n}J]Y = 1423.22113 \\
SSE &= e^{'}e = Y^{'}[I - H]Y = 159.63887
\end{aligned}
$$

where J is a $n \times n$ matrix of 1s and $H = X(X^{'}X)^{-1}X^{'}$ also size $n \times n$.

MSE=$\frac{SSE}{n-p} = 53.21296$ and MSR=$\frac{SSR}{p-1} = 711.61056$

The standard errors for the betas are

	x0	x1	x2
x0	40.2702	-6.2482	-0.9613
x1	-6.2482	1.8768	-1.0757
x2	-0.9613	-1.0757	2.5634

Table 470: Variance Matrix

$$
\begin{aligned}
s(\hat{\beta}_0) &= \sqrt{MSE(X'X)^{-1}_{1,1}} = 6.34588 \\
s(\hat{\beta}_1) &= \sqrt{MSE(X'X)^{-1}_{2,2}} = 1.369948 \\
s(\hat{\beta}_2) &= \sqrt{MSE(X'X)^{-1}_{3,3}} = 1.601055
\end{aligned}
$$

For testing an individual βk, $H_0 : \beta_k = 0$ versus $H_A : \beta_k \neq 0$ a t-test can be done where

$$t^* = \frac{\hat{\beta}_k - 0}{s(\hat{\beta}_k)} \tag{15}$$

and reject H_0, the null, if $|t^*|$ is greater than $t(1 - \frac{\alpha}{2}; n - p)$, otherwise fail to reject the null hypothesis.

The t-values - lower and upper for .95; n-p: -3.1824 and 3.1824 □

Problem 116. Solve for R-squared (R^2), coefficient of multiple determination and test:

$H_0 : \beta_1 = \beta_2 \cdots \beta_{p-1} = 0$ versus

$H_A : \beta_k \neq 0$ for at least one k where $1 \leq k \leq (p-1)$

Also, test the individual betas:

$H_0 : \beta_k = 0$ versus

$H_A : \beta_k \neq 0$ for at each k where $1 \leq k \leq 2$

The determinant of $X^T X = 1494$

	y	x1	x2
1	15.30	7.00	2.00
2	18.60	5.00	4.00
3	39.70	5.00	8.00
4	26.80	2.00	8.00
5	17.70	6.00	3.00
6	17.30	8.00	2.00

	x0	x1	x2
x0	6.00	33.00	27.00
x1	33.00	203.00	124.00
x2	27.00	124.00	161.00

Table 471: $X^{'}X$

	x0	x1	x2
x0	17307.00	-1965.00	-1389.00
x1	-1965.00	237.00	147.00
x2	-1389.00	147.00	129.00

Table 472: $det(X^{'}X)(X^{'}X)^{-1}$

	1	2	3	4	5	6
x0	774.00	1926.00	-3630.00	2265.00	1350.00	-1191.00
x1	-12.00	-192.00	396.00	-315.00	-102.00	225.00
x2	-102.00	-138.00	378.00	-63.00	-120.00	45.00

Table 473: $det(X^{'}X)(X^{'}X)^{-1}X^{'}$

	1	2	3	4	5	6
1	486.00	306.00	-102.00	-66.00	396.00	474.00
2	306.00	414.00	-138.00	438.00	360.00	114.00
3	-102.00	-138.00	1374.00	186.00	-120.00	294.00
4	-66.00	438.00	186.00	1131.00	186.00	-381.00
5	396.00	360.00	-120.00	186.00	378.00	294.00
6	474.00	114.00	294.00	-381.00	294.00	699.00

Table 474: Det(X^TX) times the Hat matrix

Solution: The regression table and other pertinent information

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
x1	1	106.72	106.72	59.75	0.0045
x2	1	319.36	319.36	178.82	0.0009
Residuals	3	5.36	1.79		

Table 475: Linear regression model output in ANOVA table.

	Estimate	Std. Error	t value	Pr(>|t|)
(Intercept)	-21.7219	4.5485	-4.78	0.0174
x1	3.7560	0.5323	7.06	0.0058
x2	5.2512	0.3927	13.37	0.0009

Table 476: Linear regression model output.

The coefficient of multiple determination equals

$R^2 = \frac{SSR}{SSTO} = 1 - \frac{SSE}{SSTO} = 0.98758,$

and the F-statistic equals

$F^* = \frac{MSR}{MSE} = 119.28536$

	x
intercept	-21.7219
$\hat{\beta}_1$	3.7560
$\hat{\beta}_2$	5.2512

Table 477: The betas=$(X^{'}X)^{-1}X^{'}Y$

	x1	x2	y	yhat	resid
1	7.00	2.00	15.30	15.07	0.23
2	5.00	4.00	18.60	18.06	0.54
3	5.00	8.00	39.70	39.07	0.63
4	2.00	8.00	26.80	27.80	-1.00
5	6.00	3.00	17.70	16.57	1.13
6	8.00	2.00	17.30	18.83	-1.53

$$\begin{aligned} SSTO &= Y^{'}Y - \frac{1}{n}Y^{'}JY = Y^{'}[I - \frac{1}{n}J]Y = 431.43333 \\ SSR &= Y^{'}[H - \frac{1}{n}J]Y = 426.07548 \\ SSE &= e^{'}e = Y^{'}[I - H]Y = 5.35785 \end{aligned}$$

where J is a $n \times n$ matrix of 1s and $H = X(X^{'}X)^{-1}X^{'}$ also size $n \times n$.

MSE=$\frac{SSE}{n-p} = 1.78595$ and MSR=$\frac{SSR}{p-1} = 213.03774$

The standard errors for the betas are

	x0	x1	x2
x0	20.6891	-2.3490	-1.6604
x1	-2.3490	0.2833	0.1757
x2	-1.6604	0.1757	0.1542

Table 478: Variance Matrix

$$
\begin{aligned}
s(\hat{\beta}_0) &= \sqrt{MSE(X'X)^{-1}_{1,1}} = 4.548522 \\
s(\hat{\beta}_1) &= \sqrt{MSE(X'X)^{-1}_{2,2}} = 0.532272 \\
s(\hat{\beta}_2) &= \sqrt{MSE(X'X)^{-1}_{3,3}} = 0.392694
\end{aligned}
$$

For testing an individual βk, $H_0 : \beta_k = 0$ versus $H_A : \beta_k \neq 0$ a t-test can be done where

$$t^* = \frac{\hat{\beta}_k - 0}{s(\hat{\beta}_k)} \tag{16}$$

and reject H_0, the null, if $|t^*|$ is greater than $t(1 - \frac{\alpha}{2}; n - p)$, otherwise fail to reject the null hypothesis.

The t-values - lower and upper for .95; n-p: -3.1824 and 3.1824 □

Problem 117. Solve for R-squared (R^2), coefficient of multiple determination and test:

$H_0 : \beta_1 = \beta_2 \cdots \beta_{p-1} = 0$ versus

$H_A : \beta_k \neq 0$ for at least one k where $1 \leq k \leq (p-1)$

Also, test the individual betas:

$H_0 : \beta_k = 0$ versus

$H_A : \beta_k \neq 0$ for at each k where $1 \leq k \leq 2$

The determinant of $X^T X = 16434$

	y	x1	x2
1	-12.50	7.00	9.00
2	17.50	0.00	5.00
3	-20.30	7.00	0.00
4	-14.10	6.00	2.00
5	-22.10	9.00	8.00
6	-10.00	5.00	4.00

	x0	x1	x2
x0	6.00	34.00	28.00
x1	34.00	240.00	167.00
x2	28.00	167.00	190.00

Table 479: $X^{'} X$

	x0	x1	x2
x0	17711.00	-1784.00	-1042.00
x1	-1784.00	356.00	-50.00
x2	-1042.00	-50.00	284.00

Table 480: $det(X^{'}X)(X^{'}X)^{-1}$

	1	2	3	4	5	6
x0	-4155.00	12501.00	5223.00	4923.00	-6681.00	4623.00
x1	258.00	-2034.00	708.00	252.00	1020.00	-204.00
x2	1164.00	378.00	-1392.00	-774.00	780.00	-156.00

Table 481: $det(X^{'}X)(X^{'}X)^{-1}X^{'}$

	1	2	3	4	5	6
1	8127.00	1665.00	-2349.00	-279.00	7479.00	1791.00
2	1665.00	14391.00	-1737.00	1053.00	-2781.00	3843.00
3	-2349.00	-1737.00	10179.00	6687.00	459.00	3195.00
4	-279.00	1053.00	6687.00	4887.00	999.00	3087.00
5	7479.00	-2781.00	459.00	999.00	8739.00	1539.00
6	1791.00	3843.00	3195.00	3087.00	1539.00	2979.00

Table 482: Det(X^TX) times the Hat matrix

Solution: The regression table and other pertinent information

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
x1	1	972.95	972.95	440.96	0.0002
x2	1	51.86	51.86	23.50	0.0167
Residuals	3	6.62	2.21		

Table 483: Linear regression model output in ANOVA table.

	Estimate	Std. Error	t value	Pr(>|t|)
(Intercept)	11.9681	1.5420	7.76	0.0044
x1	-4.7005	0.2186	-21.50	0.0002
x2	0.9467	0.1953	4.85	0.0167

Table 484: Linear regression model output.

The coefficient of multiple determination equals

$$R^2 = \frac{SSR}{SSTO} = 1 - \frac{SSE}{SSTO} = 0.99358,$$

and the F-statistic equals

$$F^* = \frac{MSR}{MSE} = 232.23301$$

	x
intercept	11.9681
$\hat{\beta}_1$	-4.7005
$\hat{\beta}_2$	0.9467

Table 485: The betas=$(X^{'}X)^{-1}X^{'}Y$

	x1	x2	y	yhat	resid
1	7.00	9.00	-12.50	-12.41	-0.09
2	0.00	5.00	17.50	16.70	0.80
3	7.00	0.00	-20.30	-20.94	0.64
4	6.00	2.00	-14.10	-14.34	0.24
5	9.00	8.00	-22.10	-22.76	0.66
6	5.00	4.00	-10.00	-7.75	-2.25

$$
\begin{aligned}
SSTO &= Y^{'}Y - \frac{1}{n}Y^{'}JY = Y^{'}[I - \frac{1}{n}J]Y = 1031.435 \\
SSR &= Y^{'}[H - \frac{1}{n}J]Y = 1024.81568 \\
SSE &= e^{'}e = Y^{'}[I - H]Y = 6.61932
\end{aligned}
$$

where J is a $n \times n$ matrix of 1s and $H = X(X^{'}X)^{-1}X^{'}$ also size $n \times n$.

MSE=$\frac{SSE}{n-p} = 2.20644$ and MSR=$\frac{SSR}{p-1} = 512.40784$

The standard errors for the betas are

	x0	x1	x2
x0	2.3779	-0.2395	-0.1399
x1	-0.2395	0.0478	-0.0067
x2	-0.1399	-0.0067	0.0381

Table 486: Variance Matrix

$$s(\hat{\beta}_0) = \sqrt{MSE(X'X)^{-1}_{1,1}} = 1.542041$$
$$s(\hat{\beta}_1) = \sqrt{MSE(X'X)^{-1}_{2,2}} = 0.218625$$
$$s(\hat{\beta}_2) = \sqrt{MSE(X'X)^{-1}_{3,3}} = 0.195269$$

For testing an individual βk, $H_0 : \beta_k = 0$ versus $H_A : \beta_k \neq 0$ a t-test can be done where

$$t^* = \frac{\hat{\beta}_k - 0}{s(\hat{\beta}_k)} \tag{17}$$

and reject H_0, the null, if $|t^*|$ is greater than $t(1 - \frac{\alpha}{2}; n - p)$, otherwise fail to reject the null hypothesis.

The t-values - lower and upper for .95; n-p: -3.1824 and 3.1824 □

Problem 118. Solve for R-squared (R^2), coefficient of multiple determination and test:

$H_0 : \beta_1 = \beta_2 \cdots \beta_{p-1} = 0$ versus

$H_A : \beta_k \neq 0$ for at least one k where $1 \leq k \leq (p-1)$

Also, test the individual betas:

$H_0 : \beta_k = 0$ versus

$H_A : \beta_k \neq 0$ for at each k where $1 \leq k \leq 2$

The determinant of $X^T X = 17064$

	y	x1	x2
1	-12.40	0.00	6.00
2	19.20	9.00	0.00
3	-18.40	7.00	8.00
4	-12.20	0.00	6.00
5	12.40	8.00	2.00
6	-18.90	2.00	8.00

	x0	x1	x2
x0	6.00	26.00	30.00
x1	26.00	198.00	88.00
x2	30.00	88.00	204.00

Table 487: $X^{'}X$

	x0	x1	x2
x0	32648.00	-2664.00	-3652.00
x1	-2664.00	324.00	252.00
x2	-3652.00	252.00	512.00

Table 488: $det(X^{'}X)(X^{'}X)^{-1}$

	1	2	3	4	5	6
x0	10736.00	8672.00	-15216.00	10736.00	4032.00	-1896.00
x1	-1152.00	252.00	1620.00	-1152.00	432.00	-0.00
x2	-580.00	-1384.00	2208.00	-580.00	-612.00	948.00

Table 489: $det(X^{'}X)(X^{'}X)^{-1}X^{'}$

	1	2	3	4	5	6
1	7256.00	368.00	-1968.00	7256.00	360.00	3792.00
2	368.00	10940.00	-636.00	368.00	7920.00	-1896.00
3	-1968.00	-636.00	13788.00	-1968.00	2160.00	5688.00
4	7256.00	368.00	-1968.00	7256.00	360.00	3792.00
5	360.00	7920.00	2160.00	360.00	6264.00	0.00
6	3792.00	-1896.00	5688.00	3792.00	-0.00	5688.00

Table 490: Det(X^TX) times the Hat matrix

Solution: The regression table and other pertinent information

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
x1	1	656.57	656.57	282.20	0.0005
x2	1	704.21	704.21	302.68	0.0004
Residuals	3	6.98	2.33		

Table 491: Linear regression model output in ANOVA table.

	Estimate	Std. Error	t value	Pr(>|t|)
(Intercept)	15.7174	2.1098	7.45	0.0050
x1	0.5114	0.2102	2.43	0.0931
x2	-4.5967	0.2642	-17.40	0.0004

Table 492: Linear regression model output.

The coefficient of multiple determination equals

$R^2 = \frac{SSR}{SSTO} = 1 - \frac{SSE}{SSTO} = 0.9949,$

and the F-statistic equals

$F^* = \frac{MSR}{MSE} = 292.43726$

	x
intercept	15.7174
$\hat{\beta}_1$	0.5114
$\hat{\beta}_2$	-4.5967

Table 493: The betas=$(X^{'}X)^{-1}X^{'}Y$

	x1	x2	y	yhat	resid
1	0.00	6.00	-12.40	-11.86	-0.54
2	9.00	0.00	19.20	20.32	-1.12
3	7.00	8.00	-18.40	-17.48	-0.92
4	0.00	6.00	-12.20	-11.86	-0.34
5	8.00	2.00	12.40	10.62	1.78
6	2.00	8.00	-18.90	-20.03	1.13

$$SSTO = Y^{'}Y - \frac{1}{n}Y^{'}JY = Y^{'}[I - \frac{1}{n}J]Y = 1367.755$$
$$SSR = Y^{'}[H - \frac{1}{n}J]Y = 1360.77517$$
$$SSE = e^{'}e = Y^{'}[I - H]Y = 6.97983$$

where J is a $n \times n$ matrix of 1s and $H = X(X^{'}X)^{-1}X^{'}$ also size $n \times n$.

MSE=$\frac{SSE}{n-p} = 2.32661$ and MSR=$\frac{SSR}{p-1} = 680.38758$

The standard errors for the betas are

	x0	x1	x2
x0	4.4514	-0.3632	-0.4979
x1	-0.3632	0.0442	0.0344
x2	-0.4979	0.0344	0.0698

Table 494: Variance Matrix

$$
\begin{aligned}
s(\hat{\beta}_0) &= \sqrt{MSE(X'X)^{-1}_{1,1}} = 2.109841 \\
s(\hat{\beta}_1) &= \sqrt{MSE(X'X)^{-1}_{2,2}} = 0.210181 \\
s(\hat{\beta}_2) &= \sqrt{MSE(X'X)^{-1}_{3,3}} = 0.264214
\end{aligned}
$$

For testing an individual βk, $H_0 : \beta_k = 0$ versus $H_A : \beta_k \neq 0$ a t-test can be done where

$$t^* = \frac{\hat{\beta}_k - 0}{s(\hat{\beta}_k)} \tag{18}$$

and reject H_0, the null, if $|t^*|$ is greater than $t(1 - \frac{\alpha}{2}; n - p)$, otherwise fail to reject the null hypothesis.

The t-values - lower and upper for .95; n-p: -3.1824 and 3.1824 □

Problem 119. Solve for R-squared (R^2), coefficient of multiple determination and test:

$H_0 : \beta_1 = \beta_2 \cdots \beta_{p-1} = 0$ versus

$H_A : \beta_k \neq 0$ for at least one k where $1 \leq k \leq (p-1)$

Also, test the individual betas:

$H_0 : \beta_k = 0$ versus

$H_A : \beta_k \neq 0$ for at each k where $1 \leq k \leq 2$

The determinant of $X^T X = 9704$

	y	x1	x2
1	-26.10	5.00	4.00
2	-29.30	5.00	6.00
3	-22.00	0.00	8.00
4	4.60	0.00	0.00
5	-21.90	3.00	7.00
6	-29.40	7.00	5.00

	x0	x1	x2
x0	6.00	20.00	30.00
x1	20.00	108.00	106.00
x2	30.00	106.00	190.00

Table 495: $X^{'}X$

	x0	x1	x2
x0	9284.00	-620.00	-1120.00
x1	-620.00	240.00	-36.00
x2	-1120.00	-36.00	248.00

Table 496: $det(X^{'}X)(X^{'}X)^{-1}$

	1	2	3	4	5	6
x0	1704.00	-536.00	324.00	9284.00	-416.00	-656.00
x1	436.00	364.00	-908.00	-620.00	-152.00	880.00
x2	-308.00	188.00	864.00	-1120.00	508.00	-132.00

Table 497: $det(X^{'}X)(X^{'}X)^{-1}X^{'}$

	1	2	3	4	5	6
1	2652.00	2036.00	-760.00	1704.00	856.00	3216.00
2	2036.00	2412.00	968.00	-536.00	1872.00	2952.00
3	-760.00	968.00	7236.00	324.00	3648.00	-1712.00
4	1704.00	-536.00	324.00	9284.00	-416.00	-656.00
5	856.00	1872.00	3648.00	-416.00	2684.00	1060.00
6	3216.00	2952.00	-1712.00	-656.00	1060.00	4844.00

Table 498: Det(X^TX) times the Hat matrix

Solution: The regression table and other pertinent information

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
x1	1	439.84	439.84	36.90	0.0090
x2	1	346.43	346.43	29.06	0.0125
Residuals	3	35.76	11.92		

Table 499: Linear regression model output in ANOVA table.

	Estimate	Std. Error	t value	Pr(>|t|)
(Intercept)	3.6279	3.3771	1.07	0.3614
x1	-2.8302	0.5430	-5.21	0.0137
x2	-2.9755	0.5520	-5.39	0.0125

Table 500: Linear regression model output.

The coefficient of multiple determination equals

$R^2 = \frac{SSR}{SSTO} = 1 - \frac{SSE}{SSTO} = 0.9565,$

and the F-statistic equals

$F^* = \frac{MSR}{MSE} = 32.97894$

	x
intercept	3.6279
$\hat{\beta}_1$	-2.8302
$\hat{\beta}_2$	-2.9755

Table 501: The betas=$(X^{'}X)^{-1}X^{'}Y$

	x1	x2	y	yhat	resid
1	5.00	4.00	-26.10	-22.42	-3.68
2	5.00	6.00	-29.30	-28.38	-0.92
3	0.00	8.00	-22.00	-20.18	-1.82
4	0.00	0.00	4.60	3.63	0.97
5	3.00	7.00	-21.90	-25.69	3.79
6	7.00	5.00	-29.40	-31.06	1.66

$$\begin{aligned} SSTO &= Y^{'}Y - \frac{1}{n}Y^{'}JY = Y^{'}[I - \frac{1}{n}J]Y = 822.02833 \\ SSR &= Y^{'}[H - \frac{1}{n}J]Y = 786.26614 \\ SSE &= e^{'}e = Y^{'}[I - H]Y = 35.76219 \end{aligned}$$

where J is a $n \times n$ matrix of 1s and $H = X(X^{'}X)^{-1}X^{'}$ also size $n \times n$.

MSE=$\frac{SSE}{n-p} = 11.92073$ and MSR=$\frac{SSR}{p-1} = 393.13307$

The standard errors for the betas are

	x0	x1	x2
x0	11.4048	-0.7616	-1.3758
x1	-0.7616	0.2948	-0.0442
x2	-1.3758	-0.0442	0.3047

Table 502: Variance Matrix

$$\begin{aligned} s(\hat{\beta}_0) &= \sqrt{MSE(X'X)^{-1}_{1,1}} = 3.377097 \\ s(\hat{\beta}_1) &= \sqrt{MSE(X'X)^{-1}_{2,2}} = 0.542977 \\ s(\hat{\beta}_2) &= \sqrt{MSE(X'X)^{-1}_{3,3}} = 0.551953 \end{aligned}$$

For testing an individual βk, $H_0 : \beta_k = 0$ versus $H_A : \beta_k \neq 0$ a t-test can be done where

$$t^* = \frac{\hat{\beta}_k - 0}{s(\hat{\beta}_k)} \tag{19}$$

and reject H_0, the null, if $|t^*|$ is greater than $t(1 - \frac{\alpha}{2}; n - p)$, otherwise fail to reject the null hypothesis.

The t-values - lower and upper for .95; n-p: -3.1824 and 3.1824 □

Problem 120. Solve for R-squared (R^2), coefficient of multiple determination and test:

$H_0 : \beta_1 = \beta_2 \cdots \beta_{p-1} = 0$ versus

$H_A : \beta_k \neq 0$ for at least one k where $1 \leq k \leq (p-1)$

Also, test the individual betas:

$H_0 : \beta_k = 0$ versus

$H_A : \beta_k \neq 0$ for at each k where $1 \leq k \leq 2$

The determinant of $X^T X = 3762$

	y	x1	x2
1	-10.50	4.00	4.00
2	-1.70	9.00	4.00
3	3.10	7.00	0.00
4	-22.30	2.00	6.00
5	-16.50	2.00	4.00
6	-12.50	3.00	3.00

	x0	x1	x2
x0	6.00	27.00	21.00
x1	27.00	163.00	81.00
x2	21.00	81.00	93.00

Table 503: $X^{'}X$

	x0	x1	x2
x0	8598.00	-810.00	-1236.00
x1	-810.00	117.00	81.00
x2	-1236.00	81.00	249.00

Table 504: $det(X^{'}X)(X^{'}X)^{-1}$

	1	2	3	4	5	6
x0	414.00	-3636.00	2928.00	-438.00	2034.00	2460.00
x1	-18.00	567.00	9.00	-90.00	-252.00	-216.00
x2	84.00	489.00	-669.00	420.00	-78.00	-246.00

Table 505: $det(X^{'}X)(X^{'}X)^{-1}X^{'}$

	1	2	3	4	5	6
1	678.00	588.00	288.00	882.00	714.00	612.00
2	588.00	3423.00	333.00	432.00	-546.00	-468.00
3	288.00	333.00	2991.00	-1068.00	270.00	948.00
4	882.00	432.00	-1068.00	1902.00	1062.00	552.00
5	714.00	-546.00	270.00	1062.00	1218.00	1044.00
6	612.00	-468.00	948.00	552.00	1044.00	1074.00

Table 506: Det(X^TX) times the Hat matrix

Solution: The regression table and other pertinent information

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
x1	1	353.38	353.38	228.96	0.0006
x2	1	82.50	82.50	53.46	0.0053
Residuals	3	4.63	1.54		

Table 507: Linear regression model output in ANOVA table.

	Estimate	Std. Error	t value	Pr(>|t|)
(Intercept)	-11.5982	1.8781	-6.18	0.0085
x1	2.1579	0.2191	9.85	0.0022
x2	-2.3368	0.3196	-7.31	0.0053

Table 508: Linear regression model output.

The coefficient of multiple determination equals

$$R^2 = \frac{SSR}{SSTO} = 1 - \frac{SSE}{SSTO} = 0.98949,$$

and the F-statistic equals

$$F^* = \frac{MSR}{MSE} = 141.2095$$

	x
intercept	-11.5982
$\hat{\beta}_1$	2.1579
$\hat{\beta}_2$	-2.3368

Table 509: The betas=$(X^{'}X)^{-1}X^{'}Y$

	x1	x2	y	yhat	resid
1	4.00	4.00	-10.50	-12.31	1.81
2	9.00	4.00	-1.70	-1.52	-0.18
3	7.00	0.00	3.10	3.51	-0.41
4	2.00	6.00	-22.30	-21.30	-1.00
5	2.00	4.00	-16.50	-16.63	0.13
6	3.00	3.00	-12.50	-12.14	-0.36

$$
\begin{aligned}
SSTO &= Y^{'}Y - \frac{1}{n}Y^{'}JY = Y^{'}[I - \frac{1}{n}J]Y = 440.51333 \\
SSR &= Y^{'}[H - \frac{1}{n}J]Y = 435.88316 \\
SSE &= e^{'}e = Y^{'}[I - H]Y = 4.63018
\end{aligned}
$$

where J is a $n \times n$ matrix of 1s and $H = X(X^{'}X)^{-1}X^{'}$ also size $n \times n$.

MSE=$\frac{SSE}{n-p} = 1.54339$ and MSR=$\frac{SSR}{p-1} = 217.94158$

The standard errors for the betas are

	x0	x1	x2
x0	3.5274	-0.3323	-0.5071
x1	-0.3323	0.0480	0.0332
x2	-0.5071	0.0332	0.1022

Table 510: Variance Matrix

$$s(\hat{\beta}_0) = \sqrt{MSE(X'X)^{-1}_{1,1}} = 1.878138$$
$$s(\hat{\beta}_1) = \sqrt{MSE(X'X)^{-1}_{2,2}} = 0.21909$$
$$s(\hat{\beta}_2) = \sqrt{MSE(X'X)^{-1}_{3,3}} = 0.319616$$

For testing an individual βk, $H_0 : \beta_k = 0$ versus $H_A : \beta_k \neq 0$ a t-test can be done where

$$t^* = \frac{\hat{\beta}_k - 0}{s(\hat{\beta}_k)} \quad (20)$$

and reject H_0, the null, if $|t^*|$ is greater than $t(1 - \frac{\alpha}{2}; n - p)$, otherwise fail to reject the null hypothesis.

The t-values - lower and upper for .95; n-p: -3.1824 and 3.1824 □

Problem 121. Solve for 95% confidence and prediction interval for $X1 = 9$ and $X2 = 2$. The prediction interval is for 4 new observations. Model assumes an intercept.

	y	x1	x2
1	6.1	4.0	3.0
2	-1.3	9.0	2.0
3	-2.5	6.0	1.0
4	-1.3	1.0	0.0
5	9.1	5.0	4.0
6	35.1	1.0	8.0

The determinant of $X^T X = 10184$

	x0	x1	x2
x0	6.00	26.00	18.00
x1	26.00	160.00	64.00
x2	18.00	64.00	94.00

Table 511: $X^{'}X$

	x0	x1	x2
x0	10944.00	-1292.00	-1216.00
x1	-1292.00	240.00	84.00
x2	-1216.00	84.00	284.00

Table 512: $det(X^{'}X)(X^{'}X)^{-1}$

	1	2	3	4	5	6
x0	2128.00	-3116.00	1976.00	9652.00	-380.00	-76.00
x1	-80.00	1036.00	232.00	-1052.00	244.00	-380.00
x2	-28.00	108.00	-428.00	-1132.00	340.00	1140.00

Table 513: $det(X^{'}X)(X^{'}X)^{-1}X^{'}$

	1	2	3	4	5	6
1	1724.00	1352.00	1620.00	2048.00	1616.00	1824.00
2	1352.00	6424.00	3208.00	-2080.00	2496.00	-1216.00
3	1620.00	3208.00	2940.00	2208.00	1424.00	-1216.00
4	2048.00	-2080.00	2208.00	8600.00	-136.00	-456.00
5	1616.00	2496.00	1424.00	-136.00	2200.00	2584.00
6	1824.00	-1216.00	-1216.00	-456.00	2584.00	8664.00

Table 514: Det(X^TX) times the Hat matrix

Solution: The regression table and other pertinent information

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
x1	1	298.51	298.51	75.05	0.0032
x2	1	710.72	710.72	178.69	0.0009
Residuals	3	11.93	3.98		

Table 515: Linear regression model output in ANOVA table.

	Estimate	Std. Error	t value	Pr(>|t|)
(Intercept)	-0.6463	2.0674	-0.31	0.7751
x1	-1.1945	0.3062	-3.90	0.0299
x2	4.4519	0.3330	13.37	0.0009

Table 516: Linear regression model output.

The coefficient of multiple determination equals

$R^2 = \frac{SSR}{SSTO} = 1 - \frac{SSE}{SSTO} = 0.98832,$

and the F-statistic equals

$F^* = \frac{MSR}{MSE} = 126.87106$

	x
intercept	-0.6463
betahat1	-1.1945
betahat2	4.4519

Table 517: The betas=$(X^{'}X)^{-1}X^{'}Y$

	x1	x2	y	yhat	resid
1	4.00	3.00	6.10	7.93	-1.83
2	9.00	2.00	-1.30	-2.49	1.19
3	6.00	1.00	-2.50	-3.36	0.86
4	1.00	0.00	-1.30	-1.84	0.54
5	5.00	4.00	9.10	11.19	-2.09
6	1.00	8.00	35.10	33.77	1.33

$$SSTO = Y^{'}Y - \frac{1}{n}Y^{'}JY = Y^{'}[I - \frac{1}{n}J]Y = 1021.15333$$
$$SSR = Y^{'}[H - \frac{1}{n}J]Y = 1009.22128$$
$$SSE = e^{'}e = Y^{'}[I - H]Y = 11.93205$$

where J is a $n \times n$ matrix of 1s and $H = X(X^{'}X)^{-1}X^{'}$ also size $n \times n$.

MSE=$\frac{SSE}{n-p} = 3.97735$ and MSR=$\frac{SSR}{p-1} = 504.61064$

	x0	x1	x2
x0	4.2742	-0.5046	-0.4749
x1	-0.5046	0.0937	0.0328
x2	-0.4749	0.0328	0.1109

Table 518: Variance Matrix

The standard errors for the betas are

$$
\begin{aligned}
s(\hat{\beta}_0) &= \sqrt{MSE(X'X)^{-1}_{1,1}} = 2.067406 \\
s(\hat{\beta}_1) &= \sqrt{MSE(X'X)^{-1}_{2,2}} = 0.306156 \\
s(\hat{\beta}_2) &= \sqrt{MSE(X'X)^{-1}_{3,3}} = 0.33304
\end{aligned}
$$

The $\hat{Y}_h = -2.49293$ and $s^2(\hat{Y}_h) = 2.508886$ and the 95% confidence limits for $E[Y_h]$=

	1	2
theci	-7.53	2.55

Table 519: The 95 percent confidence interval

	1	2
thepi	-8.45	3.46

Table 520: The 95 percent prediction interval

	1	2
theti	-3.18	3.18

Table 521: t-values - lower and upper for .95; n-p

□

Problem 122. Solve for 95% confidence and prediction interval for $X1 = 3$ and $X2 = 0$. The prediction interval is for 4 new observations. Model assumes an intercept.

	y	x1	x2
1	-17.3	5.0	7.0
2	-11.4	3.0	0.0
3	-18.4	0.0	4.0
4	-26.6	0.0	8.0
5	-7.0	5.0	2.0
6	-16.3	7.0	7.0

The determinant of $X^T X = 12728$

	x0	x1	x2
x0	6.00	20.00	28.00
x1	20.00	108.00	94.00
x2	28.00	94.00	182.00

Table 522: $X^{'}X$

	x0	x1	x2
x0	10820.00	-1008.00	-1144.00
x1	-1008.00	308.00	-4.00
x2	-1144.00	-4.00	248.00

Table 523: $det(X^{'}X)(X^{'}X)^{-1}$

	1	2	3	4	5	6
x0	-2228.00	7796.00	6244.00	1668.00	3492.00	-4244.00
x1	504.00	-84.00	-1024.00	-1040.00	524.00	1120.00
x2	572.00	-1156.00	-152.00	840.00	-668.00	564.00

Table 524: $det(X^{'}X)(X^{'}X)^{-1}X^{'}$

	1	2	3	4	5	6
1	4296.00	-716.00	60.00	2348.00	1436.00	5304.00
2	-716.00	7544.00	3172.00	-1452.00	5064.00	-884.00
3	60.00	3172.00	5636.00	5028.00	820.00	-1988.00
4	2348.00	-1452.00	5028.00	8388.00	-1852.00	268.00
5	1436.00	5064.00	820.00	-1852.00	4776.00	2484.00
6	5304.00	-884.00	-1988.00	268.00	2484.00	7544.00

Table 525: Det(X^TX) times the Hat matrix

Solution: The regression table and other pertinent information

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
x1	1	69.33	69.33	13.21	0.0359
x2	1	136.82	136.82	26.08	0.0145
Residuals	3	15.74	5.25		

Table 526: Linear regression model output in ANOVA table.

	Estimate	Std. Error	t value	Pr(>|t|)
(Intercept)	-12.9522	2.1119	-6.13	0.0087
x1	1.3215	0.3563	3.71	0.0341
x2	-1.6327	0.3197	-5.11	0.0145

Table 527: Linear regression model output.

The coefficient of multiple determination equals

$$R^2 = \frac{SSR}{SSTO} = 1 - \frac{SSE}{SSTO} = 0.92906,$$

and the F-statistic equals

$$F^* = \frac{MSR}{MSE} = 19.64596$$

	x
intercept	-12.9522
betahat1	1.3215
betahat2	-1.6327

Table 528: The betas=$(X^{'}X)^{-1}X^{'}Y$

	x1	x2	y	yhat	resid
1	5.00	7.00	-17.30	-17.77	0.47
2	3.00	0.00	-11.40	-8.99	-2.41
3	0.00	4.00	-18.40	-19.48	1.08
4	0.00	8.00	-26.60	-26.01	-0.59
5	5.00	2.00	-7.00	-9.61	2.61
6	7.00	7.00	-16.30	-15.13	-1.17

$$\begin{aligned} SSTO &= Y^{'}Y - \frac{1}{n}Y^{'}JY = Y^{'}[I - \frac{1}{n}J]Y = 221.89333 \\ SSR &= Y^{'}[H - \frac{1}{n}J]Y = 206.15321 \\ SSE &= e^{'}e = Y^{'}[I - H]Y = 15.74013 \end{aligned}$$

where J is a $n \times n$ matrix of 1s and $H = X(X^{'}X)^{-1}X^{'}$ also size $n \times n$.

MSE=$\frac{SSE}{n-p} = 5.24671$ and MSR=$\frac{SSR}{p-1} = 103.0766$

	x0	x1	x2
x0	4.4602	-0.4155	-0.4716
x1	-0.4155	0.1270	-0.0016
x2	-0.4716	-0.0016	0.1022

Table 529: Variance Matrix

The standard errors for the betas are

$$\begin{aligned} s(\hat{\beta}_0) &= \sqrt{MSE(X'X)^{-1}_{1,1}} = 2.111918 \\ s(\hat{\beta}_1) &= \sqrt{MSE(X'X)^{-1}_{2,2}} = 0.356319 \\ s(\hat{\beta}_2) &= \sqrt{MSE(X'X)^{-1}_{3,3}} = 0.319734 \end{aligned}$$

The $\hat{Y}_h = -8.987681$ and $s^2(\hat{Y}_h) = 3.109771$ and the 95% confidence limits for $E[Y_h]$=

	1	2
theci	-14.60	-3.38

Table 530: The 95 percent confidence interval

	1	2
thepi	-15.68	-2.30

Table 531: The 95 percent prediction interval

	1	2
theti	-3.18	3.18

Table 532: t-values - lower and upper for .95; n-p

□

Problem 123. Solve for 95% confidence and prediction interval for $X1 = 2$ and $X2 = 1$. The prediction interval is for 2 new observations. Model assumes an intercept.

	y	x1	x2
1	20.2	6.0	6.0
2	1.6	2.0	1.0
3	17.8	2.0	7.0
4	9.5	3.0	2.0
5	20.2	6.0	6.0
6	8.7	1.0	8.0

The determinant of $X^T X = 5576$

	x0	x1	x2
x0	6.00	20.00	30.00
x1	20.00	90.00	102.00
x2	30.00	102.00	190.00

Table 533: $X^{'}X$

	x0	x1	x2
x0	6696.00	-740.00	-660.00
x1	-740.00	240.00	-12.00
x2	-660.00	-12.00	140.00

Table 534: $det(X^{'}X)(X^{'}X)^{-1}$

	1	2	3	4	5	6
x0	-1704.00	4556.00	596.00	3156.00	-1704.00	676.00
x1	628.00	-272.00	-344.00	-44.00	628.00	-596.00
x2	108.00	-544.00	296.00	-416.00	108.00	448.00

Table 535: $det(X^{'}X)(X^{'}X)^{-1}X^{'}$

	1	2	3	4	5	6
1	2712.00	-340.00	308.00	396.00	2712.00	-212.00
2	-340.00	3468.00	204.00	2652.00	-340.00	-68.00
3	308.00	204.00	1980.00	156.00	308.00	2620.00
4	396.00	2652.00	156.00	2192.00	396.00	-216.00
5	2712.00	-340.00	308.00	396.00	2712.00	-212.00
6	-212.00	-68.00	2620.00	-216.00	-212.00	3664.00

Table 536: Det(X^TX) times the Hat matrix

Solution: The regression table and other pertinent information

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
x1	1	146.17	146.17	9.94	0.0512
x2	1	97.12	97.12	6.60	0.0825
Residuals	3	44.13	14.71		

Table 537: Linear regression model output in ANOVA table.

	Estimate	Std. Error	t value	Pr(>|t|)
(Intercept)	-2.7044	4.2031	-0.64	0.5657
x1	2.3690	0.7957	2.98	0.0587
x2	1.5615	0.6078	2.57	0.0825

Table 538: Linear regression model output.

The coefficient of multiple determination equals

$$R^2 = \frac{SSR}{SSTO} = 1 - \frac{SSE}{SSTO} = 0.84645,$$

and the F-statistic equals

$$F^* = \frac{MSR}{MSE} = 8.26873$$

	x
intercept	-2.7044
betahat1	2.3690
betahat2	1.5615

Table 539: The betas=$(X^{'}X)^{-1}X^{'}Y$

	x1	x2	y	yhat	resid
1	6.00	6.00	20.20	20.88	-0.68
2	2.00	1.00	1.60	3.60	-2.00
3	2.00	7.00	17.80	12.96	4.84
4	3.00	2.00	9.50	7.53	1.97
5	6.00	6.00	20.20	20.88	-0.68
6	1.00	8.00	8.70	12.16	-3.46

$$\begin{aligned} SSTO &= Y^{'}Y - \frac{1}{n}Y^{'}JY = Y^{'}[I - \frac{1}{n}J]Y = 287.42 \\ SSR &= Y^{'}[H - \frac{1}{n}J]Y = 243.28631 \\ SSE &= e^{'}e = Y^{'}[I - H]Y = 44.13369 \end{aligned}$$

where J is a $n \times n$ matrix of 1s and $H = X(X^{'}X)^{-1}X^{'}$ also size $n \times n$.

MSE=$\frac{SSE}{n-p} = 14.71123$ and MSR=$\frac{SSR}{p-1} = 121.64316$

	x0	x1	x2
x0	17.6661	-1.9524	-1.7413
x1	-1.9524	0.6332	-0.0317
x2	-1.7413	-0.0317	0.3694

Table 540: Variance Matrix

The standard errors for the betas are

$$s(\hat{\beta}_0) = \sqrt{MSE(X'X)^{-1}_{1,1}} = 4.203111$$
$$s(\hat{\beta}_1) = \sqrt{MSE(X'X)^{-1}_{2,2}} = 0.795735$$
$$s(\hat{\beta}_2) = \sqrt{MSE(X'X)^{-1}_{3,3}} = 0.607753$$

The $\hat{Y}_h = 3.595122$ and $s^2(\hat{Y}_h) = 9.149667$ and the 95% confidence limits for $E[Y_h]$=

	1	2
theci	-6.03	13.22

Table 541: The 95 percent confidence interval

	1	2
thepi	-9.33	16.52

Table 542: The 95 percent prediction interval

	1	2
theti	-3.18	3.18

Table 543: t-values - lower and upper for .95; n-p

□

Problem 124. Solve for 95% confidence and prediction interval for $X1 = 3$ and $X2 = 5$. The prediction interval is for 4 new observations. Model assumes an intercept.

	y	x1	x2
1	39.1	6.0	9.0
2	19.7	3.0	5.0
3	8.7	7.0	1.0
4	6.5	0.0	2.0
5	13.2	6.0	0.0
6	27.6	8.0	5.0

The determinant of $X^T X = 14392$

	x0	x1	x2
x0	6.00	30.00	22.00
x1	30.00	194.00	116.00
x2	22.00	116.00	136.00

Table 544: $X^{'}X$

	x0	x1	x2
x0	12928.00	-1528.00	-788.00
x1	-1528.00	332.00	-36.00
x2	-788.00	-36.00	264.00

Table 545: $det(X^{'}X)(X^{'}X)^{-1}$

	1	2	3	4	5	6
x0	-3332.00	4404.00	1444.00	11352.00	3760.00	-3236.00
x1	140.00	-712.00	760.00	-1600.00	464.00	948.00
x2	1372.00	424.00	-776.00	-260.00	-1004.00	244.00

Table 546: $det(X^{'}X)(X^{'}X)^{-1}X^{'}$

	1	2	3	4	5	6
1	9856.00	3948.00	-980.00	-588.00	-2492.00	4648.00
2	3948.00	4388.00	-156.00	5252.00	132.00	828.00
3	-980.00	-156.00	5988.00	-108.00	6004.00	3644.00
4	-588.00	5252.00	-108.00	10832.00	1752.00	-2748.00
5	-2492.00	132.00	6004.00	1752.00	6544.00	2452.00
6	4648.00	828.00	3644.00	-2748.00	2452.00	5568.00

Table 547: Det(X^TX) times the Hat matrix

Solution: The regression table and other pertinent information

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
x1	1	147.64	147.64	9.99	0.0509
x2	1	582.34	582.34	39.40	0.0082
Residuals	3	44.34	14.78		

Table 548: Linear regression model output in ANOVA table.

	Estimate	Std. Error	t value	Pr(>|t|)
(Intercept)	0.2186	3.6439	0.06	0.9559
x1	1.3861	0.5839	2.37	0.0982
x2	3.2684	0.5207	6.28	0.0082

Table 549: Linear regression model output.

The coefficient of multiple determination equals

$R^2 = \frac{SSR}{SSTO} = 1 - \frac{SSE}{SSTO} = 0.94273,$

and the F-statistic equals

$F^* = \frac{MSR}{MSE} = 24.6928$

	x
intercept	0.2186
betahat1	1.3861
betahat2	3.2684

Table 550: The betas=$(X^{'}X)^{-1}X^{'}Y$

	x1	x2	y	yhat	resid
1	6.00	9.00	39.10	37.95	1.15
2	3.00	5.00	19.70	20.72	-1.02
3	7.00	1.00	8.70	13.19	-4.49
4	0.00	2.00	6.50	6.76	-0.26
5	6.00	0.00	13.20	8.54	4.66
6	8.00	5.00	27.60	27.65	-0.05

$$\begin{aligned} SSTO &= Y^{'}Y - \frac{1}{n}Y^{'}JY = Y^{'}[I - \frac{1}{n}J]Y = 774.33333 \\ SSR &= Y^{'}[H - \frac{1}{n}J]Y = 729.98908 \\ SSE &= e^{'}e = Y^{'}[I - H]Y = 44.34425 \end{aligned}$$

where J is a $n \times n$ matrix of 1s and $H = X(X^{'}X)^{-1}X^{'}$ also size $n \times n$.

MSE=$\frac{SSE}{n-p} = 14.78142$ and MSR=$\frac{SSR}{p-1} = 364.99454$

	x0	x1	x2
x0	13.2778	-1.5693	-0.8093
x1	-1.5693	0.3410	-0.0370
x2	-0.8093	-0.0370	0.2711

Table 551: Variance Matrix

The standard errors for the betas are

$$
\begin{aligned}
s(\hat{\beta}_0) &= \sqrt{MSE(X'X)^{-1}_{1,1}} = 3.643872 \\
s(\hat{\beta}_1) &= \sqrt{MSE(X'X)^{-1}_{2,2}} = 0.583938 \\
s(\hat{\beta}_2) &= \sqrt{MSE(X'X)^{-1}_{3,3}} = 0.520714
\end{aligned}
$$

The $\hat{Y}_h = 20.718899$ and $s^2(\hat{Y}_h) = 4.50673$ and the 95% confidence limits for $E[Y_h]$=

	1	2
theci	13.96	27.47

Table 552: The 95 percent confidence interval

	1	2
thepi	11.60	29.83

Table 553: The 95 percent prediction interval

	1	2
theti	-3.18	3.18

Table 554: t-values - lower and upper for .95; n-p

□

Problem 125. Solve for 95% confidence and prediction interval for $X1 = 7$ and $X2 = 2$. The prediction interval is for 1 new observations. Model assumes an intercept.

	y	x1	x2
1	13.5	4.0	9.0
2	-28.4	7.0	2.0
3	-41.3	9.0	1.0
4	7.1	1.0	3.0
5	-19.5	4.0	1.0
6	16.7	3.0	8.0

The determinant of $X^T X = 12968$

	x0	x1	x2
x0	6.00	28.00	24.00
x1	28.00	172.00	90.00
x2	24.00	90.00	160.00

Table 555: $X^{'}X$

	x0	x1	x2
x0	19420.00	-2320.00	-1608.00
x1	-2320.00	384.00	132.00
x2	-1608.00	132.00	248.00

Table 556: $det(X^{'}X)(X^{'}X)^{-1}$

	1	2	3	4	5	6
x0	-4332.00	-36.00	-3068.00	12276.00	8532.00	-404.00
x1	404.00	632.00	1268.00	-1540.00	-652.00	-112.00
x2	1152.00	-188.00	-172.00	-732.00	-832.00	772.00

Table 557: $det(X^{'}X)(X^{'}X)^{-1}X^{'}$

	1	2	3	4	5	6
1	7652.00	800.00	456.00	-472.00	-1564.00	6096.00
2	800.00	4012.00	5464.00	32.00	2304.00	356.00
3	456.00	5464.00	8172.00	-2316.00	1832.00	-640.00
4	-472.00	32.00	-2316.00	8540.00	5384.00	1800.00
5	-1564.00	2304.00	1832.00	5384.00	5092.00	-80.00
6	6096.00	356.00	-640.00	1800.00	-80.00	5436.00

Table 558: Det(X^TX) times the Hat matrix

Solution: The regression table and other pertinent information

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
x1	1	2106.87	2106.87	617.05	0.0001
x2	1	838.00	838.00	245.43	0.0006
Residuals	3	10.24	3.41		

Table 559: Linear regression model output in ANOVA table.

	Estimate	Std. Error	t value	Pr(>|t|)
(Intercept)	-1.2887	2.2612	-0.57	0.6086
x1	-5.0088	0.3180	-15.75	0.0006
x2	4.0032	0.2555	15.67	0.0006

Table 560: Linear regression model output.

The coefficient of multiple determination equals

$$R^2 = \frac{SSR}{SSTO} = 1 - \frac{SSE}{SSTO} = 0.99653,$$

and the F-statistic equals

$$F^* = \frac{MSR}{MSE} = 431.24117$$

	x
intercept	-1.2887
betahat1	-5.0088
betahat2	4.0032

Table 561: The betas=$(X^{'}X)^{-1}X^{'}Y$

	x1	x2	y	yhat	resid
1	4.00	9.00	13.50	14.71	-1.21
2	7.00	2.00	-28.40	-28.34	-0.06
3	9.00	1.00	-41.30	-42.36	1.06
4	1.00	3.00	7.10	5.71	1.39
5	4.00	1.00	-19.50	-17.32	-2.18
6	3.00	8.00	16.70	15.71	0.99

$$
\begin{aligned}
SSTO &= Y^{'}Y - \frac{1}{n}Y^{'}JY = Y^{'}[I - \frac{1}{n}J]Y = 2955.115 \\
SSR &= Y^{'}[H - \frac{1}{n}J]Y = 2944.87176 \\
SSE &= e^{'}e = Y^{'}[I - H]Y = 10.24324
\end{aligned}
$$

where J is a $n \times n$ matrix of 1s and $H = X(X^{'}X)^{-1}X^{'}$ also size $n \times n$.

MSE=$\frac{SSE}{n-p} = 3.41441$ and MSR=$\frac{SSR}{p-1} = 1472.43588$

	x0	x1	x2
x0	5.1132	-0.6108	-0.4234
x1	-0.6108	0.1011	0.0348
x2	-0.4234	0.0348	0.0653

Table 562: Variance Matrix

The standard errors for the betas are

$$
\begin{aligned}
s(\hat{\beta}_0) &= \sqrt{MSE(X'X)^{-1}_{1,1}} = 2.261238 \\
s(\hat{\beta}_1) &= \sqrt{MSE(X'X)^{-1}_{2,2}} = 0.317971 \\
s(\hat{\beta}_2) &= \sqrt{MSE(X'X)^{-1}_{3,3}} = 0.255533
\end{aligned}
$$

The $\hat{Y}_h = -28.343584$ and $s^2(\hat{Y}_h) = 1.056341$ and the 95% confidence limits for $E[Y_h]$=

	1	2
theci	-31.61	-25.07

Table 563: The 95 percent confidence interval

	1	2
thepi	-35.07	-21.61

Table 564: The 95 percent prediction interval

	1	2
theti	-3.18	3.18

Table 565: t-values - lower and upper for .95; n-p

□

Problem 126. Solve for 95% confidence and prediction interval for $X1 = 9$ and $X2 = 3$. The prediction interval is for 3 new observations. Model assumes an intercept.

	y	x1	x2
1	-12.8	9.0	3.0
2	-15.6	9.0	3.0
3	19.1	4.0	5.0
4	4.0	9.0	7.0
5	-12.1	6.0	0.0
6	-9.6	7.0	2.0

The determinant of $X^T X = 3642$

	x0	x1	x2
x0	6.00	44.00	20.00
x1	44.00	344.00	151.00
x2	20.00	151.00	96.00

Table 566: $X^{'}X$

	x0	x1	x2
x0	10223.00	-1204.00	-236.00
x1	-1204.00	176.00	-26.00
x2	-236.00	-26.00	128.00

Table 567: $det(X^{'}X)(X^{'}X)^{-1}$

	1	2	3	4	5	6
x0	-1321.00	-1321.00	4227.00	-2265.00	2999.00	1323.00
x1	302.00	302.00	-630.00	198.00	-148.00	-24.00
x2	-86.00	-86.00	300.00	426.00	-392.00	-162.00

Table 568: $det(X^{'}X)(X^{'}X)^{-1}X^{'}$

	1	2	3	4	5	6
1	1139.00	1139.00	-543.00	795.00	491.00	621.00
2	1139.00	1139.00	-543.00	795.00	491.00	621.00
3	-543.00	-543.00	3207.00	657.00	447.00	417.00
4	795.00	795.00	657.00	2499.00	-1077.00	-27.00
5	491.00	491.00	447.00	-1077.00	2111.00	1179.00
6	621.00	621.00	417.00	-27.00	1179.00	831.00

Table 569: Det(X^TX) times the Hat matrix

Solution: The regression table and other pertinent information

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
x1	1	338.67	338.67	195.58	0.0008
x2	1	561.21	561.21	324.10	0.0004
Residuals	3	5.19	1.73		

Table 570: Linear regression model output in ANOVA table.

	Estimate	Std. Error	t value	Pr(>|t|)
(Intercept)	16.5303	2.2047	7.50	0.0049
x1	-4.8865	0.2893	-16.89	0.0005
x2	4.4412	0.2467	18.00	0.0004

Table 571: Linear regression model output.

The coefficient of multiple determination equals

$$R^2 = \frac{SSR}{SSTO} = 1 - \frac{SSE}{SSTO} = 0.99426,$$

and the F-statistic equals

$$F^* = \frac{MSR}{MSE} = 259.83909$$

	x
intercept	16.5303
betahat1	-4.8865
betahat2	4.4412

Table 572: The betas=$(X^{'}X)^{-1}X^{'}Y$

	x1	x2	y	yhat	resid
1	9.00	3.00	-12.80	-14.12	1.32
2	9.00	3.00	-15.60	-14.12	-1.48
3	4.00	5.00	19.10	19.19	-0.09
4	9.00	7.00	4.00	3.64	0.36
5	6.00	0.00	-12.10	-12.79	0.69
6	7.00	2.00	-9.60	-8.79	-0.81

$$\begin{aligned} SSTO &= Y^{'}Y - \frac{1}{n}Y^{'}JY = Y^{'}[I - \frac{1}{n}J]Y = 905.08 \\ SSR &= Y^{'}[H - \frac{1}{n}J]Y = 899.88514 \\ SSE &= e^{'}e = Y^{'}[I - H]Y = 5.19486 \end{aligned}$$

where J is a $n \times n$ matrix of 1s and $H = X(X^{'}X)^{-1}X^{'}$ also size $n \times n$.

MSE=$\frac{SSE}{n-p} = 1.73162$ and MSR=$\frac{SSR}{p-1} = 449.94257$

	x0	x1	x2
x0	4.8606	-0.5725	-0.1122
x1	-0.5725	0.0837	-0.0124
x2	-0.1122	-0.0124	0.0609

Table 573: Variance Matrix

The standard errors for the betas are

$$s(\hat{\beta}_0) = \sqrt{MSE(X'X)^{-1}_{1,1}} = 2.20468$$
$$s(\hat{\beta}_1) = \sqrt{MSE(X'X)^{-1}_{2,2}} = 0.289276$$
$$s(\hat{\beta}_2) = \sqrt{MSE(X'X)^{-1}_{3,3}} = 0.246696$$

The $\hat{Y}_h = -14.124547$ and $s^2(\hat{Y}_h) = 0.541547$ and the 95% confidence limits for $E[Y_h]$=

	1	2
theci	-16.47	-11.78

Table 574: The 95 percent confidence interval

	1	2
thepi	-17.49	-10.76

Table 575: The 95 percent prediction interval

	1	2
theti	-3.18	3.18

Table 576: t-values - lower and upper for .95; n-p

□

Problem 127. Solve for 95% confidence and prediction interval for $X1 = 1$ and $X2 = 9$. The prediction interval is for 2 new observations. Model assumes an intercept.

	y	x1	x2
1	46.6	3.0	9.0
2	39.8	1.0	9.0
3	40.8	3.0	9.0
4	31.7	5.0	2.0
5	37.9	0.0	8.0
6	25.5	8.0	0.0

The determinant of $X^T X = 5342$

	x0	x1	x2
x0	6.00	20.00	37.00
x1	20.00	108.00	73.00
x2	37.00	73.00	311.00

Table 577: $X^{'}X$

	x0	x1	x2
x0	28259.00	-3519.00	-2536.00
x1	-3519.00	497.00	302.00
x2	-2536.00	302.00	248.00

Table 578: $det(X^{'}X)(X^{'}X)^{-1}$

	1	2	3	4	5	6
x0	-5122.00	1916.00	-5122.00	5592.00	7971.00	107.00
x1	690.00	-304.00	690.00	-430.00	-1103.00	457.00
x2	602.00	-2.00	602.00	-530.00	-552.00	-120.00

Table 579: $det(X^{'}X)(X^{'}X)^{-1}X^{'}$

	1	2	3	4	5	6
1	2366.00	986.00	2366.00	-468.00	-306.00	398.00
2	986.00	1594.00	986.00	392.00	1900.00	-516.00
3	2366.00	986.00	2366.00	-468.00	-306.00	398.00
4	-468.00	392.00	-468.00	2382.00	1352.00	2152.00
5	-306.00	1900.00	-306.00	1352.00	3555.00	-853.00
6	398.00	-516.00	398.00	2152.00	-853.00	3763.00

Table 580: Det(X^TX) times the Hat matrix

Solution: The regression table and other pertinent information

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
x1	1	141.59	141.59	14.29	0.0324
x2	1	104.27	104.27	10.53	0.0477
Residuals	3	29.72	9.91		

Table 581: Linear regression model output in ANOVA table.

	Estimate	Std. Error	t value	Pr(>|t|)
(Intercept)	20.7207	7.2386	2.86	0.0644
x1	0.8285	0.9600	0.86	0.4516
x2	2.2002	0.6781	3.24	0.0477

Table 582: Linear regression model output.

The coefficient of multiple determination equals

$$R^2 = \frac{SSR}{SSTO} = 1 - \frac{SSE}{SSTO} = 0.89217,$$

and the F-statistic equals

$$F^* = \frac{MSR}{MSE} = 12.41072$$

	x
intercept	20.7207
betahat1	0.8285
betahat2	2.2002

Table 583: The betas=$(X^{'}X)^{-1}X^{'}Y$

	x1	x2	y	yhat	resid
1	3.00	9.00	46.60	43.01	3.59
2	1.00	9.00	39.80	41.35	-1.55
3	3.00	9.00	40.80	43.01	-2.21
4	5.00	2.00	31.70	29.26	2.44
5	0.00	8.00	37.90	38.32	-0.42
6	8.00	0.00	25.50	27.35	-1.85

$$\begin{aligned} SSTO &= Y^{'}Y - \frac{1}{n}Y^{'}JY = Y^{'}[I - \frac{1}{n}J]Y = 275.575 \\ SSR &= Y^{'}[H - \frac{1}{n}J]Y = 245.8596 \\ SSE &= e^{'}e = Y^{'}[I - H]Y = 29.7154 \end{aligned}$$

where J is a $n \times n$ matrix of 1s and $H = X(X^{'}X)^{-1}X^{'}$ also size $n \times n$.

MSE=$\frac{SSE}{n-p} = 9.90513$ and MSR=$\frac{SSR}{p-1} = 122.9298$

	x0	x1	x2
x0	52.3978	-6.5249	-4.7023
x1	-6.5249	0.9215	0.5600
x2	-4.7023	0.5600	0.4598

Table 584: Variance Matrix

The standard errors for the betas are

$$
\begin{aligned}
s(\hat{\beta}_0) &= \sqrt{MSE(X'X)^{-1}_{1,1}} = 7.238634 \\
s(\hat{\beta}_1) &= \sqrt{MSE(X'X)^{-1}_{2,2}} = 0.959967 \\
s(\hat{\beta}_2) &= \sqrt{MSE(X'X)^{-1}_{3,3}} = 0.678116
\end{aligned}
$$

The $\hat{Y}_h = 41.350805$ and $s^2(\hat{Y}_h) = 2.955594$ and the 95% confidence limits for $E[Y_h]$=

	1	2
theci	35.88	46.82

Table 585: The 95 percent confidence interval

	1	2
thepi	32.40	50.30

Table 586: The 95 percent prediction interval

	1	2
theti	-3.18	3.18

Table 587: t-values - lower and upper for .95; n-p

□

Problem 128. Solve for 95% confidence and prediction interval for $X1 = 7$ and $X2 = 7$. The prediction interval is for 4 new observations. Model assumes an intercept.

	y	x1	x2
1	-6.7	7.0	0.0
2	25.3	7.0	7.0
3	37.8	2.0	6.0
4	20.7	7.0	8.0
5	26.6	8.0	9.0
6	-0.5	8.0	2.0

The determinant of $X^T X = 9636$

	x0	x1	x2
x0	6.00	39.00	32.00
x1	39.00	279.00	205.00
x2	32.00	205.00	234.00

Table 588: $X^{'}X$

	x0	x1	x2
x0	23261.00	-2566.00	-933.00
x1	-2566.00	380.00	18.00
x2	-933.00	18.00	153.00

Table 589: $det(X^{'}X)(X^{'}X)^{-1}$

	1	2	3	4	5	6
x0	5299.00	-1232.00	12531.00	-2165.00	-5664.00	867.00
x1	94.00	220.00	-1698.00	238.00	636.00	510.00
x2	-807.00	264.00	21.00	417.00	588.00	-483.00

Table 590: $det(X^{'}X)(X^{'}X)^{-1}X^{'}$

	1	2	3	4	5	6
1	5957.00	308.00	645.00	-499.00	-1212.00	4437.00
2	308.00	2156.00	792.00	2420.00	2904.00	1056.00
3	645.00	792.00	9261.00	813.00	-864.00	-1011.00
4	-499.00	2420.00	813.00	2837.00	3492.00	573.00
5	-1212.00	2904.00	-864.00	3492.00	4716.00	600.00
6	4437.00	1056.00	-1011.00	573.00	600.00	3981.00

Table 591: Det(X^TX) times the Hat matrix

Solution: The regression table and other pertinent information

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
x1	1	485.79	485.79	35.69	0.0094
x2	1	948.46	948.46	69.69	0.0036
Residuals	3	40.83	13.61		

Table 592: Linear regression model output in ANOVA table.

	Estimate	Std. Error	t value	Pr(>|t|)
(Intercept)	21.9061	5.7319	3.82	0.0315
x1	-3.9082	0.7326	-5.33	0.0129
x2	3.8807	0.4649	8.35	0.0036

Table 593: Linear regression model output.

The coefficient of multiple determination equals

$R^2 = \frac{SSR}{SSTO} = 1 - \frac{SSE}{SSTO} = 0.97232,$

and the F-statistic equals

$F^* = \frac{MSR}{MSE} = 52.69027$

	x
intercept	21.9061
betahat1	-3.9082
betahat2	3.8807

Table 594: The betas=$(X^{'}X)^{-1}X^{'}Y$

	x1	x2	y	yhat	resid
1	7.00	0.00	-6.70	-5.45	-1.25
2	7.00	7.00	25.30	21.71	3.59
3	2.00	6.00	37.80	37.37	0.43
4	7.00	8.00	20.70	25.59	-4.89
5	8.00	9.00	26.60	25.57	1.03
6	8.00	2.00	-0.50	-1.60	1.10

$$
\begin{aligned}
SSTO &= Y^{'}Y - \frac{1}{n}Y^{'}JY = Y^{'}[I - \frac{1}{n}J]Y = 1475.08 \\
SSR &= Y^{'}[H - \frac{1}{n}J]Y = 1434.24942 \\
SSE &= e^{'}e = Y^{'}[I - H]Y = 40.83058
\end{aligned}
$$

where J is a $n \times n$ matrix of 1s and $H = X(X^{'}X)^{-1}X^{'}$ also size $n \times n$.

MSE=$\frac{SSE}{n-p} = 13.61019$ and MSR=$\frac{SSR}{p-1} = 717.12471$

	x0	x1	x2
x0	32.8546	-3.6243	-1.3178
x1	-3.6243	0.5367	0.0254
x2	-1.3178	0.0254	0.2161

Table 595: Variance Matrix

The standard errors for the betas are

$$
\begin{aligned}
s(\hat{\beta}_0) &= \sqrt{MSE(X'X)^{-1}_{1,1}} = 5.731891 \\
s(\hat{\beta}_1) &= \sqrt{MSE(X'X)^{-1}_{2,2}} = 0.732615 \\
s(\hat{\beta}_2) &= \sqrt{MSE(X'X)^{-1}_{3,3}} = 0.464868
\end{aligned}
$$

The $\hat{Y}_h = 21.713699$ and $s^2(\hat{Y}_h) = 3.045203$ and the 95% confidence limits for $E[Y_h]$=

	1	2
theci	16.16	27.27

Table 596: The 95 percent confidence interval

	1	2
thepi	13.63	29.79

Table 597: The 95 percent prediction interval

	1	2
theti	-3.18	3.18

Table 598: t-values - lower and upper for .95; n-p

□

Problem 129. Solve for 95% confidence and prediction interval for $X1 = 8$ and $X2 = 2$. The prediction interval is for 1 new observations. Model assumes an intercept.

	y	x1	x2
1	36.5	7.0	1.0
2	37.2	8.0	2.0
3	36.7	2.0	9.0
4	35.8	9.0	0.0
5	30.9	7.0	0.0
6	13.6	0.0	3.0

The determinant of $X^T X = 12264$

	x0	x1	x2
x0	6.00	33.00	15.00
x1	33.00	247.00	41.00
x2	15.00	41.00	95.00

Table 599: $X^{'}X$

	x0	x1	x2
x0	21784.00	-2520.00	-2352.00
x1	-2520.00	345.00	249.00
x2	-2352.00	249.00	393.00

Table 600: $det(X^{'}X)(X^{'}X)^{-1}$

	1	2	3	4	5	6
x0	1792.00	-3080.00	-4424.00	-896.00	4144.00	14728.00
x1	144.00	738.00	411.00	585.00	-105.00	-1773.00
x2	-216.00	426.00	1683.00	-111.00	-609.00	-1173.00

Table 601: $det(X^{'}X)(X^{'}X)^{-1}X^{'}$

	1	2	3	4	5	6
1	2584.00	2512.00	136.00	3088.00	2800.00	1144.00
2	2512.00	3676.00	2230.00	3562.00	2086.00	-1802.00
3	136.00	2230.00	11545.00	-725.00	-1547.00	625.00
4	3088.00	3562.00	-725.00	4369.00	3199.00	-1229.00
5	2800.00	2086.00	-1547.00	3199.00	3409.00	2317.00
6	1144.00	-1802.00	625.00	-1229.00	2317.00	11209.00

Table 602: Det(X^TX) times the Hat matrix

Solution: The regression table and other pertinent information

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
x1	1	205.97	205.97	34.04	0.0100
x2	1	199.19	199.19	32.92	0.0105
Residuals	3	18.15	6.05		

Table 603: Linear regression model output in ANOVA table.

	Estimate	Std. Error	t value	Pr(>|t|)
(Intercept)	6.9100	3.2783	2.11	0.1256
x1	3.3740	0.4126	8.18	0.0038
x2	2.5265	0.4403	5.74	0.0105

Table 604: Linear regression model output.

The coefficient of multiple determination equals

$$R^2 = \frac{SSR}{SSTO} = 1 - \frac{SSE}{SSTO} = 0.95712,$$

and the F-statistic equals

$$F^* = \frac{MSR}{MSE} = 33.48073$$

	x
intercept	6.9100
betahat1	3.3740
betahat2	2.5265

Table 605: The betas=$(X^{'}X)^{-1}X^{'}Y$

	x1	x2	y	yhat	resid
1	7.00	1.00	36.50	33.05	3.45
2	8.00	2.00	37.20	38.96	-1.76
3	2.00	9.00	36.70	36.40	0.30
4	9.00	0.00	35.80	37.28	-1.48
5	7.00	0.00	30.90	30.53	0.37
6	0.00	3.00	13.60	14.49	-0.89

$$\begin{aligned} SSTO &= Y^{'}Y - \frac{1}{n}Y^{'}JY = Y^{'}[I - \frac{1}{n}J]Y = 423.30833 \\ SSR &= Y^{'}[H - \frac{1}{n}J]Y = 405.15656 \\ SSE &= e^{'}e = Y^{'}[I - H]Y = 18.15178 \end{aligned}$$

where J is a $n \times n$ matrix of 1s and $H = X(X^{'}X)^{-1}X^{'}$ also size $n \times n$.

MSE=$\frac{SSE}{n-p} = 6.05059$ and MSR=$\frac{SSR}{p-1} = 202.57828$

	x0	x1	x2
x0	10.7474	-1.2433	-1.1604
x1	-1.2433	0.1702	0.1228
x2	-1.1604	0.1228	0.1939

Table 606: Variance Matrix

The standard errors for the betas are

$$
\begin{aligned}
s(\hat{\beta}_0) &= \sqrt{MSE(X'X)^{-1}_{1,1}} = 3.278323 \\
s(\hat{\beta}_1) &= \sqrt{MSE(X'X)^{-1}_{2,2}} = 0.412565 \\
s(\hat{\beta}_2) &= \sqrt{MSE(X'X)^{-1}_{3,3}} = 0.440331
\end{aligned}
$$

The $\hat{Y}_h = 38.955153$ and $s^2(\hat{Y}_h) = 1.813599$ and the 95% confidence limits for $E[Y_h]$=

	1	2
theci	34.67	43.24

Table 607: The 95 percent confidence interval

	1	2
thepi	30.03	47.88

Table 608: The 95 percent prediction interval

	1	2
theti	-3.18	3.18

Table 609: t-values - lower and upper for .95; n-p

□

Problem 130. Solve for 95% confidence and prediction interval for $X1 = 5$ and $X2 = 5$. The prediction interval is for 4 new observations. Model assumes an intercept.

	y	x1	x2
1	22.5	6.0	9.0
2	22.6	5.0	5.0
3	0.3	0.0	1.0
4	15.5	6.0	8.0
5	44.9	9.0	7.0
6	13.7	3.0	0.0

The determinant of $X^T X = 7520$

	x0	x1	x2
x0	6.00	29.00	30.00
x1	29.00	187.00	190.00
x2	30.00	190.00	220.00

Table 610: $X^{'}X$

	x0	x1	x2
x0	5040.00	-680.00	-100.00
x1	-680.00	420.00	-270.00
x2	-100.00	-270.00	281.00

Table 611: $det(X^{'}X)(X^{'}X)^{-1}$

	1	2	3	4	5	6
x0	60.00	1140.00	4940.00	160.00	-1780.00	3000.00
x1	-590.00	70.00	-950.00	-320.00	1210.00	580.00
x2	809.00	-45.00	181.00	528.00	-563.00	-910.00

Table 612: $det(X^{'}X)(X^{'}X)^{-1}X^{'}$

	1	2	3	4	5	6
1	3801.00	1155.00	869.00	2992.00	413.00	-1710.00
2	1155.00	1265.00	1095.00	1200.00	1455.00	1350.00
3	869.00	1095.00	5121.00	688.00	-2343.00	2090.00
4	2992.00	1200.00	688.00	2464.00	976.00	-800.00
5	413.00	1455.00	-2343.00	976.00	5169.00	1850.00
6	-1710.00	1350.00	2090.00	-800.00	1850.00	4740.00

Table 613: Det(X^TX) times the Hat matrix

Solution: The regression table and other pertinent information

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
x1	1	929.27	929.27	34.90	0.0097
x2	1	71.85	71.85	2.70	0.1990
Residuals	3	79.89	26.63		

Table 614: Linear regression model output in ANOVA table.

	Estimate	Std. Error	t value	Pr(>|t|)
(Intercept)	-1.0301	4.2245	-0.24	0.8231
x1	6.0289	1.2195	4.94	0.0159
x2	-1.6386	0.9975	-1.64	0.1990

Table 615: Linear regression model output.

The coefficient of multiple determination equals

$$R^2 = \frac{SSR}{SSTO} = 1 - \frac{SSE}{SSTO} = 0.9261,$$

and the F-statistic equals

$$F^* = \frac{MSR}{MSE} = 18.79798$$

	x
intercept	-1.0301
betahat1	6.0289
betahat2	-1.6386

Table 616: The betas=$(X^{'}X)^{-1}X^{'}Y$

	x1	x2	y	yhat	resid
1	6.00	9.00	22.50	20.40	2.10
2	5.00	5.00	22.60	20.92	1.68
3	0.00	1.00	0.30	-2.67	2.97
4	6.00	8.00	15.50	22.03	-6.53
5	9.00	7.00	44.90	41.76	3.14
6	3.00	0.00	13.70	17.06	-3.36

$$\begin{aligned} SSTO &= Y^{'}Y - \frac{1}{n}Y^{'}JY = Y^{'}[I - \frac{1}{n}J]Y = 1081.00833 \\ SSR &= Y^{'}[H - \frac{1}{n}J]Y = 1001.12291 \\ SSE &= e^{'}e = Y^{'}[I - H]Y = 79.88542 \end{aligned}$$

where J is a $n \times n$ matrix of 1s and $H = X(X^{'}X)^{-1}X^{'}$ also size $n \times n$.

MSE=$\frac{SSE}{n-p} = 26.62847$ and MSR=$\frac{SSR}{p-1} = 500.56145$

	x0	x1	x2
x0	17.8467	-2.4079	-0.3541
x1	-2.4079	1.4872	-0.9561
x2	-0.3541	-0.9561	0.9950

Table 617: Variance Matrix

The standard errors for the betas are

$$\begin{aligned} s(\hat{\beta}_0) &= \sqrt{MSE(X'X)^{-1}_{1,1}} = 4.224541 \\ s(\hat{\beta}_1) &= \sqrt{MSE(X'X)^{-1}_{2,2}} = 1.21952 \\ s(\hat{\beta}_2) &= \sqrt{MSE(X'X)^{-1}_{3,3}} = 0.99751 \end{aligned}$$

The $\hat{Y}_h = 20.921476$ and $s^2(\hat{Y}_h) = 4.479391$ and the 95% confidence limits for $E[Y_h]$=

	1	2
theci	14.19	27.66

Table 618: The 95 percent confidence interval

	1	2
thepi	10.30	31.54

Table 619: The 95 percent prediction interval

	1	2
theti	-3.18	3.18

Table 620: t-values - lower and upper for .95; n-p

□

Problem 131. Calculate $SSR(X_1|X_2)$, $SSR(X_3|X_2)$, $SSR(X_1|X_2, X_3)$, $SSR(X_2, X_3|X_1)$, and $MSR(X_2, X_3|X_1)$. Also, calculate $R^2_{Y1|2}$, $R^2_{Y1|23}$, $R^2_{Y2|1}$.

	y	x1	x2	x3
1	40.7	6.0	6.0	8.0
2	24.1	1.0	6.0	4.0
3	40.2	5.0	3.0	7.0
4	40.9	8.0	3.0	3.0
5	24.6	3.0	9.0	6.0

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
x1	1	253.29	253.29	78.45	0.0716
x2	1	23.42	23.42	7.26	0.2263
x3	1	37.32	37.32	11.56	0.1821
Residuals	1	3.23	3.23		

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
x2	1	179.20	179.20	4.69	0.1626
x3	1	61.70	61.70	1.62	0.3315
Residuals	2	76.36	38.18		

Solution:

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
x1	1	253.29	253.29	11.88	0.0410
Residuals	3	63.97	21.32		

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
x2	1	179.20	179.20	3.89	0.1430
Residuals	3	138.06	46.02		

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
x3	1	20.77	20.77	0.21	0.6778
Residuals	3	296.49	98.83		

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
x1	1	253.29	253.29	12.49	0.0716
x2	1	23.42	23.42	1.16	0.3949
Residuals	2	40.55	20.27		

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
x1	1	253.29	253.29	11.39	0.0777
x3	1	19.50	19.50	0.88	0.4480
Residuals	2	44.48	22.24		

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
x2	1	179.20	179.20	4.69	0.1626
x3	1	61.70	61.70	1.62	0.3315
Residuals	2	76.36	38.18		

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
x1	1	253.29	253.29	78.45	0.0716
x2	1	23.42	23.42	7.26	0.2263
x3	1	37.32	37.32	11.56	0.1821
Residuals	1	3.23	3.23		

For the general calculations:

$$\begin{aligned} SSTO &= Y'Y - \frac{1}{n}Y'JY = Y'[I - \frac{1}{n}J]Y \\ SSR &= Y'[H - \frac{1}{n}J]Y \\ SSE &= e'e = Y'[I - H]Y \end{aligned}$$

where J is a $n \times n$ matrix of 1s and $H = X(X'X)^{-1}X'$ also size $n \times n$.

$$\begin{aligned} SSR(X_1|X_2) &= SSE(X_2) - SSE(X_2, X_1) = 97.5124 \\ SSR(X_2|X_1) &= SSE(X_1) - SSE(X_2, X_1) = 23.4247 \\ SSR(X_3|X_2) &= SSE(X_2) - SSE(X_2, X_3) = 61.7038 \end{aligned}$$

$$
\begin{aligned}
SSR(X_1|X_2,X_3) &= SSE(X_2,X_3) - SSE(X_1,X_2,X_3) = 73.1277 \\
SSR(X_3|X_1,X_2) &= SSE(X_1,X_2) - SSE(X_1,X_2,X_3) = 37.3191 \\
MSR(X_2,X_3|X_1) &= (SSR(X_2|X_1) + SSR(X_3|X_1,X_2))/2 = 30.3719 \\
R^2_{Y1|2} &= \frac{SSR(X_1|X_2)}{SSE(X_2)} = 0.7063 \\
R^2_{Y2|1} &= \frac{SSR(X_2|X_1)}{SSE(X_1)} = 0.3662 \\
R^2_{Y1|23} &= \frac{SSR(X_1|X_2,X_3)}{SSE(X_2,X_3)} = 0.9577
\end{aligned}
$$

□

Problem 132. Calculate $SSR(X_1|X_2)$, $SSR(X_3|X_2)$, $SSR(X_1|X_2, X_3)$, $SSR(X_2, X_3|X_1)$, and $MSR(X_2, X_3|X_1)$. Also, calculate $R^2_{Y1|2}$, $R^2_{Y1|23}$, $R^2_{Y2|1}$.

	y	x1	x2	x3
1	-17.0	2.0	3.0	7.0
2	-0.0	1.0	0.0	7.0
3	-39.5	7.0	6.0	1.0
4	-43.4	1.0	9.0	5.0
5	-28.7	5.0	6.0	7.0

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
x1	1	256.09	256.09	38.01	0.1024
x2	1	955.17	955.17	141.79	0.0533
x3	1	30.91	30.91	4.59	0.2780
Residuals	1	6.74	6.74		

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
x2	1	1165.20	1165.20	227.43	0.0044
x3	1	73.46	73.46	14.34	0.0632
Residuals	2	10.25	5.12		

Solution:

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
x1	1	256.09	256.09	0.77	0.4438
Residuals	3	992.82	330.94		

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
x2	1	1165.20	1165.20	41.76	0.0075
Residuals	3	83.70	27.90		

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
x3	1	512.26	512.26	2.09	0.2444
Residuals	3	736.65	245.55		

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
x1	1	256.09	256.09	13.60	0.0663
x2	1	955.17	955.17	50.74	0.0191
Residuals	2	37.65	18.83		

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
x1	1	256.09	256.09	0.70	0.4915
x3	1	258.39	258.39	0.70	0.4898
Residuals	2	734.43	367.21		

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
x2	1	1165.20	1165.20	227.43	0.0044
x3	1	73.46	73.46	14.34	0.0632
Residuals	2	10.25	5.12		

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
x1	1	256.09	256.09	38.01	0.1024
x2	1	955.17	955.17	141.79	0.0533
x3	1	30.91	30.91	4.59	0.2780
Residuals	1	6.74	6.74		

For the general calculations:

$$\begin{aligned} SSTO &= Y'Y - \frac{1}{n}Y'JY = Y'[I - \frac{1}{n}J]Y \\ SSR &= Y'[H - \frac{1}{n}J]Y \\ SSE &= e'e = Y'[I - H]Y \end{aligned}$$

where J is a $n \times n$ matrix of 1s and $H = X(X'X)^{-1}X'$ also size $n \times n$.

$$\begin{aligned} SSR(X_1|X_2) &= SSE(X_2) - SSE(X_2, X_1) = 46.0521 \\ SSR(X_2|X_1) &= SSE(X_1) - SSE(X_2, X_1) = 955.1676 \\ SSR(X_3|X_2) &= SSE(X_2) - SSE(X_2, X_3) = 73.4564 \end{aligned}$$

$$
\begin{aligned}
SSR(X_1|X_2,X_3) &= SSE(X_2,X_3) - SSE(X_1,X_2,X_3) = 3.5101 \\
SSR(X_3|X_1,X_2) &= SSE(X_1,X_2) - SSE(X_1,X_2,X_3) = 30.9143 \\
MSR(X_2,X_3|X_1) &= (SSR(X_2|X_1) + SSR(X_3|X_1,X_2))/2 = 493.041 \\
R^2_{Y1|2} &= \frac{SSR(X_1|X_2)}{SSE(X_2)} = 0.5502 \\
R^2_{Y2|1} &= \frac{SSR(X_2|X_1)}{SSE(X_1)} = 0.9621 \\
R^2_{Y1|23} &= \frac{SSR(X_1|X_2,X_3)}{SSE(X_2,X_3)} = 0.3426
\end{aligned}
$$

□

Problem 133. Calculate $SSR(X_1|X_2)$, $SSR(X_3|X_2)$, $SSR(X_1|X_2, X_3)$, $SSR(X_2, X_3|X_1)$, and $MSR(X_2, X_3|X_1)$. Also, calculate $R^2_{Y1|2}$, $R^2_{Y1|23}$, $R^2_{Y2|1}$.

	y	x1	x2	x3
1	-59.9	8.0	5.0	6.0
2	-35.1	1.0	0.0	4.0
3	-29.6	2.0	8.0	5.0
4	-26.6	2.0	9.0	5.0
5	-24.3	4.0	7.0	0.0

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
x1	1	518.58	518.58	543.03	0.0273
x2	1	146.53	146.53	153.44	0.0513
x3	1	168.11	168.11	176.04	0.0479
Residuals	1	0.95	0.95		

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
x2	1	102.33	102.33	0.45	0.5732
x3	1	272.42	272.42	1.19	0.3899
Residuals	2	459.43	229.72		

Solution:

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
x1	1	518.58	518.58	4.93	0.1130
Residuals	3	315.60	105.20		

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
x2	1	102.33	102.33	0.42	0.5633
Residuals	3	731.85	243.95		

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
x3	1	282.25	282.25	1.53	0.3036
Residuals	3	551.93	183.98		

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
x1	1	518.58	518.58	6.13	0.1316
x2	1	146.53	146.53	1.73	0.3186
Residuals	2	169.07	84.53		

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
x1	1	518.58	518.58	7.75	0.1085
x3	1	181.75	181.75	2.72	0.2411
Residuals	2	133.85	66.92		

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
x2	1	102.33	102.33	0.45	0.5732
x3	1	272.42	272.42	1.19	0.3899
Residuals	2	459.43	229.72		

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
x1	1	518.58	518.58	543.03	0.0273
x2	1	146.53	146.53	153.44	0.0513
x3	1	168.11	168.11	176.04	0.0479
Residuals	1	0.95	0.95		

For the general calculations:

$$\begin{aligned} SSTO &= Y^{'}Y - \frac{1}{n}Y^{'}JY = Y^{'}[I - \frac{1}{n}J]Y \\ SSR &= Y^{'}[H - \frac{1}{n}J]Y \\ SSE &= e^{'}e = Y^{'}[I - H]Y \end{aligned}$$

where J is a $n \times n$ matrix of 1s and $H = X(X^{'}X)^{-1}X^{'}$ also size $n \times n$.

$$\begin{aligned} SSR(X_1|X_2) &= SSE(X_2) - SSE(X_2, X_1) = 562.7828 \\ SSR(X_2|X_1) &= SSE(X_1) - SSE(X_2, X_1) = 146.5291 \\ SSR(X_3|X_2) &= SSE(X_2) - SSE(X_2, X_3) = 272.4153 \end{aligned}$$

$$
\begin{aligned}
SSR(X_1|X_2,X_3) &= SSE(X_2,X_3) - SSE(X_1,X_2,X_3) = 458.4788 \\
SSR(X_3|X_1,X_2) &= SSE(X_1,X_2) - SSE(X_1,X_2,X_3) = 168.1113 \\
MSR(X_2,X_3|X_1) &= (SSR(X_2|X_1) + SSR(X_3|X_1,X_2))/2 = 157.3202 \\
R^2_{Y1|2} &= \frac{SSR(X_1|X_2)}{SSE(X_2)} = 0.769 \\
R^2_{Y2|1} &= \frac{SSR(X_2|X_1)}{SSE(X_1)} = 0.4643 \\
R^2_{Y1|23} &= \frac{SSR(X_1|X_2,X_3)}{SSE(X_2,X_3)} = 0.9979
\end{aligned}
$$

□

Problem 134. Calculate $SSR(X_1|X_2)$, $SSR(X_3|X_2)$, $SSR(X_1|X_2, X_3)$, $SSR(X_2, X_3|X_1)$, and $MSR(X_2, X_3|X_1)$. Also, calculate $R^2_{Y1|2}$, $R^2_{Y1|23}$, $R^2_{Y2|1}$.

	y	x1	x2	x3
1	2.2	1.0	2.0	4.0
2	19.6	8.0	3.0	6.0
3	6.5	4.0	7.0	6.0
4	10.4	7.0	6.0	9.0
5	7.9	0.0	0.0	8.0

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
x1	1	101.96	101.96	5.09	0.2657
x2	1	44.13	44.13	2.20	0.3775
x3	1	1.37	1.37	0.07	0.8371
Residuals	1	20.05	20.05		

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
x2	1	0.34	0.34	0.00	0.9532
x3	1	13.77	13.77	0.18	0.7130
Residuals	2	153.41	76.70		

Solution:

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
x1	1	101.96	101.96	4.67	0.1196
Residuals	3	65.55	21.85		

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
x2	1	0.34	0.34	0.01	0.9430
Residuals	3	167.17	55.72		

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
x3	1	14.10	14.10	0.28	0.6358
Residuals	3	153.41	51.14		

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
x1	1	101.96	101.96	9.52	0.0909
x2	1	44.13	44.13	4.12	0.1795
Residuals	2	21.42	10.71		

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
x1	1	101.96	101.96	3.18	0.2163
x3	1	1.52	1.52	0.05	0.8479
Residuals	2	64.03	32.02		

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
x2	1	0.34	0.34	0.00	0.9532
x3	1	13.77	13.77	0.18	0.7130
Residuals	2	153.41	76.70		

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
x1	1	101.96	101.96	5.09	0.2657
x2	1	44.13	44.13	2.20	0.3775
x3	1	1.37	1.37	0.07	0.8371
Residuals	1	20.05	20.05		

For the general calculations:

$$
\begin{aligned}
SSTO &= Y'Y - \frac{1}{n}Y'JY = Y'[I - \frac{1}{n}J]Y \\
SSR &= Y'[H - \frac{1}{n}J]Y \\
SSE &= e'e = Y'[I - H]Y
\end{aligned}
$$

where J is a $n \times n$ matrix of 1s and $H = X(X'X)^{-1}X'$ also size $n \times n$.

$$
\begin{aligned}
SSR(X_1|X_2) &= SSE(X_2) - SSE(X_2, X_1) = 145.7555 \\
SSR(X_2|X_1) &= SSE(X_1) - SSE(X_2, X_1) = 44.1323 \\
SSR(X_3|X_2) &= SSE(X_2) - SSE(X_2, X_3) = 13.7667
\end{aligned}
$$

$$
\begin{aligned}
SSR(X_1|X_2,X_3) &= SSE(X_2,X_3) - SSE(X_1,X_2,X_3) = 133.3602 \\
SSR(X_3|X_1,X_2) &= SSE(X_1,X_2) - SSE(X_1,X_2,X_3) = 1.3713 \\
MSR(X_2,X_3|X_1) &= (SSR(X_2|X_1) + SSR(X_3|X_1,X_2))/2 = 22.7518 \\
R^2_{Y1|2} &= \frac{SSR(X_1|X_2)}{SSE(X_2)} = 0.8719 \\
R^2_{Y2|1} &= \frac{SSR(X_2|X_1)}{SSE(X_1)} = 0.6733 \\
R^2_{Y1|23} &= \frac{SSR(X_1|X_2,X_3)}{SSE(X_2,X_3)} = 0.8693
\end{aligned}
$$

□

Problem 135. Calculate $SSR(X_1|X_2)$, $SSR(X_3|X_2)$, $SSR(X_1|X_2, X_3)$, $SSR(X_2, X_3|X_1)$, and $MSR(X_2, X_3|X_1)$. Also, calculate $R^2_{Y1|2}$, $R^2_{Y1|23}$, $R^2_{Y2|1}$.

	y	x1	x2	x3
1	32.4	4.0	1.0	5.0
2	13.1	2.0	4.0	0.0
3	88.4	7.0	9.0	8.0
4	39.5	3.0	0.0	8.0
5	24.4	2.0	3.0	3.0

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
x1	1	3070.77	3070.77	11341.80	0.0060
x2	1	10.33	10.33	38.17	0.1022
x3	1	285.20	285.20	1053.37	0.0196
Residuals	1	0.27	0.27		

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
x2	1	1605.81	1605.81	129.00	0.0077
x3	1	1735.86	1735.86	139.45	0.0071
Residuals	2	24.90	12.45		

Solution:

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
x1	1	3070.77	3070.77	31.14	0.0114
Residuals	3	295.80	98.60		

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
x2	1	1605.81	1605.81	2.74	0.1967
Residuals	3	1760.76	586.92		

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
x3	1	2039.66	2039.66	4.61	0.1210
Residuals	3	1326.91	442.30		

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
x1	1	3070.77	3070.77	21.51	0.0435
x2	1	10.33	10.33	0.07	0.8131
Residuals	2	285.47	142.73		

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
x1	1	3070.77	3070.77	29.92	0.0318
x3	1	90.57	90.57	0.88	0.4467
Residuals	2	205.23	102.62		

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
x2	1	1605.81	1605.81	129.00	0.0077
x3	1	1735.86	1735.86	139.45	0.0071
Residuals	2	24.90	12.45		

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
x1	1	3070.77	3070.77	11341.80	0.0060
x2	1	10.33	10.33	38.17	0.1022
x3	1	285.20	285.20	1053.37	0.0196
Residuals	1	0.27	0.27		

For the general calculations:

$$\begin{aligned} SSTO &= Y'Y - \frac{1}{n}Y'JY = Y'[I - \frac{1}{n}J]Y \\ SSR &= Y'[H - \frac{1}{n}J]Y \\ SSE &= e'e = Y'[I - H]Y \end{aligned}$$

where J is a $n \times n$ matrix of 1s and $H = X(X'X)^{-1}X'$ also size $n \times n$.

$$\begin{aligned} SSR(X_1|X_2) &= SSE(X_2) - SSE(X_2, X_1) = 1475.2915 \\ SSR(X_2|X_1) &= SSE(X_1) - SSE(X_2, X_1) = 10.3345 \\ SSR(X_3|X_2) &= SSE(X_2) - SSE(X_2, X_3) = 1735.8636 \end{aligned}$$

$$
\begin{aligned}
SSR(X_1|X_2, X_3) &= SSE(X_2, X_3) - SSE(X_1, X_2, X_3) = 24.6253 \\
SSR(X_3|X_1, X_2) &= SSE(X_1, X_2) - SSE(X_1, X_2, X_3) = 285.1974 \\
MSR(X_2, X_3|X_1) &= (SSR(X_2|X_1) + SSR(X_3|X_1, X_2))/2 = 147.766 \\
R^2_{Y1|2} &= \frac{SSR(X_1|X_2)}{SSE(X_2)} = 0.8379 \\
R^2_{Y2|1} &= \frac{SSR(X_2|X_1)}{SSE(X_1)} = 0.0349 \\
R^2_{Y1|23} &= \frac{SSR(X_1|X_2, X_3)}{SSE(X_2, X_3)} = 0.9891
\end{aligned}
$$

□

Problem 136. Calculate $SSR(X_1|X_2)$, $SSR(X_3|X_2)$, $SSR(X_1|X_2, X_3)$, $SSR(X_2, X_3|X_1)$, and $MSR(X_2, X_3|X_1)$. Also, calculate $R^2_{Y1|2}$, $R^2_{Y1|23}$, $R^2_{Y2|1}$.

	y	x1	x2	x3
1	19.2	4.0	1.0	0.0
2	27.7	5.0	1.0	3.0
3	15.5	1.0	3.0	3.0
4	23.1	5.0	4.0	9.0
5	16.5	6.0	2.0	3.0

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
x1	1	19.30	19.30	12.87	0.1731
x2	1	1.47	1.47	0.98	0.5029
x3	1	78.97	78.97	52.65	0.0872
Residuals	1	1.50	1.50		

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
x2	1	4.61	4.61	0.52	0.5458
x3	1	78.88	78.88	8.89	0.0965
Residuals	2	17.74	8.87		

Solution:

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
x1	1	19.30	19.30	0.71	0.4623
Residuals	3	81.94	27.31		

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
x2	1	4.61	4.61	0.14	0.7303
Residuals	3	96.63	32.21		

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
x3	1	9.08	9.08	0.30	0.6246
Residuals	3	92.16	30.72		

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
x1	1	19.30	19.30	0.48	0.5602
x2	1	1.47	1.47	0.04	0.8659
Residuals	2	80.47	40.23		

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
x1	1	19.30	19.30	0.50	0.5533
x3	1	4.51	4.51	0.12	0.7654
Residuals	2	77.43	38.72		

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
x2	1	4.61	4.61	0.52	0.5458
x3	1	78.88	78.88	8.89	0.0965
Residuals	2	17.74	8.87		

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
x1	1	19.30	19.30	12.87	0.1731
x2	1	1.47	1.47	0.98	0.5029
x3	1	78.97	78.97	52.65	0.0872
Residuals	1	1.50	1.50		

For the general calculations:

$$\begin{aligned} SSTO &= Y'Y - \frac{1}{n}Y'JY = Y'[I - \frac{1}{n}J]Y \\ SSR &= Y'[H - \frac{1}{n}J]Y \\ SSE &= e'e = Y'[I - H]Y \end{aligned}$$

where J is a $n \times n$ matrix of 1s and $H = X(X'X)^{-1}X'$ also size $n \times n$.

$$\begin{aligned} SSR(X_1|X_2) &= SSE(X_2) - SSE(X_2, X_1) = 16.1592 \\ SSR(X_2|X_1) &= SSE(X_1) - SSE(X_2, X_1) = 1.473 \\ SSR(X_3|X_2) &= SSE(X_2) - SSE(X_2, X_3) = 78.8832 \end{aligned}$$

$$SSR(X_1|X_2,X_3) = SSE(X_2,X_3) - SSE(X_1,X_2,X_3) = 16.245$$

$$SSR(X_3|X_1,X_2) = SSE(X_1,X_2) - SSE(X_1,X_2,X_3) = 78.969$$

$$MSR(X_2,X_3|X_1) = (SSR(X_2|X_1) + SSR(X_3|X_1,X_2))/2 = 40.221$$

$$R^2_{Y1|2} = \frac{SSR(X_1|X_2)}{SSE(X_2)} = 0.1672$$

$$R^2_{Y2|1} = \frac{SSR(X_2|X_1)}{SSE(X_1)} = 0.018$$

$$R^2_{Y1|23} = \frac{SSR(X_1|X_2,X_3)}{SSE(X_2,X_3)} = 0.9155$$

□

Problem 137. Calculate $SSR(X_1|X_2)$, $SSR(X_3|X_2)$, $SSR(X_1|X_2, X_3)$, $SSR(X_2, X_3|X_1)$, and $MSR(X_2, X_3|X_1)$. Also, calculate $R^2_{Y1|2}$, $R^2_{Y1|23}$, $R^2_{Y2|1}$.

	y	x1	x2	x3
1	24.2	7.0	1.0	2.0
2	22.8	4.0	5.0	9.0
3	43.2	4.0	9.0	8.0
4	45.1	7.0	6.0	4.0
5	14.5	0.0	3.0	0.0

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
x1	1	243.87	243.87	4619.04	0.0094
x2	1	415.84	415.84	7876.27	0.0072
x3	1	68.22	68.22	1292.06	0.0177
Residuals	1	0.05	0.05		

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
x2	1	404.85	404.85	2.59	0.2490
x3	1	10.23	10.23	0.07	0.8221
Residuals	2	312.89	156.45		

Solution:

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
x1	1	243.87	243.87	1.51	0.3066
Residuals	3	484.10	161.37		

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
x2	1	404.85	404.85	3.76	0.1479
Residuals	3	323.12	107.71		

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
x3	1	138.41	138.41	0.70	0.4629
Residuals	3	589.56	196.52		

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
x1	1	243.87	243.87	7.14	0.1161
x2	1	415.84	415.84	12.18	0.0732
Residuals	2	68.27	34.13		

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
x1	1	243.87	243.87	1.20	0.3874
x3	1	78.16	78.16	0.39	0.5982
Residuals	2	405.94	202.97		

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
x2	1	404.85	404.85	2.59	0.2490
x3	1	10.23	10.23	0.07	0.8221
Residuals	2	312.89	156.45		

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
x1	1	243.87	243.87	4619.04	0.0094
x2	1	415.84	415.84	7876.27	0.0072
x3	1	68.22	68.22	1292.06	0.0177
Residuals	1	0.05	0.05		

For the general calculations:

$$\begin{aligned} SSTO &= Y'Y - \frac{1}{n}Y'JY = Y'[I - \frac{1}{n}J]Y \\ SSR &= Y'[H - \frac{1}{n}J]Y \\ SSE &= e'e = Y'[I - H]Y \end{aligned}$$

where J is a $n \times n$ matrix of 1s and $H = X(X'X)^{-1}X'$ also size $n \times n$.

$$\begin{aligned} SSR(X_1|X_2) &= SSE(X_2) - SSE(X_2, X_1) = 254.8492 \\ SSR(X_2|X_1) &= SSE(X_1) - SSE(X_2, X_1) = 415.8362 \\ SSR(X_3|X_2) &= SSE(X_2) - SSE(X_2, X_3) = 10.2269 \end{aligned}$$

$$
\begin{aligned}
SSR(X_1|X_2,X_3) &= SSE(X_2,X_3) - SSE(X_1,X_2,X_3) = 312.8379 \\
SSR(X_3|X_1,X_2) &= SSE(X_1,X_2) - SSE(X_1,X_2,X_3) = 68.2155 \\
MSR(X_2,X_3|X_1) &= (SSR(X_2|X_1) + SSR(X_3|X_1,X_2))/2 = 242.0259 \\
R^2_{Y1|2} &= \frac{SSR(X_1|X_2)}{SSE(X_2)} = 0.7887 \\
R^2_{Y2|1} &= \frac{SSR(X_2|X_1)}{SSE(X_1)} = 0.859 \\
R^2_{Y1|23} &= \frac{SSR(X_1|X_2,X_3)}{SSE(X_2,X_3)} = 0.9998
\end{aligned}
$$

□

Problem 138. Calculate $SSR(X_1|X_2)$, $SSR(X_3|X_2)$, $SSR(X_1|X_2, X_3)$, $SSR(X_2, X_3|X_1)$, and $MSR(X_2, X_3|X_1)$. Also, calculate $R^2_{Y1|2}$, $R^2_{Y1|23}$, $R^2_{Y2|1}$.

	y	x1	x2	x3
1	-1.2	4.0	0.0	5.0
2	-17.9	1.0	7.0	3.0
3	-20.9	0.0	9.0	4.0
4	-7.6	3.0	0.0	5.0
5	-26.8	7.0	4.0	1.0

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
x1	1	3.27	3.27	0.10	0.8062
x2	1	381.40	381.40	11.52	0.1824
x3	1	9.83	9.83	0.30	0.6824
Residuals	1	33.09	33.09		

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
x2	1	229.23	229.23	13.50	0.0668
x3	1	164.39	164.39	9.68	0.0896
Residuals	2	33.97	16.99		

Solution:

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
x1	1	3.27	3.27	0.02	0.8888
Residuals	3	424.32	141.44		

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
x2	1	229.23	229.23	3.47	0.1595
Residuals	3	198.36	66.12		

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
x3	1	318.65	318.65	8.78	0.0594
Residuals	3	108.94	36.31		

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
x1	1	3.27	3.27	0.15	0.7340
x2	1	381.40	381.40	17.77	0.0519
Residuals	2	42.92	21.46		

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
x1	1	3.27	3.27	0.14	0.7403
x3	1	379.16	379.16	16.79	0.0547
Residuals	2	45.16	22.58		

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
x2	1	229.23	229.23	13.50	0.0668
x3	1	164.39	164.39	9.68	0.0896
Residuals	2	33.97	16.99		

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
x1	1	3.27	3.27	0.10	0.8062
x2	1	381.40	381.40	11.52	0.1824
x3	1	9.83	9.83	0.30	0.6824
Residuals	1	33.09	33.09		

For the general calculations:

$$\begin{aligned} SSTO &= Y'Y - \frac{1}{n}Y'JY = Y'[I - \frac{1}{n}J]Y \\ SSR &= Y'[H - \frac{1}{n}J]Y \\ SSE &= e'e = Y'[I - H]Y \end{aligned}$$

where J is a $n \times n$ matrix of 1s and $H = X(X'X)^{-1}X'$ also size $n \times n$.

$$\begin{aligned} SSR(X_1|X_2) &= SSE(X_2) - SSE(X_2, X_1) = 155.4396 \\ SSR(X_2|X_1) &= SSE(X_1) - SSE(X_2, X_1) = 381.3999 \\ SSR(X_3|X_2) &= SSE(X_2) - SSE(X_2, X_3) = 164.3891 \end{aligned}$$

$$\begin{aligned}
SSR(X_1|X_2,X_3) &= SSE(X_2,X_3) - SSE(X_1,X_2,X_3) = 0.8768 \\
SSR(X_3|X_1,X_2) &= SSE(X_1,X_2) - SSE(X_1,X_2,X_3) = 9.8262 \\
MSR(X_2,X_3|X_1) &= (SSR(X_2|X_1) + SSR(X_3|X_1,X_2))/2 = 195.6131 \\
R^2_{Y1|2} &= \frac{SSR(X_1|X_2)}{SSE(X_2)} = 0.7836 \\
R^2_{Y2|1} &= \frac{SSR(X_2|X_1)}{SSE(X_1)} = 0.8988 \\
R^2_{Y1|23} &= \frac{SSR(X_1|X_2,X_3)}{SSE(X_2,X_3)} = 0.0258
\end{aligned}$$

□

Problem 139. Calculate $SSR(X_1|X_2)$, $SSR(X_3|X_2)$, $SSR(X_1|X_2, X_3)$, $SSR(X_2, X_3|X_1)$, and $MSR(X_2, X_3|X_1)$. Also, calculate $R^2_{Y1|2}$, $R^2_{Y1|23}$, $R^2_{Y2|1}$.

	y	x1	x2	x3
1	41.8	7.0	5.0	4.0
2	45.4	4.0	8.0	8.0
3	27.7	4.0	7.0	4.0
4	39.0	6.0	9.0	5.0
5	25.1	5.0	0.0	2.0

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
x1	1	27.60	27.60	23.53	0.1294
x2	1	135.34	135.34	115.36	0.0591
x3	1	154.39	154.39	131.60	0.0554
Residuals	1	1.17	1.17		

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
x2	1	122.54	122.54	2.19	0.2767
x3	1	84.27	84.27	1.51	0.3442
Residuals	2	111.68	55.84		

Solution:

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
x1	1	27.60	27.60	0.28	0.6307
Residuals	3	290.90	96.97		

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
x2	1	122.54	122.54	1.88	0.2643
Residuals	3	195.96	65.32		

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
x3	1	206.72	206.72	5.55	0.0998
Residuals	3	111.78	37.26		

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
x1	1	27.60	27.60	0.35	0.6118
x2	1	135.34	135.34	1.74	0.3179
Residuals	2	155.56	77.78		

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
x1	1	27.60	27.60	10.01	0.0870
x3	1	285.38	285.38	103.52	0.0095
Residuals	2	5.51	2.76		

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
x2	1	122.54	122.54	2.19	0.2767
x3	1	84.27	84.27	1.51	0.3442
Residuals	2	111.68	55.84		

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
x1	1	27.60	27.60	23.53	0.1294
x2	1	135.34	135.34	115.36	0.0591
x3	1	154.39	154.39	131.60	0.0554
Residuals	1	1.17	1.17		

For the general calculations:

$$\begin{aligned} SSTO &= Y'Y - \frac{1}{n}Y'JY = Y'[I - \frac{1}{n}J]Y \\ SSR &= Y'[H - \frac{1}{n}J]Y \\ SSE &= e'e = Y'[I - H]Y \end{aligned}$$

where J is a $n \times n$ matrix of 1s and $H = X(X'X)^{-1}X'$ also size $n \times n$.

$$\begin{aligned} SSR(X_1|X_2) &= SSE(X_2) - SSE(X_2, X_1) = 40.3946 \\ SSR(X_2|X_1) &= SSE(X_1) - SSE(X_2, X_1) = 135.3367 \\ SSR(X_3|X_2) &= SSE(X_2) - SSE(X_2, X_3) = 84.2734 \end{aligned}$$

$$
\begin{aligned}
SSR(X_1|X_2, X_3) &= SSE(X_2, X_3) - SSE(X_1, X_2, X_3) = 110.5099 \\
SSR(X_3|X_1, X_2) &= SSE(X_1, X_2) - SSE(X_1, X_2, X_3) = 154.3887 \\
MSR(X_2, X_3|X_1) &= (SSR(X_2|X_1) + SSR(X_3|X_1, X_2))/2 = 144.8627 \\
R^2_{Y1|2} &= \frac{SSR(X_1|X_2)}{SSE(X_2)} = 0.2061 \\
R^2_{Y2|1} &= \frac{SSR(X_2|X_1)}{SSE(X_1)} = 0.4652 \\
R^2_{Y1|23} &= \frac{SSR(X_1|X_2, X_3)}{SSE(X_2, X_3)} = 0.9895
\end{aligned}
$$

□

Problem 140. Calculate $SSR(X_1|X_2)$, $SSR(X_3|X_2)$, $SSR(X_1|X_2, X_3)$, $SSR(X_2, X_3|X_1)$, and $MSR(X_2, X_3|X_1)$. Also, calculate $R^2_{Y1|2}$, $R^2_{Y1|23}$, $R^2_{Y2|1}$.

	y	x1	x2	x3
1	-46.4	2.0	7.0	4.0
2	-11.5	2.0	1.0	2.0
3	-26.6	7.0	3.0	8.0
4	-55.6	0.0	8.0	6.0
5	-28.8	5.0	5.0	5.0

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
x1	1	289.82	289.82	18.08	0.1470
x2	1	861.70	861.70	53.77	0.0863
x3	1	40.58	40.58	2.53	0.3572
Residuals	1	16.03	16.03		

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
x2	1	1149.57	1149.57	46.59	0.0208
x3	1	9.22	9.22	0.37	0.6033
Residuals	2	49.34	24.67		

Solution:

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
x1	1	289.82	289.82	0.95	0.4023
Residuals	3	918.31	306.10		

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
x2	1	1149.57	1149.57	58.89	0.0046
Residuals	3	58.56	19.52		

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
x3	1	148.51	148.51	0.42	0.5629
Residuals	3	1059.62	353.21		

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
x1	1	289.82	289.82	10.24	0.0853
x2	1	861.70	861.70	30.45	0.0313
Residuals	2	56.61	28.30		

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
x1	1	289.82	289.82	1.90	0.3017
x3	1	613.74	613.74	4.03	0.1825
Residuals	2	304.57	152.28		

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
x2	1	1149.57	1149.57	46.59	0.0208
x3	1	9.22	9.22	0.37	0.6033
Residuals	2	49.34	24.67		

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
x1	1	289.82	289.82	18.08	0.1470
x2	1	861.70	861.70	53.77	0.0863
x3	1	40.58	40.58	2.53	0.3572
Residuals	1	16.03	16.03		

For the general calculations:

$$
\begin{aligned}
SSTO &= Y^{'}Y - \frac{1}{n}Y^{'}JY = Y^{'}[I - \frac{1}{n}J]Y \\
SSR &= Y^{'}[H - \frac{1}{n}J]Y \\
SSE &= e^{'}e = Y^{'}[I - H]Y
\end{aligned}
$$

where J is a $n \times n$ matrix of 1s and $H = X(X^{'}X)^{-1}X^{'}$ also size $n \times n$.

$$
\begin{aligned}
SSR(X_1|X_2) &= SSE(X_2) - SSE(X_2, X_1) = 1.9522 \\
SSR(X_2|X_1) &= SSE(X_1) - SSE(X_2, X_1) = 861.701 \\
SSR(X_3|X_2) &= SSE(X_2) - SSE(X_2, X_3) = 9.2154
\end{aligned}
$$

$$
\begin{aligned}
SSR(X_1|X_2,X_3) &= SSE(X_2,X_3) - SSE(X_1,X_2,X_3) = 33.3162 \\
SSR(X_3|X_1,X_2) &= SSE(X_1,X_2) - SSE(X_1,X_2,X_3) = 40.5794 \\
MSR(X_2,X_3|X_1) &= (SSR(X_2|X_1) + SSR(X_3|X_1,X_2))/2 = 451.1402 \\
R^2_{Y1|2} &= \frac{SSR(X_1|X_2)}{SSE(X_2)} = 0.0333 \\
R^2_{Y2|1} &= \frac{SSR(X_2|X_1)}{SSE(X_1)} = 0.9384 \\
R^2_{Y1|23} &= \frac{SSR(X_1|X_2,X_3)}{SSE(X_2,X_3)} = 0.6752
\end{aligned}
$$

□

3.7. Multiple Linear Regression - SSR(X's given other X's), etc. given Regression Output

Problem 141. Calculate $SSR(X_1|X_2)$, $SSR(X_3|X_2)$, $SSR(X_1|X_2, X_3)$, $SSR(X_2, X_3|X_1)$, and $MSR(X_2, X_3|X_1)$. Also, calculate $R^2_{Y1|2}$, $R^2_{Y1|23}$, $R^2_{Y2|1}$.

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
x1	1	1319.41	1319.41	35.11	0.0004
Residuals	8	300.67	37.58		

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
x2	1	1.48	1.48	0.01	0.9340
Residuals	8	1618.60	202.32		

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
x3	1	48.42	48.42	0.25	0.6329
Residuals	8	1571.66	196.46		

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
x1	1	1319.41	1319.41	31.29	0.0008
x2	1	5.48	5.48	0.13	0.7292
Residuals	7	295.19	42.17		

	Df	Sum Sq	Mean Sq	F value	Pr(>F)

x1	1	1319.41	1319.41	46.89	0.0002
x3	1	103.71	103.71	3.69	0.0963
Residuals	7	196.96	28.14		

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
x2	1	1.48	1.48	0.01	0.9375
x3	1	51.55	51.55	0.23	0.6459
Residuals	7	1567.05	223.86		

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
x1	1	1319.41	1319.41	44.61	0.0005
x2	1	5.48	5.48	0.19	0.6819
x3	1	117.74	117.74	3.98	0.0930
Residuals	6	177.45	29.57		

Solution:
The standard errors for the betas are

$$
\begin{aligned}
SSR(X_1|X_2) &= SSE(X_2) - SSE(X_2, X_1) = 1323.4104 \\
SSR(X_2|X_1) &= SSE(X_1) - SSE(X_2, X_1) = 5.478 \\
SSR(X_3|X_2) &= SSE(X_2) - SSE(X_2, X_3) = 51.5511 \\
SSR(X_1|X_2, X_3) &= SSE(X_2, X_3) - SSE(X_1, X_2, X_3) = 1389.6003 \\
SSR(X_3|X_1, X_2) &= SSE(X_1, X_2) - SSE(X_1, X_2, X_3) = 117.7411 \\
MSR(X_2, X_3|X_1) &= (SSR(X_2|X_1) + SSR(X_3|X_1, X_2))/2 = 61.6096 \\
R^2_{Y1|2} &= \frac{SSR(X_1|X_2)}{SSE(X_2)} = 0.8176 \\
R^2_{Y2|1} &= \frac{SSR(X_2|X_1)}{SSE(X_1)} = 0.0182 \\
R^2_{Y1|23} &= \frac{SSR(X_1|X_2, X_3)}{SSE(X_2, X_3)} = 0.8868
\end{aligned}
$$

□

Problem 142. Calculate $SSR(X_1|X_2)$, $SSR(X_3|X_2)$, $SSR(X_1|X_2, X_3)$, $SSR(X_2, X_3|X_1)$, and $MSR(X_2, X_3|X_1)$. Also, calculate $R^2_{Y1|2}$, $R^2_{Y1|23}$, $R^2_{Y2|1}$.

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
x1	1	1672.01	1672.01	55.51	0.0001
Residuals	8	240.98	30.12		

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
x2	1	99.05	99.05	0.44	0.5272
Residuals	8	1813.95	226.74		

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
x3	1	6.42	6.42	0.03	0.8737
Residuals	8	1906.58	238.32		

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
x1	1	1672.01	1672.01	110.29	0.0000
x2	1	134.86	134.86	8.90	0.0204
Residuals	7	106.12	15.16		

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
x1	1	1672.01	1672.01	51.51	0.0002
x3	1	13.76	13.76	0.42	0.5357
Residuals	7	227.22	32.46		

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
x2	1	99.05	99.05	0.41	0.5447
x3	1	102.51	102.51	0.42	0.5380
Residuals	7	1711.44	244.49		

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
x1	1	1672.01	1672.01	108.30	0.0000
x2	1	134.86	134.86	8.74	0.0254
x3	1	13.49	13.49	0.87	0.3860
Residuals	6	92.63	15.44		

Solution:

The standard errors for the betas are

$$
\begin{aligned}
SSR(X_1|X_2) &= SSE(X_2) - SSE(X_2, X_1) = 1707.8294 \\
SSR(X_2|X_1) &= SSE(X_1) - SSE(X_2, X_1) = 134.8625 \\
SSR(X_3|X_2) &= SSE(X_2) - SSE(X_2, X_3) = 102.5073 \\
SSR(X_1|X_2, X_3) &= SSE(X_2, X_3) - SSE(X_1, X_2, X_3) = 1618.8108 \\
SSR(X_3|X_1, X_2) &= SSE(X_1, X_2) - SSE(X_1, X_2, X_3) = 13.4886 \\
MSR(X_2, X_3|X_1) &= (SSR(X_2|X_1) + SSR(X_3|X_1, X_2))/2 = 74.1755 \\
R^2_{Y1|2} &= \frac{SSR(X_1|X_2)}{SSE(X_2)} = 0.9415 \\
R^2_{Y2|1} &= \frac{SSR(X_2|X_1)}{SSE(X_1)} = 0.5596 \\
R^2_{Y1|23} &= \frac{SSR(X_1|X_2, X_3)}{SSE(X_2, X_3)} = 0.9459
\end{aligned}
$$

□

Problem 143. Calculate $SSR(X_1|X_2)$, $SSR(X_3|X_2)$, $SSR(X_1|X_2,X_3)$, $SSR(X_2,X_3|X_1)$, and $MSR(X_2,X_3|X_1)$. Also, calculate $R^2_{Y1|2}$, $R^2_{Y1|23}$, $R^2_{Y2|1}$.

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
x1	1	1.13	1.13	0.00	0.9511
Residuals	8	2255.14	281.89		

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
x2	1	1431.23	1431.23	13.88	0.0058
Residuals	8	825.04	103.13		

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
x3	1	1111.34	1111.34	7.77	0.0237
Residuals	8	1144.93	143.12		

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
x1	1	1.13	1.13	0.01	0.9110
x2	1	1667.39	1667.39	19.86	0.0029
Residuals	7	587.75	83.96		

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
x1	1	1.13	1.13	0.01	0.9326
x3	1	1228.40	1228.40	8.37	0.0232
Residuals	7	1026.74	146.68		

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
x2	1	1431.23	1431.23	24.42	0.0017
x3	1	414.79	414.79	7.08	0.0325
Residuals	7	410.25	58.61		

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
x1	1	1.13	1.13	0.15	0.7120
x2	1	1667.39	1667.39	221.74	0.0000
x3	1	542.63	542.63	72.16	0.0001
Residuals	6	45.12	7.52		

Solution:
The standard errors for the betas are

$$
\begin{aligned}
SSR(X_1|X_2) &= SSE(X_2) - SSE(X_2, X_1) = 237.2869 \\
SSR(X_2|X_1) &= SSE(X_1) - SSE(X_2, X_1) = 1667.3867 \\
SSR(X_3|X_2) &= SSE(X_2) - SSE(X_2, X_3) = 414.7863 \\
SSR(X_1|X_2, X_3) &= SSE(X_2, X_3) - SSE(X_1, X_2, X_3) = 365.135 \\
SSR(X_3|X_1, X_2) &= SSE(X_1, X_2) - SSE(X_1, X_2, X_3) = 542.6345 \\
MSR(X_2, X_3|X_1) &= (SSR(X_2|X_1) + SSR(X_3|X_1, X_2))/2 = 1105.0106 \\
R^2_{Y1|2} &= \frac{SSR(X_1|X_2)}{SSE(X_2)} = 0.2876 \\
R^2_{Y2|1} &= \frac{SSR(X_2|X_1)}{SSE(X_1)} = 0.7394 \\
R^2_{Y1|23} &= \frac{SSR(X_1|X_2, X_3)}{SSE(X_2, X_3)} = 0.89
\end{aligned}
$$

□

Problem 144. Calculate $SSR(X_1|X_2)$, $SSR(X_3|X_2)$, $SSR(X_1|X_2, X_3)$, $SSR(X_2, X_3|X_1)$, and $MSR(X_2, X_3|X_1)$. Also, calculate $R^2_{Y1|2}$, $R^2_{Y1|23}$, $R^2_{Y2|1}$.

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
x1	1	799.69	799.69	70.45	0.0000
Residuals	8	90.82	11.35		

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
x2	1	6.87	6.87	0.06	0.8093
Residuals	8	883.64	110.45		

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
x3	1	197.74	197.74	2.28	0.1692
Residuals	8	692.77	86.60		

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
x1	1	799.69	799.69	61.70	0.0001
x2	1	0.09	0.09	0.01	0.9371
Residuals	7	90.73	12.96		

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
x1	1	799.69	799.69	73.97	0.0001
x3	1	15.14	15.14	1.40	0.2752
Residuals	7	75.67	10.81		

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
x2	1	6.87	6.87	0.07	0.7942
x3	1	228.71	228.71	2.44	0.1619
Residuals	7	654.93	93.56		

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
x1	1	799.69	799.69	70.09	0.0002
x2	1	0.09	0.09	0.01	0.9333
x3	1	22.27	22.27	1.95	0.2119
Residuals	6	68.46	11.41		

Solution:
The standard errors for the betas are

$$
\begin{aligned}
SSR(X_1|X_2) &= SSE(X_2) - SSE(X_2, X_1) = 792.9106 \\
SSR(X_2|X_1) &= SSE(X_1) - SSE(X_2, X_1) = 0.0868 \\
SSR(X_3|X_2) &= SSE(X_2) - SSE(X_2, X_3) = 228.7103 \\
SSR(X_1|X_2, X_3) &= SSE(X_2, X_3) - SSE(X_1, X_2, X_3) = 586.4695 \\
SSR(X_3|X_1, X_2) &= SSE(X_1, X_2) - SSE(X_1, X_2, X_3) = 22.2692 \\
MSR(X_2, X_3|X_1) &= (SSR(X_2|X_1) + SSR(X_3|X_1, X_2))/2 = 11.178 \\
R^2_{Y1|2} &= \frac{SSR(X_1|X_2)}{SSE(X_2)} = 0.8973 \\
R^2_{Y2|1} &= \frac{SSR(X_2|X_1)}{SSE(X_1)} = 0.001 \\
R^2_{Y1|23} &= \frac{SSR(X_1|X_2, X_3)}{SSE(X_2, X_3)} = 0.8955
\end{aligned}
$$

□

Problem 145. Calculate $SSR(X_1|X_2)$, $SSR(X_3|X_2)$, $SSR(X_1|X_2, X_3)$, $SSR(X_2, X_3|X_1)$, and $MSR(X_2, X_3|X_1)$. Also, calculate $R^2_{Y1|2}$, $R^2_{Y1|23}$, $R^2_{Y2|1}$.

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
x1	1	0.53	0.53	0.02	0.8925
Residuals	8	218.23	27.28		

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
x2	1	22.29	22.29	0.91	0.3686
Residuals	8	196.47	24.56		

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
x3	1	82.34	82.34	4.83	0.0592
Residuals	8	136.43	17.05		

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
x1	1	0.53	0.53	0.02	0.8944
x2	1	21.99	21.99	0.78	0.4052
Residuals	7	196.24	28.03		

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
x1	1	0.53	0.53	0.03	0.8733
x3	1	82.36	82.36	4.24	0.0784
Residuals	7	135.87	19.41		

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
x2	1	22.29	22.29	1.15	0.3196
x3	1	60.49	60.49	3.11	0.1210
Residuals	7	135.98	19.43		

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
x1	1	0.53	0.53	0.02	0.8829
x2	1	21.99	21.99	0.98	0.3610
x3	1	61.26	61.26	2.72	0.1500
Residuals	6	134.98	22.50		

Solution:
The standard errors for the betas are

$$
\begin{aligned}
SSR(X_1|X_2) &= SSE(X_2) - SSE(X_2, X_1) = 0.2298 \\
SSR(X_2|X_1) &= SSE(X_1) - SSE(X_2, X_1) = 21.9889 \\
SSR(X_3|X_2) &= SSE(X_2) - SSE(X_2, X_3) = 60.4917 \\
SSR(X_1|X_2, X_3) &= SSE(X_2, X_3) - SSE(X_1, X_2, X_3) = 0.9982 \\
SSR(X_3|X_1, X_2) &= SSE(X_1, X_2) - SSE(X_1, X_2, X_3) = 61.2602 \\
MSR(X_2, X_3|X_1) &= (SSR(X_2|X_1) + SSR(X_3|X_1, X_2))/2 = 41.6246 \\
R^2_{Y1|2} &= \frac{SSR(X_1|X_2)}{SSE(X_2)} = 0.0012 \\
R^2_{Y2|1} &= \frac{SSR(X_2|X_1)}{SSE(X_1)} = 0.1008 \\
R^2_{Y1|23} &= \frac{SSR(X_1|X_2, X_3)}{SSE(X_2, X_3)} = 0.0073
\end{aligned}
$$

□

Problem 146. Calculate $SSR(X_1|X_2)$, $SSR(X_3|X_2)$, $SSR(X_1|X_2, X_3)$, $SSR(X_2, X_3|X_1)$, and $MSR(X_2, X_3|X_1)$. Also, calculate $R^2_{Y1|2}$, $R^2_{Y1|23}$, $R^2_{Y2|1}$.

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
x1	1	187.14	187.14	0.95	0.3581
Residuals	8	1574.94	196.87		

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
x2	1	1311.84	1311.84	23.31	0.0013
Residuals	8	450.25	56.28		

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
x3	1	151.86	151.86	0.75	0.4104
Residuals	8	1610.23	201.28		

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
x1	1	187.14	187.14	3.94	0.0874
x2	1	1242.83	1242.83	26.20	0.0014
Residuals	7	332.11	47.44		

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
x1	1	187.14	187.14	0.98	0.3545
x3	1	242.33	242.33	1.27	0.2964
Residuals	7	1332.62	190.37		

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
x2	1	1311.84	1311.84	40.82	0.0004
x3	1	225.29	225.29	7.01	0.0331
Residuals	7	224.96	32.14		

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
x1	1	187.14	187.14	53.55	0.0003
x2	1	1242.83	1242.83	355.61	0.0000
x3	1	311.14	311.14	89.03	0.0001
Residuals	6	20.97	3.49		

Solution:
The standard errors for the betas are

$$
\begin{aligned}
SSR(X_1|X_2) &= SSE(X_2) - SSE(X_2, X_1) = 118.1367 \\
SSR(X_2|X_1) &= SSE(X_1) - SSE(X_2, X_1) = 1242.8308 \\
SSR(X_3|X_2) &= SSE(X_2) - SSE(X_2, X_3) = 225.2862 \\
SSR(X_1|X_2, X_3) &= SSE(X_2, X_3) - SSE(X_1, X_2, X_3) = 203.9949 \\
SSR(X_3|X_1, X_2) &= SSE(X_1, X_2) - SSE(X_1, X_2, X_3) = 311.1443 \\
MSR(X_2, X_3|X_1) &= (SSR(X_2|X_1) + SSR(X_3|X_1, X_2))/2 = 776.9876 \\
R^2_{Y1|2} &= \frac{SSR(X_1|X_2)}{SSE(X_2)} = 0.2624 \\
R^2_{Y2|1} &= \frac{SSR(X_2|X_1)}{SSE(X_1)} = 0.7891 \\
R^2_{Y1|23} &= \frac{SSR(X_1|X_2, X_3)}{SSE(X_2, X_3)} = 0.9068
\end{aligned}
$$

□

Problem 147. Calculate $SSR(X_1|X_2)$, $SSR(X_3|X_2)$, $SSR(X_1|X_2, X_3)$, $SSR(X_2, X_3|X_1)$, and $MSR(X_2, X_3|X_1)$. Also, calculate $R^2_{Y1|2}$, $R^2_{Y1|23}$, $R^2_{Y2|1}$.

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
x1	1	575.63	575.63	23.56	0.0013
Residuals	8	195.45	24.43		

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
x2	1	0.00	0.00	0.00	0.9959
Residuals	8	771.08	96.39		

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
x3	1	40.23	40.23	0.44	0.5256
Residuals	8	730.86	91.36		

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
x1	1	575.63	575.63	21.06	0.0025
x2	1	4.13	4.13	0.15	0.7091
Residuals	7	191.32	27.33		

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
x1	1	575.63	575.63	43.73	0.0003
x3	1	103.30	103.30	7.85	0.0265
Residuals	7	92.15	13.16		

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
x2	1	0.00	0.00	0.00	0.9960
x3	1	47.56	47.56	0.46	0.5193
Residuals	7	723.52	103.36		

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
x1	1	575.63	575.63	39.43	0.0008
x2	1	4.13	4.13	0.28	0.6140
x3	1	103.74	103.74	7.11	0.0372
Residuals	6	87.59	14.60		

Solution:
The standard errors for the betas are

$$
\begin{aligned}
SSR(X_1|X_2) &= SSE(X_2) - SSE(X_2, X_1) = 579.757 \\
SSR(X_2|X_1) &= SSE(X_1) - SSE(X_2, X_1) = 4.1266 \\
SSR(X_3|X_2) &= SSE(X_2) - SSE(X_2, X_3) = 47.5608 \\
SSR(X_1|X_2, X_3) &= SSE(X_2, X_3) - SSE(X_1, X_2, X_3) = 635.9321 \\
SSR(X_3|X_1, X_2) &= SSE(X_1, X_2) - SSE(X_1, X_2, X_3) = 103.7359 \\
MSR(X_2, X_3|X_1) &= (SSR(X_2|X_1) + SSR(X_3|X_1, X_2))/2 = 53.9312 \\
R^2_{Y1|2} &= \frac{SSR(X_1|X_2)}{SSE(X_2)} = 0.7519 \\
R^2_{Y2|1} &= \frac{SSR(X_2|X_1)}{SSE(X_1)} = 0.0211 \\
R^2_{Y1|23} &= \frac{SSR(X_1|X_2, X_3)}{SSE(X_2, X_3)} = 0.8789
\end{aligned}
$$

□

Problem 148. Calculate $SSR(X_1|X_2)$, $SSR(X_3|X_2)$, $SSR(X_1|X_2,X_3)$, $SSR(X_2,X_3|X_1)$, and $MSR(X_2,X_3|X_1)$. Also, calculate $R^2_{Y1|2}$, $R^2_{Y1|23}$, $R^2_{Y2|1}$.

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
x1	1	406.63	406.63	2.58	0.1466
Residuals	8	1258.79	157.35		

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
x2	1	436.96	436.96	2.85	0.1301
Residuals	8	1228.46	153.56		

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
x3	1	895.50	895.50	9.30	0.0158
Residuals	8	769.92	96.24		

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
x1	1	406.63	406.63	3.22	0.1157
x2	1	375.25	375.25	2.97	0.1283
Residuals	7	883.54	126.22		

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
x1	1	406.63	406.63	3.87	0.0897
x3	1	524.23	524.23	5.00	0.0605
Residuals	7	734.57	104.94		

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
x2	1	436.96	436.96	38.21	0.0005
x3	1	1148.42	1148.42	100.43	0.0000
Residuals	7	80.04	11.43		

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
x1	1	406.63	406.63	30.83	0.0014
x2	1	375.25	375.25	28.45	0.0018
x3	1	804.40	804.40	60.99	0.0002
Residuals	6	79.14	13.19		

Solution:
The standard errors for the betas are

$$
\begin{aligned}
SSR(X_1|X_2) &= SSE(X_2) - SSE(X_2, X_1) = 344.9188 \\
SSR(X_2|X_1) &= SSE(X_1) - SSE(X_2, X_1) = 375.2511 \\
SSR(X_3|X_2) &= SSE(X_2) - SSE(X_2, X_3) = 1148.4193 \\
SSR(X_1|X_2, X_3) &= SSE(X_2, X_3) - SSE(X_1, X_2, X_3) = 0.9033 \\
SSR(X_3|X_1, X_2) &= SSE(X_1, X_2) - SSE(X_1, X_2, X_3) = 804.4039 \\
MSR(X_2, X_3|X_1) &= (SSR(X_2|X_1) + SSR(X_3|X_1, X_2))/2 = 589.8275 \\
R^2_{Y1|2} &= \frac{SSR(X_1|X_2)}{SSE(X_2)} = 0.2808 \\
R^2_{Y2|1} &= \frac{SSR(X_2|X_1)}{SSE(X_1)} = 0.2981 \\
R^2_{Y1|23} &= \frac{SSR(X_1|X_2, X_3)}{SSE(X_2, X_3)} = 0.0113
\end{aligned}
$$

□

Problem 149. Calculate $SSR(X_1|X_2)$, $SSR(X_3|X_2)$, $SSR(X_1|X_2, X_3)$, $SSR(X_2, X_3|X_1)$, and $MSR(X_2, X_3|X_1)$. Also, calculate $R^2_{Y1|2}$, $R^2_{Y1|23}$, $R^2_{Y2|1}$.

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
x1	1	2436.00	2436.00	152.54	0.0000
Residuals	8	127.76	15.97		

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
x2	1	149.50	149.50	0.50	0.5015
Residuals	8	2414.26	301.78		

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
x3	1	128.62	128.62	0.42	0.5339
Residuals	8	2435.14	304.39		

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
x1	1	2436.00	2436.00	147.73	0.0000
x2	1	12.33	12.33	0.75	0.4158
Residuals	7	115.43	16.49		

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
x1	1	2436.00	2436.00	145.31	0.0000
x3	1	10.41	10.41	0.62	0.4565
Residuals	7	117.35	16.76		

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
x2	1	149.50	149.50	0.44	0.5292
x3	1	25.18	25.18	0.07	0.7937
Residuals	7	2389.08	341.30		

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
x1	1	2436.00	2436.00	199.83	0.0000
x2	1	12.33	12.33	1.01	0.3534
x3	1	42.29	42.29	3.47	0.1118
Residuals	6	73.14	12.19		

Solution:
The standard errors for the betas are

$$\begin{aligned}
SSR(X_1|X_2) &= SSE(X_2) - SSE(X_2, X_1) = 2298.83 \\
SSR(X_2|X_1) &= SSE(X_1) - SSE(X_2, X_1) = 12.3302 \\
SSR(X_3|X_2) &= SSE(X_2) - SSE(X_2, X_3) = 25.1812 \\
SSR(X_1|X_2, X_3) &= SSE(X_2, X_3) - SSE(X_1, X_2, X_3) = 2315.9342 \\
SSR(X_3|X_1, X_2) &= SSE(X_1, X_2) - SSE(X_1, X_2, X_3) = 42.2854 \\
MSR(X_2, X_3|X_1) &= (SSR(X_2|X_1) + SSR(X_3|X_1, X_2))/2 = 27.3078 \\
R^2_{Y1|2} &= \frac{SSR(X_1|X_2)}{SSE(X_2)} = 0.9522 \\
R^2_{Y2|1} &= \frac{SSR(X_2|X_1)}{SSE(X_1)} = 0.0965 \\
R^2_{Y1|23} &= \frac{SSR(X_1|X_2, X_3)}{SSE(X_2, X_3)} = 0.9694
\end{aligned}$$

□

Problem 150. Calculate $SSR(X_1|X_2)$, $SSR(X_3|X_2)$, $SSR(X_1|X_2, X_3)$, $SSR(X_2, X_3|X_1)$, and $MSR(X_2, X_3|X_1)$. Also, calculate $R^2_{Y1|2}$, $R^2_{Y1|23}$, $R^2_{Y2|1}$.

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
x1	1	216.45	216.45	1.28	0.2911
Residuals	8	1355.12	169.39		

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
x2	1	1161.35	1161.35	22.65	0.0014
Residuals	8	410.21	51.28		

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
x3	1	236.97	236.97	1.42	0.2675
Residuals	8	1334.59	166.82		

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
x1	1	216.45	216.45	8.20	0.0242
x2	1	1170.35	1170.35	44.34	0.0003
Residuals	7	184.77	26.40		

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
x1	1	216.45	216.45	1.36	0.2824
x3	1	237.59	237.59	1.49	0.2620
Residuals	7	1117.53	159.65		

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
x2	1	1161.35	1161.35	20.43	0.0027
x3	1	12.35	12.35	0.22	0.6552
Residuals	7	397.86	56.84		

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
x1	1	216.45	216.45	7.55	0.0334
x2	1	1170.35	1170.35	40.83	0.0007
x3	1	12.79	12.79	0.45	0.5291
Residuals	6	171.98	28.66		

Solution:
The standard errors for the betas are

$$
\begin{aligned}
SSR(X_1|X_2) &= SSE(X_2) - SSE(X_2, X_1) = 225.4455 \\
SSR(X_2|X_1) &= SSE(X_1) - SSE(X_2, X_1) = 1170.3491 \\
SSR(X_3|X_2) &= SSE(X_2) - SSE(X_2, X_3) = 12.3526 \\
SSR(X_1|X_2, X_3) &= SSE(X_2, X_3) - SSE(X_1, X_2, X_3) = 225.8784 \\
SSR(X_3|X_1, X_2) &= SSE(X_1, X_2) - SSE(X_1, X_2, X_3) = 12.7854 \\
MSR(X_2, X_3|X_1) &= (SSR(X_2|X_1) + SSR(X_3|X_1, X_2))/2 = 591.5673 \\
R^2_{Y1|2} &= \frac{SSR(X_1|X_2)}{SSE(X_2)} = 0.5496 \\
R^2_{Y2|1} &= \frac{SSR(X_2|X_1)}{SSE(X_1)} = 0.8637 \\
R^2_{Y1|23} &= \frac{SSR(X_1|X_2, X_3)}{SSE(X_2, X_3)} = 0.5677
\end{aligned}
$$

□

3.8. Multiple Linear Regression - SSR(X's given other X's), etc. given Regression Output

Problem 151. Calculate $SSR(X_1|X_2)$, $SSR(X_3|X_2)$, $SSR(X_1|X_2, X_3)$, $SSR(X_2, X_3|X_1)$, and $MSR(X_2, X_3|X_1)$. Also, calculate $R^2_{Y1|2}$, $R^2_{Y1|23}$, $R^2_{Y2|1}$.

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
x1	1	3591.25	3591.25	23.65	0.0013
Residuals	8	1214.78	151.85		

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
x2	1	3665.95	3665.95	25.72	0.0010
Residuals	8	1140.08	142.51		

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
x3	1	169.09	169.09	0.29	0.6038
Residuals	8	4636.94	579.62		

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
x1	1	3591.25	3591.25	214.54	0.0000
x2	1	1097.60	1097.60	65.57	0.0001
Residuals	7	117.17	16.74		

	Df	Sum Sq	Mean Sq	F value	Pr(>F)

x1	1	3591.25	3591.25	23.87	0.0018
x3	1	161.49	161.49	1.07	0.3347
Residuals	7	1053.29	150.47		

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
x2	1	3665.95	3665.95	22.98	0.0020
x3	1	23.55	23.55	0.15	0.7122
Residuals	7	1116.53	159.50		

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
x1	1	3591.25	3591.25	358.12	0.0000
x2	1	1097.60	1097.60	109.45	0.0000
x3	1	57.01	57.01	5.68	0.0545
Residuals	6	60.17	10.03		

Solution:
The standard errors for the betas are

$$\begin{aligned}
SSR(X_1|X_2) &= SSE(X_2) - SSE(X_2, X_1) = 1022.907 \\
SSR(X_2|X_1) &= SSE(X_1) - SSE(X_2, X_1) = 1097.6048 \\
SSR(X_3|X_2) &= SSE(X_2) - SSE(X_2, X_3) = 23.552 \\
SSR(X_1|X_2, X_3) &= SSE(X_2, X_3) - SSE(X_1, X_2, X_3) = 1056.3602 \\
SSR(X_3|X_1, X_2) &= SSE(X_1, X_2) - SSE(X_1, X_2, X_3) = 57.0052 \\
MSR(X_2, X_3|X_1) &= (SSR(X_2|X_1) + SSR(X_3|X_1, X_2))/2 = 577.305 \\
R^2_{Y1|2} &= \frac{SSR(X_1|X_2)}{SSE(X_2)} = 0.8972 \\
R^2_{Y2|1} &= \frac{SSR(X_2|X_1)}{SSE(X_1)} = 0.9035 \\
R^2_{Y1|23} &= \frac{SSR(X_1|X_2, X_3)}{SSE(X_2, X_3)} = 0.9461
\end{aligned}$$

□

Problem 152. Calculate $SSR(X_1|X_2)$, $SSR(X_3|X_2)$, $SSR(X_1|X_2, X_3)$, $SSR(X_2, X_3|X_1)$, and $MSR(X_2, X_3|X_1)$. Also, calculate $R^2_{Y1|2}$, $R^2_{Y1|23}$, $R^2_{Y2|1}$.

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
x1	1	359.71	359.71	2.79	0.1336
Residuals	8	1032.51	129.06		

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
x2	1	1300.45	1300.45	113.37	0.0000
Residuals	8	91.77	11.47		

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
x3	1	126.89	126.89	0.80	0.3966
Residuals	8	1265.33	158.17		

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
x1	1	359.71	359.71	28.74	0.0011
x2	1	944.90	944.90	75.50	0.0001
Residuals	7	87.61	12.52		

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
x1	1	359.71	359.71	4.00	0.0857
x3	1	402.61	402.61	4.47	0.0722
Residuals	7	629.89	89.98		

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
x2	1	1300.45	1300.45	104.45	0.0000
x3	1	4.61	4.61	0.37	0.5619
Residuals	7	87.15	12.45		

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
x1	1	359.71	359.71	24.94	0.0025
x2	1	944.90	944.90	65.51	0.0002
x3	1	1.07	1.07	0.07	0.7947
Residuals	6	86.54	14.42		

Solution:
The standard errors for the betas are

$$
\begin{aligned}
SSR(X_1|X_2) &= SSE(X_2) - SSE(X_2, X_1) = 4.1612 \\
SSR(X_2|X_1) &= SSE(X_1) - SSE(X_2, X_1) = 944.8985 \\
SSR(X_3|X_2) &= SSE(X_2) - SSE(X_2, X_3) = 4.6132 \\
SSR(X_1|X_2, X_3) &= SSE(X_2, X_3) - SSE(X_1, X_2, X_3) = 0.6158 \\
SSR(X_3|X_1, X_2) &= SSE(X_1, X_2) - SSE(X_1, X_2, X_3) = 1.0678 \\
MSR(X_2, X_3|X_1) &= (SSR(X_2|X_1) + SSR(X_3|X_1, X_2))/2 = 472.9831 \\
R^2_{Y1|2} &= \frac{SSR(X_1|X_2)}{SSE(X_2)} = 0.0453 \\
R^2_{Y2|1} &= \frac{SSR(X_2|X_1)}{SSE(X_1)} = 0.9152 \\
R^2_{Y1|23} &= \frac{SSR(X_1|X_2, X_3)}{SSE(X_2, X_3)} = 0.0071
\end{aligned}
$$

□

Problem 153. Calculate $SSR(X_1|X_2)$, $SSR(X_3|X_2)$, $SSR(X_1|X_2, X_3)$, $SSR(X_2, X_3|X_1)$, and $MSR(X_2, X_3|X_1)$. Also, calculate $R^2_{Y1|2}$, $R^2_{Y1|23}$, $R^2_{Y2|1}$.

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
x1	1	851.03	851.03	3.49	0.0987
Residuals	8	1950.76	243.84		

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
x2	1	784.22	784.22	3.11	0.1158
Residuals	8	2017.57	252.20		

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
x3	1	1915.71	1915.71	17.30	0.0032
Residuals	8	886.08	110.76		

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
x1	1	851.03	851.03	4.00	0.0858
x2	1	459.63	459.63	2.16	0.1853
Residuals	7	1491.13	213.02		

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
x1	1	851.03	851.03	38.06	0.0005
x3	1	1794.25	1794.25	80.25	0.0000
Residuals	7	156.50	22.36		

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
x2	1	784.22	784.22	7.12	0.0321
x3	1	1246.19	1246.19	11.31	0.0120
Residuals	7	771.38	110.20		

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
x1	1	851.03	851.03	36.51	0.0009
x2	1	459.63	459.63	19.72	0.0044
x3	1	1351.29	1351.29	57.98	0.0003
Residuals	6	139.84	23.31		

Solution:
The standard errors for the betas are

$$
\begin{aligned}
SSR(X_1|X_2) &= SSE(X_2) - SSE(X_2, X_1) = 526.4393 \\
SSR(X_2|X_1) &= SSE(X_1) - SSE(X_2, X_1) = 459.6284 \\
SSR(X_3|X_2) &= SSE(X_2) - SSE(X_2, X_3) = 1246.1875 \\
SSR(X_1|X_2, X_3) &= SSE(X_2, X_3) - SSE(X_1, X_2, X_3) = 631.5378 \\
SSR(X_3|X_1, X_2) &= SSE(X_1, X_2) - SSE(X_1, X_2, X_3) = 1351.2859 \\
MSR(X_2, X_3|X_1) &= (SSR(X_2|X_1) + SSR(X_3|X_1, X_2))/2 = 905.4572 \\
R^2_{Y1|2} &= \frac{SSR(X_1|X_2)}{SSE(X_2)} = 0.2609 \\
R^2_{Y2|1} &= \frac{SSR(X_2|X_1)}{SSE(X_1)} = 0.2356 \\
R^2_{Y1|23} &= \frac{SSR(X_1|X_2, X_3)}{SSE(X_2, X_3)} = 0.8187
\end{aligned}
$$

□

Problem 154. Calculate $SSR(X_1|X_2)$, $SSR(X_3|X_2)$, $SSR(X_1|X_2, X_3)$, $SSR(X_2, X_3|X_1)$, and $MSR(X_2, X_3|X_1)$. Also, calculate $R^2_{Y1|2}$, $R^2_{Y1|23}$, $R^2_{Y2|1}$.

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
x1	1	92.79	92.79	0.26	0.6267
Residuals	8	2903.05	362.88		

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
x2	1	1547.09	1547.09	8.54	0.0192
Residuals	8	1448.75	181.09		

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
x3	1	864.18	864.18	3.24	0.1094
Residuals	8	2131.66	266.46		

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
x1	1	92.79	92.79	0.54	0.4857
x2	1	1703.93	1703.93	9.95	0.0161
Residuals	7	1199.12	171.30		

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
x1	1	92.79	92.79	0.31	0.5941
x3	1	818.05	818.05	2.75	0.1414
Residuals	7	2085.01	297.86		

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
x2	1	1547.09	1547.09	47.67	0.0002
x3	1	1221.58	1221.58	37.64	0.0005
Residuals	7	227.18	32.45		

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
x1	1	92.79	92.79	10.13	0.0190
x2	1	1703.93	1703.93	186.03	0.0000
x3	1	1144.17	1144.17	124.92	0.0000
Residuals	6	54.96	9.16		

Solution:
The standard errors for the betas are

$$
\begin{aligned}
SSR(X_1|X_2) &= SSE(X_2) - SSE(X_2, X_1) = 249.6304 \\
SSR(X_2|X_1) &= SSE(X_1) - SSE(X_2, X_1) = 1703.9318 \\
SSR(X_3|X_2) &= SSE(X_2) - SSE(X_2, X_3) = 1221.5784 \\
SSR(X_1|X_2, X_3) &= SSE(X_2, X_3) - SSE(X_1, X_2, X_3) = 172.2193 \\
SSR(X_3|X_1, X_2) &= SSE(X_1, X_2) - SSE(X_1, X_2, X_3) = 1144.1673 \\
MSR(X_2, X_3|X_1) &= (SSR(X_2|X_1) + SSR(X_3|X_1, X_2))/2 = 1424.0495 \\
R^2_{Y1|2} &= \frac{SSR(X_1|X_2)}{SSE(X_2)} = 0.1723 \\
R^2_{Y2|1} &= \frac{SSR(X_2|X_1)}{SSE(X_1)} = 0.5869 \\
R^2_{Y1|23} &= \frac{SSR(X_1|X_2, X_3)}{SSE(X_2, X_3)} = 0.7581
\end{aligned}
$$

□

Problem 155. Calculate $SSR(X_1|X_2)$, $SSR(X_3|X_2)$, $SSR(X_1|X_2, X_3)$, $SSR(X_2, X_3|X_1)$, and $MSR(X_2, X_3|X_1)$. Also, calculate $R^2_{Y1|2}$, $R^2_{Y1|23}$, $R^2_{Y2|1}$.

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
x1	1	684.94	684.94	2.19	0.1774
Residuals	8	2505.58	313.20		

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
x2	1	2260.99	2260.99	19.46	0.0023
Residuals	8	929.53	116.19		

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
x3	1	1828.31	1828.31	10.74	0.0112
Residuals	8	1362.21	170.28		

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
x1	1	684.94	684.94	7.96	0.0257
x2	1	1903.09	1903.09	22.11	0.0022
Residuals	7	602.49	86.07		

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
x1	1	684.94	684.94	3.54	0.1020
x3	1	1150.21	1150.21	5.94	0.0449
Residuals	7	1355.36	193.62		

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
x2	1	2260.99	2260.99	427.38	0.0000
x3	1	892.49	892.49	168.70	0.0000
Residuals	7	37.03	5.29		

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
x1	1	684.94	684.94	128.79	0.0000
x2	1	1903.09	1903.09	357.85	0.0000
x3	1	570.58	570.58	107.29	0.0000
Residuals	6	31.91	5.32		

Solution:
The standard errors for the betas are

$$
\begin{aligned}
SSR(X_1|X_2) &= SSE(X_2) - SSE(X_2, X_1) = 327.0361 \\
SSR(X_2|X_1) &= SSE(X_1) - SSE(X_2, X_1) = 1903.0856 \\
SSR(X_3|X_2) &= SSE(X_2) - SSE(X_2, X_3) = 892.4941 \\
SSR(X_1|X_2, X_3) &= SSE(X_2, X_3) - SSE(X_1, X_2, X_3) = 5.1237 \\
SSR(X_3|X_1, X_2) &= SSE(X_1, X_2) - SSE(X_1, X_2, X_3) = 570.5818 \\
MSR(X_2, X_3|X_1) &= (SSR(X_2|X_1) + SSR(X_3|X_1, X_2))/2 = 1236.8337 \\
R^2_{Y1|2} &= \frac{SSR(X_1|X_2)}{SSE(X_2)} = 0.3518 \\
R^2_{Y2|1} &= \frac{SSR(X_2|X_1)}{SSE(X_1)} = 0.7595 \\
R^2_{Y1|23} &= \frac{SSR(X_1|X_2, X_3)}{SSE(X_2, X_3)} = 0.1384
\end{aligned}
$$

□

Problem 156. Calculate $SSR(X_1|X_2)$, $SSR(X_3|X_2)$, $SSR(X_1|X_2, X_3)$, $SSR(X_2, X_3|X_1)$, and $MSR(X_2, X_3|X_1)$. Also, calculate $R^2_{Y1|2}$, $R^2_{Y1|23}$, $R^2_{Y2|1}$.

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
x1	1	2044.46	2044.46	35.82	0.0003
Residuals	8	456.67	57.08		

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
x2	1	277.34	277.34	1.00	0.3471
Residuals	8	2223.79	277.97		

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
x3	1	571.65	571.65	2.37	0.1622
Residuals	8	1929.48	241.18		

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
x1	1	2044.46	2044.46	38.28	0.0005
x2	1	82.83	82.83	1.55	0.2531
Residuals	7	373.84	53.41		

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
x1	1	2044.46	2044.46	31.95	0.0008
x3	1	8.69	8.69	0.14	0.7233
Residuals	7	447.98	64.00		

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
x2	1	277.34	277.34	2.93	0.1308
x3	1	1560.55	1560.55	16.47	0.0048
Residuals	7	663.24	94.75		

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
x1	1	2044.46	2044.46	66.77	0.0002
x2	1	82.83	82.83	2.71	0.1511
x3	1	190.13	190.13	6.21	0.0470
Residuals	6	183.71	30.62		

Solution:
The standard errors for the betas are

$$
\begin{aligned}
SSR(X_1|X_2) &= SSE(X_2) - SSE(X_2, X_1) = 1849.9491 \\
SSR(X_2|X_1) &= SSE(X_1) - SSE(X_2, X_1) = 82.8294 \\
SSR(X_3|X_2) &= SSE(X_2) - SSE(X_2, X_3) = 1560.5488 \\
SSR(X_1|X_2, X_3) &= SSE(X_2, X_3) - SSE(X_1, X_2, X_3) = 479.5301 \\
SSR(X_3|X_1, X_2) &= SSE(X_1, X_2) - SSE(X_1, X_2, X_3) = 190.1298 \\
MSR(X_2, X_3|X_1) &= (SSR(X_2|X_1) + SSR(X_3|X_1, X_2))/2 = 136.4796 \\
R^2_{Y1|2} &= \frac{SSR(X_1|X_2)}{SSE(X_2)} = 0.8319 \\
R^2_{Y2|1} &= \frac{SSR(X_2|X_1)}{SSE(X_1)} = 0.1814 \\
R^2_{Y1|23} &= \frac{SSR(X_1|X_2, X_3)}{SSE(X_2, X_3)} = 0.723
\end{aligned}
$$

□

Problem 157. Calculate $SSR(X_1|X_2)$, $SSR(X_3|X_2)$, $SSR(X_1|X_2, X_3)$, $SSR(X_2, X_3|X_1)$, and $MSR(X_2, X_3|X_1)$. Also, calculate $R^2_{Y1|2}$, $R^2_{Y1|23}$, $R^2_{Y2|1}$.

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
x1	1	524.65	524.65	4.76	0.0608
Residuals	8	882.63	110.33		

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
x2	1	46.58	46.58	0.27	0.6150
Residuals	8	1360.70	170.09		

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
x3	1	984.14	984.14	18.61	0.0026
Residuals	8	423.14	52.89		

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
x1	1	524.65	524.65	4.26	0.0779
x2	1	20.40	20.40	0.17	0.6962
Residuals	7	862.23	123.18		

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
x1	1	524.65	524.65	32.26	0.0008
x3	1	768.77	768.77	47.27	0.0002
Residuals	7	113.85	16.26		

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
x2	1	46.58	46.58	0.79	0.4043
x3	1	946.66	946.66	16.00	0.0052
Residuals	7	414.04	59.15		

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
x1	1	524.65	524.65	38.87	0.0008
x2	1	20.40	20.40	1.51	0.2650
x3	1	781.25	781.25	57.88	0.0003
Residuals	6	80.98	13.50		

Solution:
The standard errors for the betas are

$$
\begin{aligned}
SSR(X_1|X_2) &= SSE(X_2) - SSE(X_2, X_1) = 498.471 \\
SSR(X_2|X_1) &= SSE(X_1) - SSE(X_2, X_1) = 20.3977 \\
SSR(X_3|X_2) &= SSE(X_2) - SSE(X_2, X_3) = 946.6569 \\
SSR(X_1|X_2, X_3) &= SSE(X_2, X_3) - SSE(X_1, X_2, X_3) = 333.0599 \\
SSR(X_3|X_1, X_2) &= SSE(X_1, X_2) - SSE(X_1, X_2, X_3) = 781.2457 \\
MSR(X_2, X_3|X_1) &= (SSR(X_2|X_1) + SSR(X_3|X_1, X_2))/2 = 400.8217 \\
R^2_{Y1|2} &= \frac{SSR(X_1|X_2)}{SSE(X_2)} = 0.3663 \\
R^2_{Y2|1} &= \frac{SSR(X_2|X_1)}{SSE(X_1)} = 0.0231 \\
R^2_{Y1|23} &= \frac{SSR(X_1|X_2, X_3)}{SSE(X_2, X_3)} = 0.8044
\end{aligned}
$$

□

Problem 158. Calculate $SSR(X_1|X_2)$, $SSR(X_3|X_2)$, $SSR(X_1|X_2, X_3)$, $SSR(X_2, X_3|X_1)$, and $MSR(X_2, X_3|X_1)$. Also, calculate $R^2_{Y1|2}$, $R^2_{Y1|23}$, $R^2_{Y2|1}$.

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
x1	1	1738.73	1738.73	27.10	0.0008
Residuals	8	513.22	64.15		

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
x2	1	378.54	378.54	1.62	0.2393
Residuals	8	1873.41	234.18		

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
x3	1	522.18	522.18	2.42	0.1588
Residuals	8	1729.77	216.22		

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
x1	1	1738.73	1738.73	27.00	0.0013
x2	1	62.41	62.41	0.97	0.3577
Residuals	7	450.82	64.40		

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
x1	1	1738.73	1738.73	166.81	0.0000
x3	1	440.26	440.26	42.24	0.0003
Residuals	7	72.97	10.42		

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
x2	1	378.54	378.54	1.77	0.2248
x3	1	378.63	378.63	1.77	0.2247
Residuals	7	1494.78	213.54		

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
x1	1	1738.73	1738.73	183.67	0.0000
x2	1	62.41	62.41	6.59	0.0425
x3	1	394.01	394.01	41.62	0.0007
Residuals	6	56.80	9.47		

Solution:

The standard errors for the betas are

$$
\begin{aligned}
SSR(X_1|X_2) &= SSE(X_2) - SSE(X_2, X_1) = 1422.5965 \\
SSR(X_2|X_1) &= SSE(X_1) - SSE(X_2, X_1) = 62.4084 \\
SSR(X_3|X_2) &= SSE(X_2) - SSE(X_2, X_3) = 378.6282 \\
SSR(X_1|X_2, X_3) &= SSE(X_2, X_3) - SSE(X_1, X_2, X_3) = 1437.9827 \\
SSR(X_3|X_1, X_2) &= SSE(X_1, X_2) - SSE(X_1, X_2, X_3) = 394.0144 \\
MSR(X_2, X_3|X_1) &= (SSR(X_2|X_1) + SSR(X_3|X_1, X_2))/2 = 228.2114 \\
R^2_{Y1|2} &= \frac{SSR(X_1|X_2)}{SSE(X_2)} = 0.7594 \\
R^2_{Y2|1} &= \frac{SSR(X_2|X_1)}{SSE(X_1)} = 0.1216 \\
R^2_{Y1|23} &= \frac{SSR(X_1|X_2, X_3)}{SSE(X_2, X_3)} = 0.962
\end{aligned}
$$

□

Problem 159. Calculate $SSR(X_1|X_2)$, $SSR(X_3|X_2)$, $SSR(X_1|X_2, X_3)$, $SSR(X_2, X_3|X_1)$, and $MSR(X_2, X_3|X_1)$. Also, calculate $R^2_{Y1|2}$, $R^2_{Y1|23}$, $R^2_{Y2|1}$.

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
x1	1	898.85	898.85	2.89	0.1275
Residuals	8	2486.93	310.87		

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
x2	1	2046.17	2046.17	12.22	0.0081
Residuals	8	1339.61	167.45		

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
x3	1	2309.50	2309.50	17.17	0.0032
Residuals	8	1076.28	134.54		

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
x1	1	898.85	898.85	22.72	0.0020
x2	1	2210.02	2210.02	55.87	0.0001
Residuals	7	276.91	39.56		

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
x1	1	898.85	898.85	8.89	0.0205
x3	1	1778.85	1778.85	17.59	0.0041
Residuals	7	708.08	101.15		

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
x2	1	2046.17	2046.17	18.59	0.0035
x3	1	569.13	569.13	5.17	0.0571
Residuals	7	770.48	110.07		

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
x1	1	898.85	898.85	46.49	0.0005
x2	1	2210.02	2210.02	114.32	0.0000
x3	1	160.92	160.92	8.32	0.0279
Residuals	6	115.99	19.33		

Solution:
The standard errors for the betas are

$$
\begin{aligned}
SSR(X_1|X_2) &= SSE(X_2) - SSE(X_2, X_1) = 1062.6917 \\
SSR(X_2|X_1) &= SSE(X_1) - SSE(X_2, X_1) = 2210.016 \\
SSR(X_3|X_2) &= SSE(X_2) - SSE(X_2, X_3) = 569.1303 \\
SSR(X_1|X_2, X_3) &= SSE(X_2, X_3) - SSE(X_1, X_2, X_3) = 654.4816 \\
SSR(X_3|X_1, X_2) &= SSE(X_1, X_2) - SSE(X_1, X_2, X_3) = 160.9201 \\
MSR(X_2, X_3|X_1) &= (SSR(X_2|X_1) + SSR(X_3|X_1, X_2))/2 = 1185.468 \\
R^2_{Y1|2} &= \frac{SSR(X_1|X_2)}{SSE(X_2)} = 0.7933 \\
R^2_{Y2|1} &= \frac{SSR(X_2|X_1)}{SSE(X_1)} = 0.8887 \\
R^2_{Y1|23} &= \frac{SSR(X_1|X_2, X_3)}{SSE(X_2, X_3)} = 0.8495
\end{aligned}
$$

□

Problem 160. Calculate $SSR(X_1|X_2)$, $SSR(X_3|X_2)$, $SSR(X_1|X_2, X_3)$, $SSR(X_2, X_3|X_1)$, and $MSR(X_2, X_3|X_1)$. Also, calculate $R^2_{Y1|2}$, $R^2_{Y1|23}$, $R^2_{Y2|1}$.

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
x1	1	3267.99	3267.99	34.01	0.0004
Residuals	8	768.79	96.10		

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
x2	1	2254.75	2254.75	10.12	0.0130
Residuals	8	1782.03	222.75		

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
x3	1	1905.40	1905.40	7.15	0.0282
Residuals	8	2131.38	266.42		

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
x1	1	3267.99	3267.99	43.89	0.0003
x2	1	247.57	247.57	3.32	0.1110
Residuals	7	521.22	74.46		

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
x1	1	3267.99	3267.99	50.43	0.0002
x3	1	315.18	315.18	4.86	0.0632
Residuals	7	453.61	64.80		

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
x2	1	2254.75	2254.75	29.51	0.0010
x3	1	1247.24	1247.24	16.33	0.0049
Residuals	7	534.79	76.40		

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
x1	1	3267.99	3267.99	206.09	0.0000
x2	1	247.57	247.57	15.61	0.0075
x3	1	426.07	426.07	26.87	0.0020
Residuals	6	95.14	15.86		

Solution:
The standard errors for the betas are

$$
\begin{aligned}
SSR(X_1|X_2) &= SSE(X_2) - SSE(X_2, X_1) = 1260.8135 \\
SSR(X_2|X_1) &= SSE(X_1) - SSE(X_2, X_1) = 247.5733 \\
SSR(X_3|X_2) &= SSE(X_2) - SSE(X_2, X_3) = 1247.2401 \\
SSR(X_1|X_2, X_3) &= SSE(X_2, X_3) - SSE(X_1, X_2, X_3) = 439.6461 \\
SSR(X_3|X_1, X_2) &= SSE(X_1, X_2) - SSE(X_1, X_2, X_3) = 426.0727 \\
MSR(X_2, X_3|X_1) &= (SSR(X_2|X_1) + SSR(X_3|X_1, X_2))/2 = 336.823 \\
R^2_{Y1|2} &= \frac{SSR(X_1|X_2)}{SSE(X_2)} = 0.7075 \\
R^2_{Y2|1} &= \frac{SSR(X_2|X_1)}{SSE(X_1)} = 0.322 \\
R^2_{Y1|23} &= \frac{SSR(X_1|X_2, X_3)}{SSE(X_2, X_3)} = 0.8221
\end{aligned}
$$

□

4. The Tables

The Cumulative Standardized Normal Distribution Negative

The Cumulative Standardized Normal Distribution Positive

Critical Values of t-distribution

For a given d.f., entry represents the critical value of t *corresponding to a specified upper-tail area* (α)

$d.f./\alpha$	**0.25**	**0.10**	**0.05**	**0.025**	**0.01**	**0.005**
1.00	1.00	3.08	6.31	12.71	31.82	63.66
2.00	0.82	1.89	2.92	4.30	6.96	9.92
3.00	0.76	1.64	2.35	3.18	4.54	5.84
4.00	0.74	1.53	2.13	2.78	3.75	4.60
5.00	0.73	1.48	2.02	2.57	3.36	4.03
6.00	0.72	1.44	1.94	2.45	3.14	3.71
7.00	0.71	1.41	1.89	2.36	3.00	3.50
8.00	0.71	1.40	1.86	2.31	2.90	3.36
9.00	0.70	1.38	1.83	2.26	2.82	3.25
10.00	0.70	1.37	1.81	2.23	2.76	3.17
11.00	0.70	1.36	1.80	2.20	2.72	3.11
12.00	0.70	1.36	1.78	2.18	2.68	3.05
13.00	0.69	1.35	1.77	2.16	2.65	3.01
14.00	0.69	1.35	1.76	2.14	2.62	2.98
15.00	0.69	1.34	1.75	2.13	2.60	2.95
16.00	0.69	1.34	1.75	2.12	2.58	2.92
17.00	0.69	1.33	1.74	2.11	2.57	2.90
18.00	0.69	1.33	1.73	2.10	2.55	2.88
19.00	0.69	1.33	1.73	2.09	2.54	2.86
20.00	0.69	1.33	1.72	2.09	2.53	2.85
21.00	0.69	1.32	1.72	2.08	2.52	2.83
22.00	0.69	1.32	1.72	2.07	2.51	2.82
23.00	0.69	1.32	1.71	2.07	2.50	2.81

24.00	0.68	1.32	1.71	2.06	2.49	2.80
25.00	0.68	1.32	1.71	2.06	2.49	2.79
26.00	0.68	1.31	1.71	2.06	2.48	2.78
27.00	0.68	1.31	1.70	2.05	2.47	2.77
28.00	0.68	1.31	1.70	2.05	2.47	2.76
29.00	0.68	1.31	1.70	2.05	2.46	2.76
30.00	0.68	1.31	1.70	2.04	2.46	2.75
∞	0.67	1.28	1.64	1.96	2.33	2.58

Critical Values of χ^2 distribution

For a specified degrees of freedom $(d.f.)$, the critical value of χ^2 corresponding to a specified upper-tail area (α)

	UPPER-TAIL AREAS							
$d.f./\alpha$	**0.990**	**0.975**	**0.950**	**0.9000**	**0.100**	**0.050**	**0.025**	**0.010**
1	0.000	0.001	0.004	0.016	2.706	3.841	5.024	6.635
2	0.020	0.051	0.103	0.211	4.605	5.991	7.378	9.210
3	0.115	0.216	0.352	0.584	6.251	7.815	9.348	11.345
4	0.297	0.484	0.711	1.064	7.779	9.488	11.143	13.277
5	0.554	0.831	1.145	1.610	9.236	11.070	12.833	15.086
6	0.872	1.237	1.635	2.204	10.645	12.592	14.449	16.812
7	1.239	1.690	2.167	2.833	12.017	14.067	16.013	18.475
8	1.646	2.180	2.733	3.490	13.362	15.507	17.535	20.090
9	2.088	2.700	3.325	4.168	14.684	16.919	19.023	21.666
10	2.558	3.247	3.940	4.865	15.987	18.307	20.483	23.209
11	3.053	3.816	4.575	5.578	17.275	19.675	21.920	24.725
12	3.571	4.404	5.226	6.304	18.549	21.026	23.337	26.217
13	4.107	5.009	5.892	7.042	19.812	22.362	24.736	27.688
14	4.660	5.629	6.571	7.790	21.064	23.685	26.119	29.141
15	5.229	6.262	7.261	8.547	22.307	24.996	27.488	30.578
16	5.812	6.908	7.962	9.312	23.542	26.296	28.845	32.000

Critical Values of F-distribution

Upper Critical Values of Spearman's Rank Correlation Coefficient R_s

Note: In the table below, the critical values give significance levels as close as possible to but not exceeding the nominal α.

	Nominal α					
n	0.10	0.05	0.025	0.01	0.005	0.001
4	1.000	1.000	-	-	-	-
5	0.800	0.900	1.000	1.000	-	-
6	0.657	0.829	0.886	0.943	1.000	-
7	0.571	0.714	0.786	0.893	0.929	1.000
8	0.524	0.643	0.738	0.833	0.881	0.952
9	0.483	0.600	0.700	0.783	0.833	0.917
10	0.455	0.564	0.648	0.745	0.794	0.879
11	0.427	0.536	0.618	0.709	0.755	0.845
12	0.406	0.503	0.587	0.678	0.727	0.818
13	0.385	0.484	0.560	0.648	0.703	0.791
14	0.367	0.464	0.538	0.626	0.679	0.771
15	0.354	0.446	0.521	0.604	0.654	0.750
16	0.341	0.429	0.503	0.582	0.635	0.729
17	0.328	0.414	0.488	0.566	0.618	0.711
18	0.317	0.401	0.472	0.550	0.600	0.692
19	0.309	0.391	0.460	0.535	0.584	0.675
20	0.299	0.380	0.447	0.522	0.570	0.662
21	0.292	0.370	0.436	0.509	0.556	0.647
22	0.284	0.361	0.425	0.497	0.544	0.633
23	0.278	0.353	0.416	0.486	0.532	0.621
24	0.271	0.344	0.407	0.476	0.521	0.609
25	0.265	0.337	0.398	0.466	0.511	0.597
26	0.259	0.331	0.390	0.457	0.501	0.586
27	0.255	0.324	0.383	0.449	0.492	0.576
28	0.250	0.318	0.375	0.441	0.483	0.567
29	0.245	0.312	0.368	0.433	0.475	0.558

	0	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0	0.5000	0.4960	0.4920	0.4880	0.4840	0.4801	0.4761	0.4721	0.4681	0.4641
-0.1	0.4602	0.4562	0.4522	0.4483	0.4443	0.4404	0.4364	0.4325	0.4286	0.4247
-0.2	0.4207	0.4168	0.4129	0.4090	0.4052	0.4013	0.3974	0.3936	0.3897	0.3859
-0.3	0.3821	0.3783	0.3745	0.3707	0.3669	0.3632	0.3594	0.3557	0.3520	0.3483
-0.4	0.3446	0.3409	0.3372	0.3336	0.3300	0.3264	0.3228	0.3192	0.3156	0.3121
-0.5	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2810	0.2776
-0.6	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2483	0.2451
-0.7	0.2420	0.2389	0.2358	0.2327	0.2296	0.2266	0.2236	0.2206	0.2177	0.2148
-0.8	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867
-0.9	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635	0.1611
-1	0.1587	0.1562	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379
-1.1	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170
-1.2	0.1151	0.1131	0.1112	0.1093	0.1075	0.1056	0.1038	0.1020	0.1003	0.0985
-1.3	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823
-1.4	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0721	0.0708	0.0694	0.0681
-1.5	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0571	0.0559
-1.6	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
-1.7	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367
-1.8	0.0359	0.0351	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294
-1.9	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233
-2	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183
-2.1	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143
-2.2	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122	0.0119	0.0116	0.0113	0.0110
-2.3	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084
-2.4	0.0082	0.0080	0.0078	0.0075	0.0073	0.0071	0.0069	0.0068	0.0066	0.0064
-2.5	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048
-2.6	0.0047	0.0045	0.0044	0.0043	0.0041	0.0040	0.0039	0.0038	0.0037	0.0036
-2.7	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026
-2.8	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020	0.0019
-2.9	0.0019	0.0018	0.0018	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014
-3	0.0013	0.0013	0.0013	0.0012	0.0012	0.0011	0.0011	0.0011	0.0010	0.0010

	0	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990

	1	2	3	4	5	6	7	8	9	10
1	647.79	799.50	864.16	899.58	921.85	937.11	948.22	956.66	963.28	968.63
2	38.51	39.00	39.17	39.25	39.30	39.33	39.36	39.37	39.39	39.40
3	17.44	16.04	15.44	15.10	14.88	14.73	14.62	14.54	14.47	14.42
4	12.22	10.65	9.98	9.60	9.36	9.20	9.07	8.98	8.90	8.84
5	10.01	8.43	7.76	7.39	7.15	6.98	6.85	6.76	6.68	6.62
6	8.81	7.26	6.60	6.23	5.99	5.82	5.70	5.60	5.52	5.46
7	8.07	6.54	5.89	5.52	5.29	5.12	4.99	4.90	4.82	4.76
8	7.57	6.06	5.42	5.05	4.82	4.65	4.53	4.43	4.36	4.30
9	7.21	5.71	5.08	4.72	4.48	4.32	4.20	4.10	4.03	3.96
10	6.94	5.46	4.83	4.47	4.24	4.07	3.95	3.85	3.78	3.72

Table 853: *For a given numerator d.f. (across) and denominator d.f. (down), entry represents the F-values for the* $P(F^* \leq F) = 0.975$. Note: $F(A; df_1, df_2) = \frac{1}{F((1-A); df_2, df_1)}$ and $P[F(df_1, df_2) \leq F(A; df_1, df_2)] = A$ where $A = .975$ for the given table.

	1	2	3	4	5	6	7	8	9	10
1	161.45	199.50	215.71	224.58	230.16	233.99	236.77	238.88	240.54	241.88
2	18.51	19.00	19.16	19.25	19.30	19.33	19.35	19.37	19.38	19.40
3	10.13	9.55	9.28	9.12	9.01	8.94	8.89	8.85	8.81	8.79
4	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6.00	5.96
5	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.77	4.74
6	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.10	4.06
7	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68	3.64
8	5.32	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.39	3.35
9	5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.18	3.14
10	4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02	2.98

Table 854: *For a given numerator d.f. (across) and denominator d.f. (down), entry represents the F-values for the* $P(F^* \leq F) = 0.95$. Note: $F(A; df_1, df_2) = \frac{1}{F((1-A); df_2, df_1)}$ and $P[F(df_1, df_2) \leq F(A; df_1, df_2)] = A$ where $A = .95$ for the given table.